50 Full-Length PSAT Math Practice Tests

Digital PSAT MATH Complete Prep

+ The Ultimate Step by Step Guide to Preparing for the PSAT Math Test

100 Skill Lessons

Step-by-Step Explanations

2027

Timed Online Tests

Instant Scoring





PREVIEW

DIGITAL PSAT MATH COMPLETE PREP

50 Full-Length PSAT Math Practice Tests

All 50 Interactive Online
10 Also Included in Print

For PSAT/NMSQT and PSAT 10

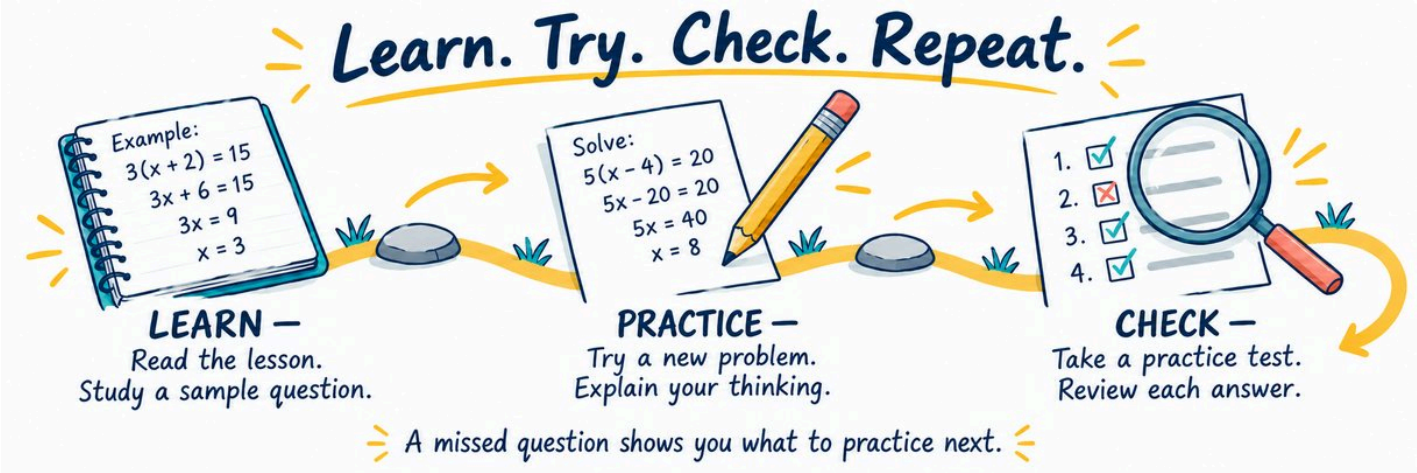
Algebra • Advanced Math • Problem Solving and Data Analysis
Geometry and Trigonometry • Digital-Test Strategies
Lessons, PSAT Math Style Questions, chapter practice,
and detailed explanations



Learn. Try. Check.

Reza Nazari

Welcome to Digital PSAT Math



Imagine a student at the kitchen table on Sunday afternoon, pencil ready for a first PSAT Math practice test. The opening questions go well. Then a word problem takes three readings, a graph looks unfamiliar, and the clock starts to feel much louder. The student circles two questions and keeps going.

Later, with the timer off, one explanation makes the word problem click: the equation needs to describe the amount left after a purchase. The student tries it again without looking. This time, the setup makes sense. Now there is a place to begin.

The PSAT can feel demanding. Familiar mathematics can look different in a word problem, and choosing a method takes time when several approaches seem possible and the clock is running. Give yourself room to learn before expecting speed.

You have 50 full-length PSAT Math practice tests to help you prepare. All 50 are interactive online, and 10 are also included in print. Lessons and detailed explanations help you understand the mathematics behind the answers, with fresh tests to try as your understanding grows.

Begin where you need help. Keep your scratch work, including unfinished attempts, so you can find the first step you could not explain when you review a difficult question. Work through that step slowly, then try again with the answer covered.

Scan or tap the code for the PSAT study hub. The next pages introduce the exam and help you plan your preparation.

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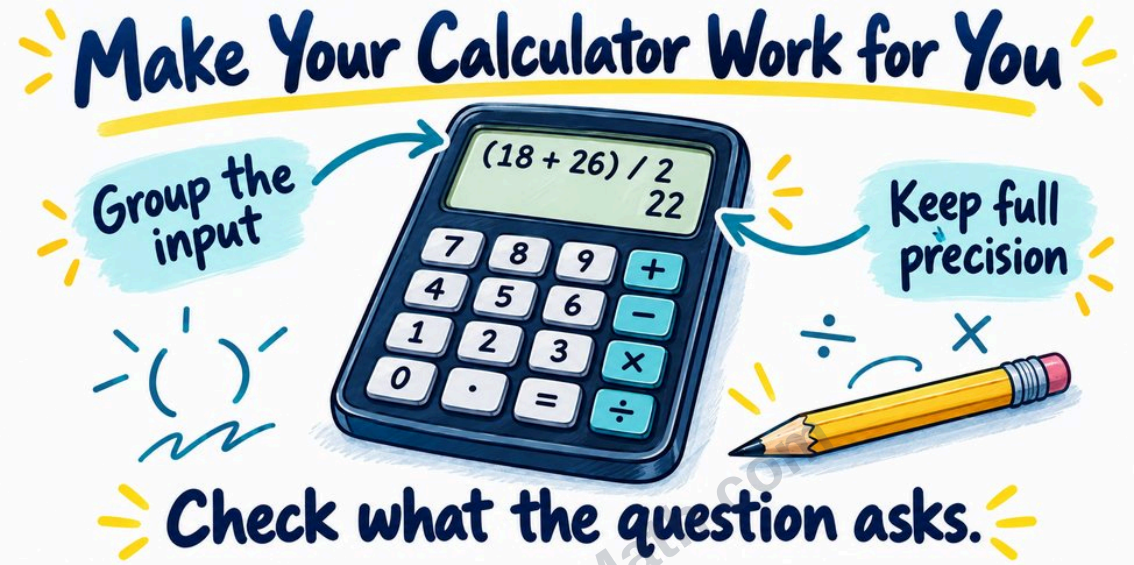
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Using Your Calculator Effectively

A calculator can save several lines of arithmetic, show where two curves meet, or help you check a result. The useful skill is deciding what to enter and what the result means. Work through this lesson with your calculator open. Try each entry yourself before reading its explanation.



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Start with the calculator you will use

Bluebook includes Desmos with graphing and scientific options, available throughout Math. You may switch between them or use a permitted handheld calculator. Current College Board policy prohibits calculators with computer algebra system (CAS) functionality. Check the policy for your own model before test day.

Use Bluebook's test preview or the assessment-specific Desmos calculators for practice. The ordinary Desmos website can include features unavailable on the exam. Find the expression area, keypad, graph settings, and controls for clearing an entry. Practice selecting an intersection to display its coordinates. If you bring a handheld, learn its fraction and exponent keys in advance.

Decide what the calculation should accomplish

For $3x + 7 = 22$, two short algebra steps give $x = 5$. For a quadratic and a line, a graph may reveal the relevant intersections quickly. Before typing, write the relationship you need on scratch paper and estimate the size or sign of the answer. A negative price or a probability above 1 should prompt another look at the setup, even when the entry itself was evaluated correctly.

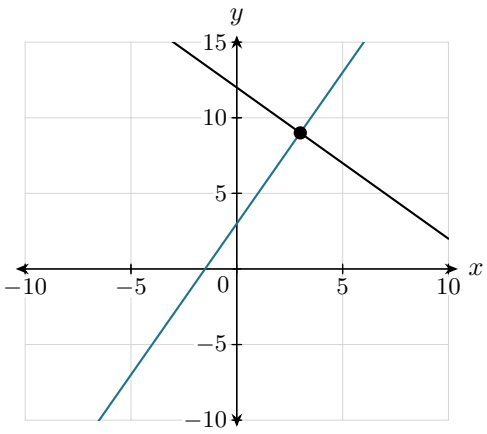
Read a Graph as Mathematics

An intersection solves an equality

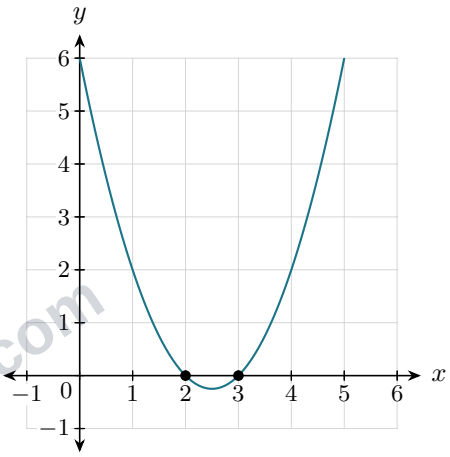
To solve $2x+3=-x+12$, enter $y=2x+3$ and $y=-x+12$ on separate expression lines in the graphing calculator. Select the intersection to read $(3,9)$. Both expressions equal 9 when $x=3$. Report 3 when asked for x, or 9 when asked for the common output.

Check by substitution: $2(3)+3=9$ and $-3+12=9$. This catches an incorrectly entered sign or coefficient. For a system asking for both variables, the ordered pair is $(3,9)$.

Equal outputs: $(3,9)$



Zero outputs: $x=2$ **and** $x=3$



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Count the roots, then read the requested quantity

For $x^2-5x+6=0$, the graph meets the x-axis at $(2,0)$ and $(3,0)$. Its roots are 2 and 3. Factoring confirms $(x-2)(x-3)=0$. The greater root is 3, their sum is 5, and their product is 6. Those are different possible answers to different questions about the same graph.

Change a window that hides useful information

The lines $y=x$ and $y=40-x$ meet at $(20,20)$. A window showing only -10 through 10 on each axis misses the intersection. Zoom out or set axis bounds that include the expected point. Then select the point and read its coordinates. The graph's apparent steepness also depends on the axis scales, so calculate a slope from coordinates when you need its value.

A graph is a guide to the allowed solutions. Recheck restrictions such as $x>0$, a whole-number count, or a nonzero denominator. An intersection outside a stated interval cannot answer a question restricted to that interval. When a displayed coordinate is rounded, use algebra or an exact fraction if the question needs an exact value.

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PSAT Math Test-Taking Strategies

Practice choosing a method and deciding when to return to a question. Reserve part of each timed module for omissions and work you can improve.

Work within the module you have

Each standard digital Math module has 22 questions and 35 minutes. You can revisit questions in the current module, but cannot return after leaving it. Your first-module performance helps determine your second module on the official adaptive exam. Focus on the current question. Its apparent difficulty cannot tell you your score or reliably identify your route.

Unscored pretest items are not identified, so answer every question. This book uses fixed modules and scores all 44 items for practice. Also try Bluebook practice for the official adaptive experience.

Make a first pass, then return with a plan

Read each question and choose a method. If repeated reading produces no first step, mark it and move on. For multiple-choice items, eliminate choices you can reject and enter your best current answer. Note the question number and where you stopped on scratch paper.

Return first to marked questions you have an approach for. A partly finished equation may be quicker than an unfamiliar setup. Continue when your solution is advancing. If the same steps lead nowhere, try another question.

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Use a pacing plan as a practice tool

Thirty-five minutes divided among 22 questions averages about 95 seconds per question. Some questions need only a short observation, leaving time for longer ones. Try the plan below during a practice module and adjust it from your results. These are suggested time budgets, not official checkpoints or a requirement to rush.

Phase	Approximate budget	What to do
First pass	25 minutes	Reach every question, answering accessible ones and marking work worth revisiting.
Return	8 minutes	Complete promising marked work and check uncertain entries.
Final scan	2 minutes	Find blanks in the question menu and confirm your responses are recorded.

Bluebook's timer gives a five-minute alert even if you hide the timer. Choose a few planned checks rather than looking at it after every line of arithmetic. If you are behind your practice plan, shorten a stalled attempt and keep moving through the module. When approved timing accommodations apply, build your plan around those directions instead.



Be Ready for Test Day

Confirm your school's arrival time and room instructions. Charge your device, pack your supplies, and keep your usual sleep and meal routine.

Prepare the device and the sign-in process

Complete the school readiness check and follow its PSAT sign-in instructions. Prepare a personal device with Bluebook and exam setup as directed. Confirm approved accommodations and assistive-technology settings with your coordinator beforehand.

Pack with the current school-day checklist

Bring or confirm	A useful check before leaving home
Testing device	Fully charged and prepared, unless your school supplies it.
Power cord or charger	Pack the one that fits. An outlet may not be available.
Pen or pencil	Staff supply scratch paper; leave your own paper at home.
Optional handheld	Confirm it is permitted, working, and familiar. Bluebook also has a calculator.
Photo ID when required	Bring approved ID if homeschooled or testing away from your regular school.
Break supplies	Pack a drink or snack for the designated break area.

Check calculator policy, remove prohibited algebra programs and stored documents, and clear saved formulas as PSAT/NMSQT rules require. Ask staff before accessing backup equipment.

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In the room, follow the proctor's directions

Store prohibited electronics, including phones and smartwatches, as instructed throughout testing and breaks. Close other apps. Use the supplied scratch paper and enter answers in Bluebook. Raise your hand for technical problems and follow the proctor's directions.

During the break and after the test

Follow break times and locations. Have a drink or snack where permitted, use the restroom if needed, and return on time. Keep phones stored and test questions out of conversations. Check the question menu for omissions before a module ends. After testing, keep your device open for submission and alert the proctor to problems. Wait for dismissal before closing it or leaving. Follow school directions and approved accommodations.

Know Your PSAT

This book prepares you for the Math section of the PSAT/NMSQT and PSAT 10. Before you begin the lessons, take a moment to see where math fits into the assessment. Both tests include Reading and Writing and Math. Each section has two timed modules.

Section	Questions	Standard testing time
Reading and Writing	54; 27 per module	64 minutes; 32 per module
Math	44; 22 per module	70 minutes; 35 per module

Use this book for your math preparation. You will begin with the number skills that support algebra, then work through equations, functions, data, geometry, and digital-test decisions. When a topic is familiar, try its questions first. When you need an explanation, read the lesson and follow the calculation one line at a time.

The Math section

You can use a permitted calculator throughout both Math modules. Questions use four choices or a numerical response. Unless indicated otherwise, figures are drawn to scale and lie in a plane, variables and expressions are real, and a function's domain contains the real inputs with real outputs.

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	Module 1	Module 2
Questions	22	22
Standard time	35 minutes	35 minutes
Calculator	Permitted throughout	Permitted throughout
Response formats	Four-choice and numeric response	Four-choice and numeric response

On the official exam, your performance in the first module determines which second module you receive. Each module contains 20 operational questions and two unscored pretest questions that are not identified to you. The book’s forms use two fixed modules and score all 44 items for practice. Neither the printed forms nor the companion’s fixed modules recreate College Board’s adaptive routing.

The PSAT-specific specification table gives approximate domain shares of 35 percent Algebra, 32.5 percent Advanced Math, 20 percent Problem-Solving and Data Analysis, and 12.5 percent Geometry and Trigonometry. Algebra includes linear equations, systems, and inequalities. Advanced Math builds on those ideas with nonlinear equations and functions. Problem-Solving and Data Analysis asks you to interpret quantities, data, and probability. Geometry and Trigonometry covers measurement, lines, angles, triangles, and right-triangle trigonometry. You will meet all four areas in the lessons and practice tests.

Multiple-choice and numeric answers

For a multiple-choice question, choose exactly one of A, B, C, or D. For a numeric response, enter the requested value, not an equation or a unit. Equivalent fractions and decimals may represent the same answer. On the official digital exam, positive entries allow up to five characters and negative entries up to six, including the minus sign. Fractions and decimal points count



as characters. A repeating decimal must be entered to the available precision by rounding or truncating correctly; $\frac{2}{3}$, .6666, and .6667 are examples. Do not enter mixed numbers, commas, percent signs, or units. Practice entering responses in Bluebook before test day.

Calculators and graphs

Bluebook includes an embedded Desmos graphing calculator. You may also bring a permitted handheld calculator. The current policy prohibits calculators with computer algebra system functionality. Use the calculator when it helps, and check that a graphing window or rounded display has not hidden part of the problem.

Using the online practice tests

Each printed test has a link to its interactive version. The two directory pages that follow printed Test 10 give you the forty additional tests, numbered 11–50. Scan a code or tap it in the digital book. Your responses save in the same browser, and the timer continues if you close a tab. You can mark questions, eliminate choices, use calculator tools, and consult the reference sheet. Results and explanations appear after Module 2.

Finding the lesson you need

Start with the topic behind a missed question, then work through its PSAT Math Style Questions with the explanations covered. The lessons group related skills so that you can move from the setup to the calculation and interpretation. Use Chapter 1 to repair foundational number skills as needed. Use the PSAT questions and practice in Chapters 2–16 to build and connect the tested skills. Chapter 12 develops sample-based population estimates and probability. Chapter 15 connects circle measurements with area comparisons and inverse measurement problems. Chapter 17 covers digital tools and response entry. Lesson practice may ask you to explain a method; the full tests use only four-choice or numerical responses.

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A Flexible Practice Plan

All 50 full-length tests are interactive online, and 10 are also printed here. Use a QR code or tap it in the digital book to open a test. Between tests, revisit uncertain answers with the explanations covered and check that you can explain the reasoning.

If the basics are new, start with foundation lessons and untimed chapter practice. When you are ready, adapt this four-week plan to your test date and study time. The lessons follow Algebra, Advanced Math, Problem-Solving and Data Analysis, and Geometry and Trigonometry. Chapter 12 develops population estimates. Chapter 15 connects circle measurements with inverse problems. In Chapter 11, focus on interpreting spread and changes to data. The standard-deviation calculation explains the measure rather than adding a formula to memorize.

1. **Week 1: Find your starting point.** Take a timed test. Study the lessons for missed questions and correct guesses before trying another test.
2. **Week 2: Build your skills.** Try the PSAT Math Style Questions with their explanations covered, then work through the chapter questions in difficult areas. Take one or two fresh tests, allowing time to review their explanations.
3. **Week 3: Practice your pace.** Try a test on screen as well as on paper. Notice where you spend too long, and practice choosing a method before calculating.

4. **Week 4: Bring it together.** Take fresh tests under your planned testing conditions. Revisit recurring mistakes and uncertain answers, and make time for rest before test day.

Allow 35 minutes per module and read the explanations after finishing both. Practice with the calculator and response-entry tools in Bluebook before test day.

PART I

Math Skill Lessons

1. Numerical Foundations

Number systems, ordering, and interval notation

A temperature of -3 degrees and a temperature of 2 degrees sit on opposite sides of zero. On a number line, positions increase as you move right. That gives $-3 < 2$. The symbol $<$ means "less than," and $>$ means "greater than." A longer string such as $-3 < 0 < 2$ states two comparisons at once.

Number families

The counting numbers $1, 2, 3, \ldots$ are positive integers. Whole numbers include zero. Integers also include the negatives: $\ldots, -2, -1, 0, 1, 2, \ldots$. A rational number can be written as $\frac{a}{b}$ with integers a, b and $b \neq 0$. Every integer is rational because $n = \frac{n}{1}$. In $\frac{a}{b}$, a is the numerator and b is the denominator. The fraction means a divided by b, so $\frac{3}{4}$ describes three of four equal parts of one whole. It is rational but is not an integer. An irrational number cannot be written as a ratio of integers. Examples include $\sqrt{2}$ and π. Rational and irrational numbers together form the real numbers, which are the positions on the number line.

A number may belong to several families. Zero is whole, integer, rational and real. Calling it an integer does not rule out those other descriptions. Some books include zero among the natural numbers and some begin at 1. Follow a definition supplied by a question rather than relying on that label alone.

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Comparing positions

Among negative numbers, the one farther left is smaller: $-8 < -3$. You can compare positive fractions by giving them a common denominator. For instance, split each sixth into two equal pieces and each fourth into three. Both amounts are now counted in twelfths: $\frac{5}{6} = \frac{5(2)}{6(2)} = \frac{10}{12}$ and $\frac{3}{4} = \frac{3(3)}{4(3)} = \frac{9}{12}$. Ten of those same-sized pieces exceed nine, so $\frac{5}{6} > \frac{3}{4}$. Their negatives reverse that order: $-\frac{5}{6} < -\frac{3}{4}$. The square of a number is that number multiplied by itself, and $\sqrt{19}$ denotes the nonnegative number whose square is 19. To compare a square root with integers, locate the number under the root between consecutive perfect squares. Since $16 < 19 < 25$, the positive square root satisfies $4 < \sqrt{19} < 5$.

For an approximation, first ask whether its precision settles the comparison. Rounding two numbers to the same decimal does not establish equality. The exact values $\frac{1}{3}$ and 0.33 differ because $\frac{1}{3} = 0.333\ldots$.

Describing a set with endpoints

An inequality describes which numbers are allowed. Here x is a placeholder for any number under consideration. The symbol $\leq$ means "less than or equal to," and $\geq$ means "greater than or equal to." For example, $x \geq 3$ allows 3 itself as well as every larger number. The statement $-2 \leq x < 5$ allows -2, excludes 5, and includes everything between them. Its interval notation is $[-2, 5)$. A bracket includes an endpoint. A parenthesis excludes it. Thus $(1, 4)$ excludes both endpoints, while $[1, 4]$ includes both.



The filled endpoint represents -2 and the open endpoint represents 5. A single interval includes all real numbers between its endpoints, including fractions. If a question asks only for integers in that interval, list those integers separately.

An unbounded set such as $x \geq 3$ has interval notation $[3, \infty)$. Infinity describes an unlimited direction, so it always takes a parenthesis. For $x < -1$ or $x \geq 2$, write $(-\infty, -1) \cup [2, \infty)$. The union symbol $\cup$ joins the two allowed sets. Check each finite endpoint against the original inequality before deciding which mark to use.

PSAT Math Style Question

How many integers x satisfy $-\sqrt{10} < x \leq 2$?

A. 4 **B.** 5

C. 6 **D.** 7

Since $3 < \sqrt{10} < 4$, the lower bound $-\sqrt{10}$ lies between -4 and -3. The permitted integers are $-3, -2, -1, 0, 1, 2$, giving 6 integers, choice C. The strict lower inequality still allows -3 because -3 is greater than $-\sqrt{10}$.

PSAT Math Style Question

A number t is greater than -4 and at most 1. Which inequality describes all possible values of t?

A. $-4 \leq t \leq 1$ **B.** $-4 < t < 1$

C. $-4 \leq t < 1$ **D.** $-4 < t \leq 1$

The phrase "greater than -4" gives $t > -4$, and "at most 1" gives $t \leq 1$. Both conditions must hold, so $-4 < t \leq 1$, choice D. The endpoint 1 is allowed, while -4 is excluded.

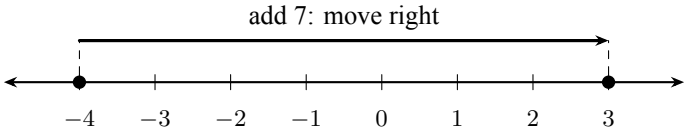
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Signed numbers and order of operations

If an account contains 6 dollars and a charge removes 9 dollars, the balance is $6 - 9 = -3$ dollars. Negative numbers describe positions below a chosen zero. You can add them by tracking direction on a number line rather than trying to memorize a separate rule for every appearance of a minus sign.

Adding and subtracting signed numbers

Adding a positive number moves right. Adding a negative number moves left. Starting at -4 and adding 7 gives 3 because the first 4 units reach zero and the remaining 3 move to its right.



The arrow begins at -4 and ends at 3. The seven unit intervals show the positive change. For $-4 + (-7)$, both movements are left, giving -11.

Numbers with the same sign combine their sizes and keep that sign. With opposite signs, subtract the smaller size from the larger and keep the sign of the number with the larger size: $8 + (-13) = -5$. Here the negative movement exceeds the positive one by 5.

Subtracting a number adds its opposite. Thus $5 - (-3) = 5 + 3 = 8$, whereas $5 - 3 = 2$. Rewrite a difficult subtraction before calculating: $-6 - (-10) = -6 + 10 = 4$. Parentheses make it clear whether a minus sign belongs to a number or tells you to subtract an entire expression.

Products, quotients and signs

Multiplying numbers with opposite signs gives a negative product. Two negative factors give a positive product. You can see why by extending a consistent pattern: $(-3)(2) = -6$, $(-3)(1) = -3$, $(-3)(0) = 0$, and $(-3)(-1) = 3$. Each decrease of 1 in the second factor increases the product by 3.

For several nonzero factors, an even number of negative factors gives a positive product and an odd number gives a negative product. Division follows the same sign rules. Since $(-4)(3) = -12$, $\frac{-12}{3} = -4$. Division by zero is undefined. Zero divided by a nonzero number is zero.

Choosing the order of calculation

A number next to parentheses means multiplication: $2(3) = 2 \times 3 = 6$. Two parentheses next to one another also mean multiply their values. Grouping symbols tell you what to calculate together. Work inside parentheses first, then evaluate exponents. Multiply and divide from left to right. Finally add and subtract from left to right. Multiplication and division have equal priority, as do addition and subtraction. In $18 \div 3 \times 2$, the quotient $18 \div 3 = 6$ comes first, then $6 \times 2 = 12$.

An exponent applies to its base. In $(-3)^2$, the parentheses make -3 the base, so $(-3)(-3) = 9$. In -3^2, the exponent applies to 3 and the leading negative remains outside: $-(3^2) = -9$. These expressions have different values.

A fraction bar groups the whole numerator and the whole denominator. In $\frac{12-4}{2+2}$, first calculate 8 above and 4 below, giving 2. You cannot treat the expression as $12 - \frac{4}{2} + 2$. When substituting a negative value, put it in parentheses so both its sign and the operation remain visible. For $2x^2 - 3x$ at $x = -2$, write $2(-2)^2 - 3(-2) = 8 + 6 = 14$.

Work one operation at a time if several signs occur together. Writing the intermediate line is useful when a calculator display conceals whether the negative sign was inside an exponent.

PSAT Math Style Question

What is the value of $-3^2 + 2(-4 + 7)$?

A. -15

B. -3

C. 3

D. 15

The exponent applies to 3, so $-3^2 = -9$. Inside the parentheses, $-4 + 7 = 3$, and $2(3) = 6$. The expression is $-9 + 6 = -3$, choice B. Using $(-3)^2$ would change the original expression and lead to 15.

PSAT Math Style Question

For $a = -2$ and $b = 5$, what is the value of $\frac{a-b}{a+3}$?

A. -7

B. $-\frac{7}{5}$

C. 3

D. 7

Substitute with parentheses: the numerator is $(-2) - 5 = -7$ and the denominator is $(-2) + 3 = 1$. Their quotient is -7, choice A. Calculating the entire denominator first prevents the fraction bar from being treated as division by a alone.

Absolute value and distance on a number line

The positions -5 and 5 are each 5 units from zero. Absolute value records that distance, written $|-5| = 5$ and $|5| = 5$. Also, $|0| = 0$. A distance cannot be negative, so the absolute value of a real number is always nonnegative.

Evaluating an absolute value

First calculate the expression between the bars. Then take its distance from zero. For $|3 - 8|$, the expression inside is -5, so the value is 5. In $-|3 - 8|$, the negative sign is outside the bars. The absolute value is still 5, and the whole expression is -5. Bars group an expression but also apply the distance operation.

For a real number a, $|a| = a$ when $a \geq 0$ and $|a| = -a$ when $a < 0$. The second rule makes a negative input positive. For example, if $a = -7$, then $-a = 7$. It does not say that an absolute value can be negative.

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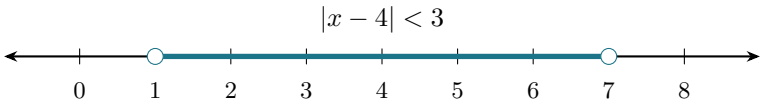
Distance between two positions

The distance from a to b is $|a-b|$. The order of subtraction does not affect the distance because $|a-b| = |b-a|$. From -3 to 4, the difference is $4-(-3) = 7$, giving a distance of 7. If both positions are negative, use the same method: the distance from -8 to -2 is $|-2-(-8)| = |6| = 6$.

The expression $|x-4|$ therefore means the distance from x to 4. The equation $|x-4| = 3$ asks for the positions 3 units to either side of 4. They are $x = 1$ and $x = 7$. Substitute both to check: $|1-4| = 3$ and $|7-4| = 3$. When the required distance is zero, the two positions coincide and there is one solution. A required negative distance has no real solution.

Allowed distances

If $|x-4| < 3$, the position must lie between 1 and 7, excluding the endpoints. Thus $1 < x < 7$. With $|x-4| \leq 3$, both endpoints are included, giving $1 \leq x \leq 7$. The statement $|x-4| > 3$ asks for positions outside the middle interval, so $x < 1$ or $x > 7$. For $\geq$, include 1 and 7. These descriptions come from distance, which makes it easier to check whether the middle or the outside is allowed.



Checking combinations

Absolute value does not distribute over addition. With $a = -6$ and $b = 2$, $|a+b| = |-4| = 4$, while $|a| + |b| = 6 + 2 = 8$. Adding movements in opposite directions allows cancellation. In general, $|a+b| \leq |a| + |b|$. For multiplication, $|ab| = |a||b|$, and for $b \neq 0$, $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$. These follow from the sizes of the factors and the sign rules for products and quotients.

PSAT Math Style Question

Which inequality describes all real numbers x whose distance from -2 is less than 5?

A. $-7 < x < 3$

B. $-3 < x < 7$

C. $-7 \leq x \leq 3$

D. $x < -7$ or $x > 3$

Starting at -2, moving 5 units left reaches -7 and moving 5 units right reaches 3. A distance less than 5 places x strictly between those endpoints. Therefore $-7 < x < 3$, choice A. Neither endpoint is included because each is exactly 5 units from -2.

PSAT Math Style Question

For $p = -6$ and $q = 2$, what is the value of $|p-q| - |p+q|$?

A. -8

B. -4

C. 4

D. 12

First calculate the expressions inside the bars: $p-q = -8$ and $p+q = -4$. Their absolute values are 8 and 4, so the difference is $8-4 = 4$, choice C. Taking the absolute values before forming $p-q$ would change the question.

Rational and irrational numbers: closure and counterexamples

The decimal 0.625 is rational because $0.625 = \frac{625}{1000} = \frac{5}{8}$. A terminating decimal always has a power of 10 as a possible denominator. A repeating decimal is rational too. For instance, if $x = 0.777\ldots$, then $10x = 7.777\ldots$. Subtracting gives $9x = 7$, so $x = \frac{7}{9}$. For a longer repeating block, multiply by a power of 10 that shifts one full block, then subtract to eliminate the repeating part.

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An irrational number has a decimal expansion that neither terminates nor eventually repeats. A decimal that is simply long is not enough evidence: $\frac{1}{7}$ has a long repeating block and is rational. Likewise, the decimal shown on a calculator is a rounded display. It does not change an irrational exact value into a rational one.

Recognizing square roots

Square roots of perfect squares are integers and therefore rational: $\sqrt{49} = 7$. The positive square root of a positive integer that is not a perfect square is irrational. Thus $\sqrt{50}$ is irrational even though it can be simplified to $5\sqrt{2}$. Multiplying $\sqrt{2}$ by 5 changes its value but does not make it rational.

The fact that $\sqrt{2}$ is irrational can be justified with factors. Suppose it were $\frac{a}{b}$ with positive integers a, b, chosen so that no integer greater than 1 divides both. Such a fraction is in lowest terms: any shared factor has already been divided out. Squaring would give $a^2 = 2b^2$. Then a^2 is even, which requires a to be even. Writing $a = 2k$ gives $b^2 = 2k^2$, so b must be even too. That contradicts the fraction being in lowest terms, because both numbers would have a factor of 2. The same prime-factor reasoning explains the result for other non-square positive integers.

Operations on rational numbers

A set is closed under an operation when applying that operation to members of the set always gives another member. Rational numbers are closed under addition, subtraction and multiplication. If a, b, c, d are integers with $b, d \neq 0$, then $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$, a ratio of integers with a nonzero denominator. The same reasoning works for subtraction and multiplication. Division of rational numbers stays rational when the divisor is nonzero.

A claim about integers may need different conditions. Integers are closed under addition and multiplication, but division can leave the set: $\frac{1}{2}$ is not an integer. Zero makes unrestricted division impossible in either set.

Combining rational and irrational values

Adding a rational number to an irrational number always gives an irrational result. If $r + s$ were rational with r rational, subtracting r would force s to be rational. This contradicts the assumption that s is irrational. The same argument applies to subtraction.

The product of a nonzero rational number and an irrational number is irrational. A supposedly rational product could be divided by the nonzero rational factor, forcing the irrational factor to be rational. The word nonzero matters: $0\sqrt{2} = 0$ is rational.

Two irrational numbers can produce either kind of result. Their sum may be rational, as $\sqrt{2} + (-\sqrt{2}) = 0$, or irrational, as $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$. Their product may be rational, as $\sqrt{2}\sqrt{2} = 2$, or irrational, as $\sqrt{2}\sqrt{3} = \sqrt{6}$. For the second product, both factors are positive, and squaring their product gives $2 \cdot 3 = 6$. Their product is therefore the positive square root of 6, which is irrational because 6 is not a perfect square. A single counterexample is enough to disprove a statement that claims something always happens. Several successful examples cannot establish such a universal claim.

Use exact values when testing a statement. Replacing $\sqrt{2}$ by 1.414 turns a question about irrational numbers into one about a terminating, rational approximation.

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PSAT Math Style Question

If r is rational and s is irrational, which of the following must be irrational?

A. $r + s$ **B.** rs

C. s^2 **D.** r^2

The sum $r + s$ must be irrational, choice A. If it were rational, subtracting the rational number r would make s rational, contradicting the given information. The product in B could be zero when $r = 0$. In C, $s = \sqrt{2}$ gives $s^2 = 2$, and r^2 in D is rational for every rational r.

PSAT Math Style Question

Which pair of irrational numbers has a rational product?

A. $\sqrt{2}$ and $\sqrt{3}$

B. $\sqrt{2}$ and $3\sqrt{2}$

C. $\sqrt{3}$ and $\sqrt{5}$

D. $\sqrt{2}$ and $\sqrt{7}$

The product in B is $\sqrt{2}(3\sqrt{2}) = 3(2) = 6$, which is rational. Each factor is irrational. The other products are $\sqrt{6}$, $\sqrt{15}$ and $\sqrt{14}$, all square roots of positive non-square integers. Choice B is the required pair.

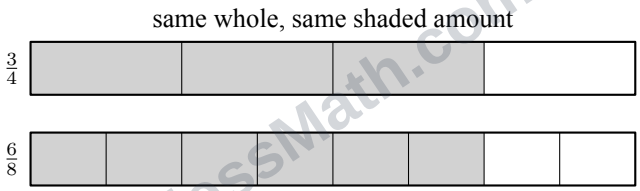
Equivalent fractions and mixed numbers

If a sandwich is cut into four equal pieces and you take three, you have $\frac{3}{4}$ of the sandwich. The denominator, 4, tells how many equal parts make one whole. The numerator, 3, counts the parts taken. The pieces must be equal, and the whole must stay the same when you compare fractions. Half of a large pizza can be more food than half of a small pizza.

A fraction also means division: $\frac{a}{b}$ is the result of dividing a by b. Its denominator cannot be zero. The fraction $\frac{0}{3}$ equals zero, while $\frac{3}{0}$ is undefined. A negative sign can go in front of the fraction, in its numerator, or in its denominator: $-\frac{2}{5} = \frac{-2}{5} = \frac{2}{-5}$. Two negative signs give a positive value, so $\frac{-2}{-5} = \frac{2}{5}$.

Changing the pieces without changing the amount

Cut each quarter of the sandwich into two equal pieces. The three quarters become six eighths. The amount of sandwich stays unchanged: $\frac{3}{4} = \frac{6}{8}$.



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The bars have equal total length. The same shaded length covers three quarters or six eighths. In symbols, multiply the numerator and denominator by the same nonzero number. For example, $\frac{5}{7} = \frac{5 \cdot 3}{7 \cdot 3} = \frac{15}{21}$. This works because the multiplier $\frac{3}{3}$ equals 1.

Simplifying reverses that process. Divide the numerator and denominator by a common factor. In $\frac{18}{24}$, both numbers are divisible by 6, giving $\frac{3}{4}$. No integer greater than 1 divides both 3 and 4, so the result is in lowest terms. You can divide in stages if a common factor is easier to spot: $\frac{18}{24} = \frac{9}{12} = \frac{3}{4}$. Both routes preserve the value. You may cancel common factors, but you cannot erase digits or cancel terms joined by addition.

Equivalent fractions can answer a missing-number question. If $\frac{7}{12} = \frac{n}{60}$, the denominator has been multiplied by 5. Multiply the numerator by 5 as well, so $n = 35$. Another check is cross multiplication: equality of $\frac{a}{b}$ and $\frac{c}{d}$ with nonzero denominators gives $ad = bc$. This follows by multiplying both sides by bd.

Improper fractions and mixed numbers

A positive proper fraction is less than 1. For a positive fraction with a positive denominator, an improper fraction has a numerator at least as large as its denominator. For a negative fraction, classify its magnitude in the same way. The fraction $\frac{17}{5}$ contains three whole groups of five fifths, with two fifths left. Thus $\frac{17}{5} = 3\frac{2}{5}$. Divide to find the whole-number part, then place the remainder over the original denominator.

For the reverse conversion, count all the fractional pieces. In $3\frac{2}{5}$, each whole contributes five fifths. The three wholes contribute 15 fifths, and the additional two fifths give $\frac{3(5)+2}{5} = \frac{17}{5}$. A mixed number represents addition, not multiplication. For a negative mixed number, the sign applies to the entire amount: $-2\frac{1}{3} = -\left(2 + \frac{1}{3}\right) = -\frac{7}{3}$. Converting before calculating helps keep that sign attached.

Comparing fractions

Give fractions a common denominator or compare them to a useful benchmark. For $\frac{5}{8}$ and $\frac{2}{3}$, use twenty-fourths: $\frac{15}{24} < \frac{16}{24}$, so $\frac{5}{8} < \frac{2}{3}$. The denominators alone do not decide which fraction is larger. With the same positive numerator, however, a larger positive denominator means smaller pieces and a smaller fraction. Thus $\frac{3}{7} < \frac{3}{5}$. For negative fractions, the value farther left on the number line is smaller: $-\frac{3}{4} < -\frac{1}{2}$.

In a PSAT question, equivalent forms may describe a measurement or distinguish close answer choices. Keep the fraction exact until the question asks for a decimal approximation. A rounded decimal can hide a small difference between two fractions.

PSAT Math Style Question

A roll of ribbon is $4\frac{3}{8}$ yards long. Which fraction represents this length in yards?

A. $\frac{7}{8}$ **B.** $\frac{3}{2}$

C. $\frac{35}{8}$ **D.** 4

Each yard contains eight eighths. Four yards therefore contain 32 eighths, and the extra three eighths bring the total to 35 eighths. The length is $\frac{4(8)+3}{8} = \frac{35}{8}$ yards, choice C. Dividing 35 by 8 returns 4 with remainder 3, which checks the mixed number. Adding 4 and 3 in the numerator would count four eighths instead of four whole yards.

PSAT Math Style Question

Which of the following lists the fractions in increasing order?

A. $\frac{5}{8}, \frac{2}{3}, \frac{7}{10}$ **B.** $\frac{2}{3}, \frac{5}{8}, \frac{7}{10}$

C. $\frac{7}{10}, \frac{2}{3}, \frac{5}{8}$ **D.** $\frac{5}{8}, \frac{7}{10}, \frac{2}{3}$

A common denominator of 120 makes the comparison exact. The fractions become $\frac{75}{120}$, $\frac{80}{120}$, and $\frac{84}{120}$. Their numerators increase in that order, so $\frac{5}{8} < \frac{2}{3} < \frac{7}{10}$, choice A. Comparing 5, 2, and 7 by themselves would ignore that the pieces have different sizes.

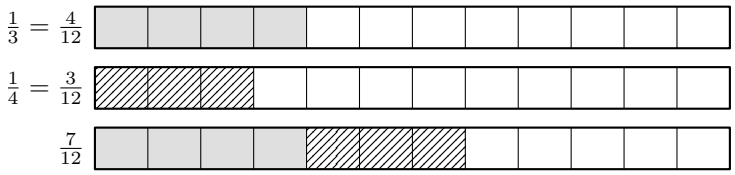
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Adding and subtracting fractions

Three eighths of a mile plus two eighths of a mile makes five eighths of a mile. The pieces have the same size, so add the counts: $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$. Keep the denominator. Adding it would change eighths into sixteenths and give the wrong distance. Subtraction follows the same idea: $\frac{7}{8} - \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$.

Making the units match

To add $\frac{1}{3}$ and $\frac{1}{4}$, first express both amounts using pieces of the same size. Twelfths work because both 3 and 4 divide 12. Replace $\frac{1}{3}$ with $\frac{4}{12}$ and $\frac{1}{4}$ with $\frac{3}{12}$. Now the sum is $\frac{7}{12}$.



All three bars use the same whole divided into twelfths. Four pieces and three pieces combine to cover seven pieces. Each replacement multiplies the numerator and denominator by the same number, so it preserves the original amount.

A common denominator is any positive common multiple of the denominators. The least common denominator is their least positive common multiple. With denominators 6 and 8, the least common denominator is 24. Their product, 48, also works, but it leaves more simplification. List multiples when the numbers are small. For larger numbers, factor them and include each prime factor as many times as either denominator needs it. Since $12 = 2^2 \cdot 3$ and $18 = 2 \cdot 3^2$, their least common multiple is $2^2 \cdot 3^2 = 36$.

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For example, $\frac{5}{12} - \frac{1}{18} = \frac{15}{36} - \frac{2}{36} = \frac{13}{36}$. Write both equivalent fractions before subtracting if you tend to lose the multiplier. Subtracting the denominators, or changing a denominator without changing its numerator, changes the problem.

Signs and mixed numbers

A signed fraction follows the same addition rules as a signed integer. In $-\frac{2}{3} + \frac{1}{4}$, use twelfths to get $-\frac{8}{12} + \frac{3}{12} = -\frac{5}{12}$. The negative amount has greater magnitude. Subtracting a negative reverses its direction: $\frac{1}{6} - \left(-\frac{1}{4}\right) = \frac{1}{6} + \frac{1}{4} = \frac{5}{12}$. Keep parentheses around a negative fraction until the subtraction sign has been handled.

Mixed numbers can be converted to improper fractions before calculation. For $2\frac{1}{6} - 1\frac{3}{4}$, the conversion gives $\frac{13}{6} - \frac{7}{4} = \frac{26}{12} - \frac{21}{12} = \frac{5}{12}$. This route works even when the fractional part of the first mixed number is too small to subtract the second fractional part.

You can also regroup one whole. Write $2\frac{1}{6} = 1\frac{7}{6}$, then subtract the one whole and $\frac{3}{4}$ from $1 + \frac{7}{6}$. The remaining fractional difference is $\frac{14}{12} - \frac{9}{12} = \frac{5}{12}$. Choose the approach whose signs and pieces you can track reliably. For several terms, a single common denominator keeps the arithmetic together: $\frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{6}{12} + \frac{4}{12} - \frac{3}{12} = \frac{7}{12}$.

Fractions with letters and fractions in context

A letter can stand for an unknown number. In $3x$, the 3 tells you to multiply x by 3. Three copies of x minus two copies leave one copy: $3x - 2x = x$. Multiplying a sum by 3 multiplies each term, so $3(x - 1) = 3x - 3$. A negative multiplier also applies to each term: $-2(x + 2) = -2x - 4$. You can check this last equality at $x = 1$: both expressions give -6. These rules let you handle an entire numerator without losing a sign.

The same rule applies when a numerator contains a letter. For $\frac{x}{3} + \frac{x}{5}$, use fifteenths: $\frac{5x}{15} + \frac{3x}{15} = \frac{8x}{15}$. This is true for every real x. If the denominator contains a variable, state its restrictions. For example, $\frac{1}{x} + \frac{2}{3x} = \frac{3}{3x} + \frac{2}{3x} = \frac{5}{3x}$ requires $x \neq 0$. The fraction cannot be evaluated at zero before or after simplification.

In a word problem, identify the whole and the measurement unit before adding. Fractions of one tank can be added as fractions of that tank. A fraction of a tank and a number of liters need a conversion first. Estimate the result too. A sum of two positive fractions must be greater than either fraction, while subtracting a positive amount must reduce the starting value.

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PSAT Math Style Question

A student spends $\frac{3}{8}$ hour reading and $\frac{5}{12}$ hour writing. What is the total time, in hours?

A. $\frac{2}{5}$ **B.** $\frac{2}{3}$

C. $\frac{19}{24}$ **D.** $\frac{23}{24}$

Both times are parts of an hour, so they can be added after their fractional pieces match. In twenty-fourths, $\frac{3}{8} = \frac{9}{24}$ and $\frac{5}{12} = \frac{10}{24}$. The total is $\frac{19}{24}$ hour, choice C. Adding the original numerators and denominators would give $\frac{8}{20} = \frac{2}{5}$, which cannot be right because it is smaller than the writing time alone.

PSAT Math Style Question

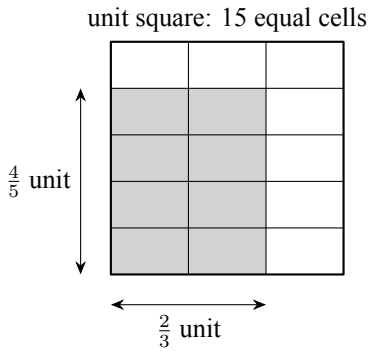
For every real number x, which expression is equivalent to $\frac{x-1}{4} - \frac{x+2}{6}$?

A. $\frac{x+1}{12}$ **B.** $\frac{x-7}{12}$

C. $\frac{-3}{2}$ **D.** $\frac{5x+1}{12}$

Use a denominator of 12. The first numerator becomes $3(x - 1)$ and the second becomes $2(x + 2)$, giving $\frac{3(x-1)-2(x+2)}{12}$. Multiply each term inside the parentheses: $3(x - 1) = 3x - 3$ and $-2(x + 2) = -2x - 4$. Combine the three copies of x minus two copies, then combine the constants: $3x - 3 - 2x - 4 = x - 7$. The result is $\frac{x-7}{12}$, choice B. At $x = 0$, the original expression is $-\frac{1}{4} - \frac{1}{3} = -\frac{7}{12}$, agreeing with this result.

Multiplying and dividing fractions



Half of three fourths is three eighths. You can see this by dividing each fourth into two equal pieces: $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$. Multiplication finds a fraction of an amount. If a rectangle has side lengths $\frac{2}{3}$ unit and $\frac{4}{5}$ unit, dividing a unit square into three columns and five rows gives fifteen equal small rectangles. Select two of the three columns and four of the five rows. Their overlap contains $2 \cdot 4 = 8$ of the 15 equal small rectangles, so its area is $\frac{8}{15}$ square unit.

The shaded rectangle spans two columns and four rows. Its eight cells each have area $\frac{1}{15}$ square unit.

These examples explain the multiplication rule: multiply the numerators and multiply the denominators, then simplify. Thus $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ when b and d are nonzero. A whole number can be written over 1. For example, $6 \cdot \frac{2}{9} = \frac{6}{1} \cdot \frac{2}{9} = \frac{12}{9} = \frac{4}{3}$.

Reducing before multiplying

In $\frac{14}{15} \cdot \frac{25}{21}$, multiplying first gives $\frac{350}{315}$. Smaller numbers are easier to handle. Divide the factors 14 and 21 by 7, and divide 25 and 15 by 5. The product becomes $\frac{2}{3} \cdot \frac{5}{3} = \frac{10}{9}$. Each reduction removes a factor shared by the complete numerator and denominator. You can reduce across the two fractions because their numerators are multiplied together and their denominators are multiplied together.

Cancellation applies to factors. For example, the 3 in $\frac{3+2}{3}$ cannot be canceled with one term of the sum. The numerator is 5, so that fraction equals $\frac{5}{3}$. Treat a sum in parentheses as a complete quantity until you have evaluated or factored it.

Convert mixed numbers before multiplication. In $1\frac{1}{2} \cdot 2\frac{2}{3}$, use $\frac{3}{2} \cdot \frac{8}{3} = 4$. Multiplying the whole parts and fractional parts separately would omit contributions. Signed multiplication follows the integer rules: two negative factors give a positive result, while one negative factor gives a negative result. Consequently $-\frac{3}{5} \cdot \frac{10}{9} = -\frac{2}{3}$.

Division counts groups

How many quarter-liter servings fit in $\frac{3}{2}$ liters? Each liter contains four quarters, so one and a half liters contains six quarters. This is the division $\frac{3}{2} \div \frac{1}{4} = 6$. Division can also describe sharing: $\frac{3}{4} \div 3 = \frac{1}{4}$ gives the amount each person receives when three people share $\frac{3}{4}$ of a pizza equally.

The reciprocal of a nonzero number is the number that multiplies with it to give 1. The reciprocal of $\frac{c}{d}$ is $\frac{d}{c}$, provided both c and d are nonzero. Dividing by $\frac{c}{d}$ means multiplying by its reciprocal: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$. To see why, call the quotient q. The equation $q \cdot \frac{c}{d} = \frac{a}{b}$ becomes $q = \frac{a}{b} \cdot \frac{d}{c}$ after both sides are multiplied by $\frac{d}{c}$.

Only the divisor is inverted. In $\frac{5}{6} \div \frac{10}{9}$, keep $\frac{5}{6}$ and multiply by $\frac{9}{10}$ to obtain $\frac{3}{4}$. Dividing by zero is undefined because zero has no reciprocal. Zero divided by a nonzero number is zero.

Nested fractions and reasonable results

A fraction bar already means division. The expression $\frac{\frac{2}{3}}{\frac{5}{6}}$ equals $\frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \cdot \frac{6}{5} = \frac{4}{5}$. If the numerator or denominator contains addition, perform that addition first. For $\frac{\frac{1}{2}+\frac{1}{3}}{\frac{5}{6}}$, the numerator is $\frac{5}{6}$, so the entire fraction equals 1.

Multiplying a positive amount by a fraction between 0 and 1 makes it smaller. Dividing by such a fraction makes it larger because smaller groups fit more times. These comparisons help check your calculations. In a serving or cutting problem, also distinguish the numerical quotient from the number of complete groups. A quotient of $6\frac{1}{2}$ means six full groups and half of another, so only six complete groups fit.

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PSAT Math Style Question

A bottle contains $2\frac{1}{4}$ liters of juice. Each serving uses $\frac{3}{8}$ liter. How many full servings can be poured?

A. 3 **B.** 5

C. 6 **D.** 8

Divide the available juice by the amount in one serving. Since $2\frac{1}{4} = \frac{9}{4}$, the count is $\frac{9}{4} \div \frac{3}{8} = \frac{9}{4} \cdot \frac{8}{3} = 6$, choice C. Canceling common factors keeps the numbers small. Six servings use $6 \cdot \frac{3}{8} = \frac{18}{8} = \frac{9}{4}$ liters, so all the juice is accounted for.

PSAT Math Style Question

Of the students at a school, $\frac{3}{5}$ participate in a sport. Of those who participate in a sport, $\frac{7}{12}$ play a team sport. What fraction of all students participate in a sport but do not play a team sport?

A. $\frac{1}{4}$ **B.** $\frac{7}{20}$

C. $\frac{11}{12}$ **D.** $\frac{12}{35}$

Within the sports group, the fraction who do not play a team sport is $1 - \frac{7}{12} = \frac{5}{12}$. That group is $\frac{5}{12}$ of the $\frac{3}{5}$ who participate in a sport. Multiply to find the fraction of the whole school: $\frac{3}{5} \cdot \frac{5}{12} = \frac{3}{12} = \frac{1}{4}$, choice A. The fraction $\frac{5}{12}$ describes the students remaining within the sports group. Multiplying by $\frac{3}{5}$ then expresses that smaller group as a fraction of the whole school.

Decimals, place value, rounding, and estimation

The number 4.37 means four wholes, three tenths, and seven hundredths. Each position has one tenth the value of the position immediately to its left. Moving right from the decimal point gives tenths, hundredths, thousandths, and ten-thousandths. Thus $0.064 = \frac{6}{100} + \frac{4}{1000} = \frac{64}{1000}$.

Zeros can mark positions without adding value. The decimals 0.6, 0.60, and 0.600 are equal. In 0.06, however, the zero in the tenths position moves the 6 into the hundredths position. That value is one tenth of 0.6. To compare positive decimals, align their decimal points and add trailing zeros: $0.47 = 0.470 < 0.506$. Compare from left to right until the first differing digit appears. For negatives, reverse the magnitude comparison. Since $0.506 > 0.470$, we have $-0.506 < -0.470$.

Calculating with decimals

For addition and subtraction, line up decimal points so that hundredths combine with hundredths and tenths with tenths. In $3.4 - 0.87$, write $3.40 - 0.87$. To subtract 7 hundredths from zero hundredths, regroup one tenth as 10 hundredths. Then regroup one whole as 10 tenths so there are enough tenths to subtract 8. The top amount is now 2 wholes, 13 tenths and 10 hundredths. Subtracting 0 wholes, 8 tenths and 7 hundredths leaves 2 wholes, 5 tenths and 3 hundredths, or 2.53. Regrouping preserves the total amount, and the added trailing zero preserves the original value 3.4.

For multiplication, multiply the whole-number digit strings first and then account for their place values. In $0.24 \cdot 0.3$, calculate $24 \cdot 3 = 72$. The first factor has two decimal places and the second has one, so the product has three: 0.072. Equivalently, $\frac{24}{100} \cdot \frac{3}{10} = \frac{72}{1000}$. A decimal point chosen by appearance alone can move the result by a factor of 10 or 100.

For division, multiply both the dividend and divisor by the same power of 10 until the divisor is a whole number. The quotient stays unchanged. Thus $4.68 \div 0.12 = 468 \div 12 = 39$. If the divisor is 0.5, multiplying both numbers by 10 is enough. Multiplying only one of them changes the quotient.

Rounding to a stated place

Rounding replaces a value with the closest value at a specified precision. To round 8.376 to the nearest hundredth, locate the hundredths digit, 7, and inspect the next digit, 6. A next digit of 5 or greater increases the retained digit by 1. The result is 8.38.

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A next digit below 5 leaves it unchanged. For instance, 8.374 rounds to 8.37. Carrying can cross several positions: 2.996 rounds to 3.00 to the nearest hundredth.

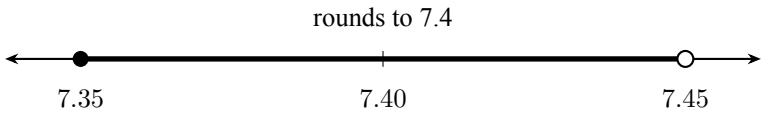
A stated precision matters. The value 6.248 rounds to 6.2 to the nearest tenth, 6.25 to the nearest hundredth, and 6 to the nearest whole number. Under the usual school convention, a halfway magnitude rounds away from zero, so -2.65 rounds to -2.7 to the nearest tenth. If a question supplies a different tie convention, follow it. Keeping negative halfway values explicit prevents a sign rule from being guessed.

Round once at the end of a multi-step calculation unless intermediate rounding is requested. Rounding 4.46 and 4.46 to whole numbers first gives a sum of 8. Their exact sum is 8.92, which rounds to 9. The early rounding lost enough to change the final answer.

Estimating and checking a rounded result

An estimate uses nearby easy values to judge size. For $19.8 \cdot 4.9$, using $20 \cdot 5$ gives about 100. An exact answer near 10 or 1,000 would signal a decimal-place error. Both chosen factors exceed the originals, so this estimate is an upper estimate for this positive product.

A value recorded as 7.4 to the nearest tenth could have come from any value at least 7.35 and less than 7.45, under the usual positive rounding rule. The lower endpoint rounds up to 7.4, while 7.45 rounds up to 7.5. This interval describes rounding uncertainty, not the accuracy of a measuring instrument.



Each marked step is 0.05. The filled lower endpoint rounds to 7.4, while the open upper endpoint rounds to 7.5. In a PSAT item, keep track of whether a displayed decimal is exact, approximate, or a rounded measurement.

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PSAT Math Style Question

A machine uses water at a constant rate of 0.375 liter per hour. How many liters does it use in 6.3 hours, rounded to the nearest tenth?

A. 0.24 **B.** 2.04

C. 2.4 **D.** 24

Multiply the hourly rate by the time: $0.375(6.3) = 2.3625$ liters. To round to the nearest tenth, look at the hundredths digit, 6. It rounds the tenths digit up from 3 to 4, giving 2.4 liters, choice C. An estimate of $0.4(6) = 2.4$ also supports the size of the result.

PSAT Math Style Question

A positive length L rounds to 12.6 centimeters to the nearest tenth. Using the convention that a halfway value rounds up, which interval contains exactly the possible original values of L?

A. $[12.5, 12.7)$ **B.** $[12.55, 12.65)$

C. $(12.55, 12.65]$ **D.** $[12.6, 12.7)$

Values beginning at 12.55 round up to 12.6, so the lower endpoint is included. Values below 12.65 still round to 12.6, but 12.65 itself rounds up to 12.7. The possible values form $[12.55, 12.65)$, choice B. The interval extends half of a tenth on each side of 12.6, and the stated halfway convention determines the two endpoint marks.

Converting fractions, decimals, and percents

The fraction $\frac{3}{4}$, the decimal 0.75, and the percent 75% name the same number. Each form serves a different purpose. Fractions preserve exact division, decimals fit place-value calculations, and percentages compare an amount with a base of 100. A percent always needs an understood whole. If 75% of 40 students attend an event, the whole is the group of 40 students, and 30 attend.

Between fractions and decimals

A fraction bar means division, so divide the numerator by the denominator. For $\frac{3}{8}$, $3 \div 8 = 0.375$. Long division produces the digits in order. Eight goes into 30 tenths three times, leaving 6 tenths. It goes into 60 hundredths seven times, leaving 4 hundredths. It goes into 40 thousandths five times with no remainder. The decimal terminates when the remainder reaches zero.

To convert a terminating decimal to a fraction, count its decimal places and use the matching power of 10 as the denominator. Thus $0.48 = \frac{48}{100} = \frac{12}{25}$ and $0.006 = \frac{6}{1000} = \frac{3}{500}$. Include the whole-number part when it is present: $2.35 = \frac{235}{100} = \frac{47}{20} = 2\frac{7}{20}$. The sign stays attached, so $-0.125 = -\frac{125}{1000} = -\frac{1}{8}$.

A fraction in lowest terms has a terminating decimal exactly when its denominator has no prime factors except 2 and 5. Powers of 10 contain only those prime factors. For example, $\frac{7}{40}$ terminates because $40 = 2^3 \cdot 5$, whereas $\frac{7}{12}$ repeats because its reduced denominator contains a factor of 3. Reduce before applying the test: $\frac{3}{15} = \frac{1}{5}$ terminates even though the original denominator contains 3. To convert a repeating decimal exactly, use the subtraction method in the rational-number lesson. A rounded display such as 0.333 is different from the exact repeating decimal $0.\overline{3}$.

Percent means hundredths

The symbol % means "per hundred." Therefore $23\% = \frac{23}{100} = 0.23$. Converting a percent to a decimal divides its numerical label by 100. Converting a decimal to a percent multiplies by 100 and attaches the symbol. Thus $0.08 = 8\%$, while $0.008 = 0.8\%$. A percent below 1 is possible. In particular, $0.5\% = 0.005 = \frac{1}{200}$.

Percentages can exceed 100. The percent 160% equals 1.6, or $\frac{8}{5}$, which is one and three fifths of the whole. A negative percent can represent a signed change, although a negative fraction of people would have no meaning in a counting context. Interpret the quantity as well as its notation.

To turn a fraction into a percent, either divide first or create a denominator of 100 when convenient. Since $\frac{7}{20} = \frac{35}{100}$, it equals 35%. For $\frac{5}{6}$, division gives $0.8\overline{3}$, so its exact percent is $83\frac{1}{3}\%$. If the question asks for the nearest whole percent, use 83%. Labeling an approximation as exact would lose the repeating part.

A percent containing a fraction is still divided by 100 as a whole quantity. For example, $12\frac{1}{2}\% = \frac{25}{2} \cdot \frac{1}{100} = \frac{1}{8}$. Reading it as $12 + \frac{1}{200}$ would treat the whole-number part as a number rather than part of the percent.

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Choosing a useful form

For a calculation involving thirds or sixths, an exact fraction can avoid rounding. For comparisons with a decimal, convert every value to a common form. For $\frac{2}{5}$, 0.38, and 41%, the decimal values are 0.40, 0.38, and 0.41. Their sizes can now be compared directly. When a question requests a particular form or rounding precision, finish in that form even if a different representation made the calculation easier.

PSAT Math Style Question

A survey found that 37.5% of the respondents preferred a certain design. Which fraction of the respondents preferred that design?

A. $\frac{3}{80}$

B. $\frac{3}{8}$

C. $\frac{15}{4}$

D. $\frac{37}{50}$

Divide the percent label by 100 to get 0.375. Writing this decimal over 1,000 gives $\frac{375}{1000}$. Divide numerator and denominator by 125 to obtain $\frac{3}{8}$, choice B. Since the percentage is between 25% and 50%, the answer should lie between $\frac{1}{4}$ and $\frac{1}{2}$.

PSAT Math Style Question

Three samples contain the same chemical at concentrations of $\frac{7}{20}$, 0.36, and 34.5%, respectively. Which lists these concentrations in increasing order?

A. $\frac{7}{20}$, 34.5%, 0.36

B. 0.36, $\frac{7}{20}$, 34.5%

C. 34.5%, 0.36, $\frac{7}{20}$

D. 34.5%, $\frac{7}{20}$, 0.36

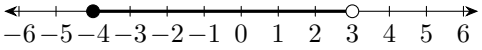
Write all three concentrations as decimals. The fraction $\frac{7}{20} = 0.350$, the given decimal is 0.360, and $34.5\% = 0.345$. Increasing order is 0.345, 0.350, 0.360, so the original forms appear as 34.5%, $\frac{7}{20}$, 0.36, choice D. The percent's number 34.5 cannot be compared directly with 0.36 until the percent sign is interpreted.

Chapter 1 Practice

Answers: page 257.

Number systems, ordering, and interval notation

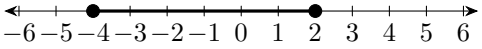
1. The marked interval shows the allowable values of x. How many integer values in the interval also satisfy $x^2 < 10$?



2. A storage unit must remain at least $-3°$C and below $4°$C. A sensor reports only integer temperatures. How many different reported temperatures meet the requirement?
3. A real number r satisfies $r^2 = 31$ and $r < 0$. Between which two consecutive integers does r lie?
4. Which interval describes all real numbers that are greater than -2 and at most 5, but are not in $[1, 3)$?
5. A machine accepts a measurement x only when $2 \le x < \sqrt{31}$. What is the greatest accepted integer?
6. A point has coordinate $p = -\frac{3}{2}$. A second point is $\frac{7}{4}$ units to its right. What is the second point's coordinate?

Signed numbers and order of operations

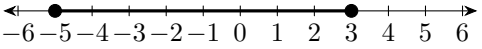
7. A robot starts at the left marked point, moves to the right marked point, then travels twice that distance to the left. What is its final coordinate?



8. An account starts at $-\$18$. Three deposits of \$14 and two withdrawals of \$9 are made. What is the final balance?
9. A score starts at 12. Each correct answer adds 4 points and each incorrect answer subtracts 3 points. After 5 correct and 7 incorrect answers, what is the score?
10. A student evaluates $36 \div (-3) \times (-2)$ as 6 by first multiplying the last two numbers. What is the correct value?
11. For $t = -4$, what is the difference $(-t)^2 - (-t^2)$?
12. A formula gives $S = \frac{a-b}{c+1} - d^2$. What is S when $a = -6$, $b = 10$, $c = 3$, and $d = -2$?
13. A model gives $h(t) = -2t^2 + 8t + 3$. What is $h(-1) - h(2)$?

Absolute value and distance on a number line

14. Two stations are at the marked positions on a straight road. A vehicle travels from the left station to the right station and returns halfway. How far is its final position from the left station?



15. A freezer's temperature changes from $-12°$C to $-5°$C. The cost model is $C = 3|T_2 - T_1| + 8$. What is C for this change?
16. A point x is 6 units from -2 on a number line and lies to the right of 1. What is x?
17. A measurement m is accepted when $|m - 8| \le 2.5$. What is the length of the interval of accepted measurements?
18. Which compound inequality describes the measurements x that are within 3 units of 2 and also at least 1?
19. An alarm activates when $|T - 20| > 4$. What is the least whole-number temperature above 20 that activates it?
20. For what value of k does $|x - 4| = k - 2$ have exactly one real solution?

Rational and irrational numbers: closure and counterexamples

21. The side length of a square is $\sqrt{18}$ meters. Is its area rational or irrational?
22. The numbers $a = \sqrt{12}$ and $b = \sqrt{3}$ are both irrational. What is the exact value of $\frac{a}{b}$?
23. For nonzero rational r, can $r + \sqrt{5}$ be rational? State yes or no.

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24. Let $a = \sqrt{3}$ and $b = 5 - \sqrt{3}$. What is $a + b$?
25. Give two irrational numbers whose sum is irrational.
26. Explain why multiplying an irrational number by 0 is a counterexample to the claim that every rational times an irrational is irrational.
27. Is $4 + \sqrt{5}$ rational or irrational? Explain.
28. An exact real number has decimal expansion $0.121221222122221\ldots$, with one additional 2 between successive 1s each time. Is it rational or irrational?

Equivalent fractions and mixed numbers

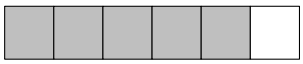
29. The bar represents one whole and is divided into equal parts. A tank is filled to the shaded fraction of its capacity. If it currently holds 21 liters, what is its capacity?



30. A recipe uses $\frac{9}{14}$ cup of oil per batch. A kitchen measures oil in seventieths of a cup. How many seventieths are needed for 3 batches?
31. A roll contains $5\frac{4}{7}$ meters of ribbon. After a $\frac{6}{7}$-meter piece is cut, how many complete $\frac{1}{7}$-meter pieces remain available?
32. A container holds $\frac{53}{12}$ liters. What is the least whole-number capacity, in liters, of a second container that can hold all of it?
33. A point has coordinate $-3\frac{2}{5}$. A second point is $\frac{8}{5}$ units to its right. What is the second coordinate?
34. Containers A and B each have capacity 60 liters. A is $\frac{7}{12}$ full and B is $\frac{3}{5}$ full. How many more liters does B contain than A?
35. Place $-\frac{5}{6}$, $-\frac{3}{4}$, and $-\frac{2}{3}$ in increasing order.
36. A container holds $\frac{15}{4}$ liters. How many whole liters and what remaining fraction of a liter does it hold?

Adding and subtracting fractions

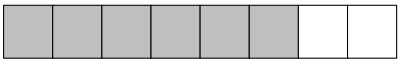
37. The shaded part of this bar is the filled portion of a tank. Water equal to $\frac{3}{8}$ of the tank's total capacity is removed. What fraction of its capacity remains filled?



38. A route is $\frac{5}{6}$ mile long. A cyclist covers $\frac{1}{4}$ mile, then $\frac{1}{3}$ mile. How much of the route remains?
39. Two machines complete $\frac{7}{15}$ and $\frac{1}{6}$ of the same order in one hour, without overlapping work. What fraction of the order remains after that hour?
40. A diver starts $\frac{3}{5}$ meter below a reference level, rises $\frac{7}{10}$ meter, then descends $\frac{1}{4}$ meter. What is the final signed position relative to the level?
41. A sensor reading changes from $-\frac{5}{6}$ to $\frac{2}{9}$. What is the signed change, final minus initial?
42. A $3\frac{1}{4}$-meter cable is shortened by $1\frac{5}{6}$ meters and then extended by $\frac{2}{3}$ meter. What is its final length?
43. Simplify $\frac{2x+1}{3} - \frac{x-2}{4}$.
44. A tank is $\frac{7}{10}$ full. Water equal to $\frac{1}{4}$ of the tank's total capacity is removed. What fraction of the tank's capacity remains filled?

Multiplying and dividing fractions

45. The whole bar represents 1 liter and is divided into equal parts. The shaded liquid is used in batches of $\frac{3}{16}$ liter. How many complete batches can be made?



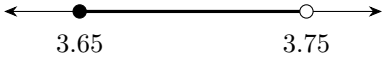
46. A water sample has mass $\frac{21}{10}$ kilogram. A test uses $\frac{4}{7}$ of the sample. How many kilograms are left?
47. A recipe needs $1\frac{5}{7}$ cups of flour per batch. How many cups are needed for $2\frac{1}{3}$ batches?
48. A pump delivers $\frac{14}{15}$ liter per minute. How many minutes does it take to deliver $\frac{7}{9}$ liter at this constant rate?
49. A liquid costs $\frac{9}{8}$ dollar per liter. A customer pays $\frac{3}{4}$ dollar. How many liters are purchased?
50. A tank receives $\frac{2}{3}$ liter and then loses $\frac{1}{6}$ liter. The net amount equals $1\frac{1}{4}$ measuring-cupfuls. What is each cup's capacity in liters?
51. A board is $5\frac{1}{2}$ feet long. Each shelf requires $\frac{3}{4}$ foot. Ignoring cutting loss, how many complete shelves can be made, and how much board remains?
52. A container is $\frac{4}{5}$ full. Then $\frac{3}{8}$ of the water currently in it is poured out. What fraction of the container's capacity remains filled?

Decimals, place value, rounding, and estimation

53. A mass recorded as 12.074 kilograms is written as $12 + \frac{a}{10} + \frac{b}{100} + \frac{c}{1000}$, where a, b, c are digits. What is $10a + b + c$?
54. Three material samples have masses 0.809, 0.89, and 0.8 kilogram. What is the difference between the greatest and least masses?
55. A 5.06-meter board is cut into a 2.879-meter piece and a 1.48-meter piece. Ignoring cutting loss, how many meters remain?
56. A material has density 1.2 grams per cubic centimeter. A sample has volume 0.045 cubic centimeter. How many such samples have a total mass of 0.27 gram?

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57. The marked interval represents all positive lengths that round to k to the nearest tenth, using the usual school rounding rule. What is k?



58. Round 9.995 to the nearest hundredth using the usual school rounding convention.
59. A positive measurement rounds to 4.2 to the nearest tenth. State the interval of possible original measurements.
60. Four identical items cost \$3.48 each. What is their total cost, rounded to the nearest dollar? Calculate before rounding.

Converting fractions, decimals, and percents

61. Of 320 submitted forms, $\frac{7}{16}$ were accepted. What percentage of the forms were not accepted?
62. A solution contains 0.072 gram of a substance in every gram of solution. How many grams of the substance are in $\frac{125}{9}$ grams of solution?
63. A club has 96 members, and 0.625 of them attended a meeting. How many additional members would have to attend for attendance to reach 75%?
64. A 2.4% fee is charged on a \$750 transaction. The fee is represented by $750f$. What is f, expressed as a fraction in lowest terms?
65. An output is 175% of its previous value. If the previous value was 48, what is the new output?
66. A test has 24 questions of equal weight. A student answers 14 correctly. What is the score as a percentage, rounded to the nearest tenth?
67. Which of $\frac{9}{24}$ and $\frac{5}{14}$ has a terminating decimal? Explain using reduced denominators.
68. Write $6\frac{1}{4}\%$ as a fraction in lowest terms.

Mixed Review

69. A measurement x satisfies $-2.5 < x \le \frac{7}{2}$ and $x^2 \ge 4$. How many integer values are possible?

A. 2 **B.** 3

C. 4 **D.** 6

70. A container is $\frac{5}{8}$ full. After 9 liters are removed, it is $\frac{1}{4}$ full. What is its capacity, in liters?

A. 15 **B.** 24

C. 36 **D.** 72

71. A balance is $-\$24$. Four equal deposits and a withdrawal of \$18 leave a balance of \$46. How large is each deposit?
72. The table shows the fractions of a batch used by three teams. What fraction of the batch remains?

Team	Fraction used
A	$\frac{3}{8}$
B	$\frac{1}{3}$
C	$\frac{1}{6}$

73. A positive mass rounds to 8.4 grams to the nearest tenth, with halfway values rounded up. What interval contains exactly its possible original values?

A. $[8.3, 8.5)$ **B.** $(8.35, 8.45]$

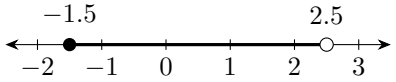
C. $[8.35, 8.45)$ **D.** $[8.35, 8.45]$

74. A runner completes $2\frac{2}{5}$ miles in $\frac{3}{5}$ hour at a constant speed. How many miles does the runner complete in $\frac{3}{4}$ hour?
75. Which is greatest: $\frac{7}{12}$, 0.58, or 58.2%? Justify your comparison.
76. A positive integer n satisfies $\sqrt{50} < n < \sqrt{90}$. What is the greatest possible value of $n^2 - 1$?

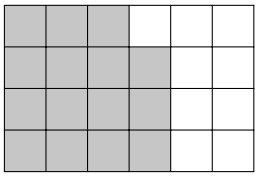
A. 48 **B.** 63

C. 80 **D.** 99

77. The marked interval contains the possible real values of x. What is the greatest integer value that $2x$ can have?



78. The tank diagram contains equal-area cells. Each cell represents $\frac{3}{8}$ liter, and shaded cells contain liquid. How many more liters are needed to fill the tank?



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2. Expressions and Linear Equations

Translating verbal relationships and defining variables

A variable is a letter that stands for a number. If n counts notebooks, a price of 3 dollars per notebook becomes $3n$ dollars. Writing a variable definition first keeps its meaning fixed. The expression $3n$ represents a cost, while the equation $3n = 18$ states that the cost equals 18 dollars.

Reading relationships in their stated order

"Five less than twice a number" means start with twice the number, $2x$, and subtract 5: $2x - 5$. "Five minus twice a number" instead gives $5 - 2x$. Try $x = 4$ if the word order feels uncertain. The first gives 3, the second -3. In "the quotient of a number and 6," the number is the dividend, giving $\frac{x}{6}$. Multiplication need not appear with a multiplication sign: $6x$ means $6 \cdot x$.

A total combines contributions. Three adult tickets at a dollars each and two child tickets at c dollars each cost $3a + 2c$. A difference depends on which amount is subtracted from which. If Mia has 7 more points than Leo, define Leo's points as p and write Mia's as $p + 7$. This relationship does not say Mia has seven times as many.

From a sentence to an equation

An equation connects two descriptions of the same quantity. A delivery charge of 8 dollars plus 2 dollars per mile gives $C = 8 + 2m$. The fixed fee is paid once. The per-mile charge is multiplied by the number of miles. State $m \geq 0$ in this context, and restrict it to whole numbers only if the situation requires whole miles.

For consecutive integers, define the first as n. The next two are $n + 1$ and $n + 2$. Consecutive even integers differ by 2, so they are $2k$, $2k + 2$, and $2k + 4$ for integer k. A phrase such as "at least 12" includes 12 and becomes ≥ 12. "Fewer than 12" excludes 12 and becomes < 12.

Before solving a model, check its units and test an easy value. In $C = 8 + 2m$, zero miles gives the fixed fee of 8 dollars. This checks the placement of the constant. A reasonable equation must also represent the requested quantity, not another quantity with similar numbers.

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PSAT Math Style Question

A gym charges a one-time registration fee of r dollars and 12 dollars per visit. Which expression gives the cost of v visits?

A. $12r + v$

B. $r + 12v$

C. $12(r + v)$

D. $rv + 12$

The registration fee contributes r once. Each of the v visits contributes 12 dollars, giving $12v$. Adding the two costs yields $r + 12v$, choice B. At zero visits, the expression returns the registration fee.

PSAT Math Style Question

The sum of three consecutive odd integers is 63. If n is the smallest integer, which equation represents this information?

A. $3n = 63$

B. $n + (n + 1) + (n + 2) = 63$

C. $n + (n + 2) + (n + 4) = 63$

D. $n + (n + 3) + (n + 6) = 63$

Consecutive odd integers are two apart. Starting at n, the next are $n + 2$ and $n + 4$. Their sum is 63, giving choice C. Solving would give $3n + 6 = 63$ and $n = 19$, whose next odd integers are 21 and 23.

Use estimates within their limits

A random sample of registered borrowers supports an estimate about registered borrowers. It does not automatically describe all town residents. A voluntary website poll may overrepresent visitors with a particular interest, even if it has many responses. Increasing its size does not repair that selection problem.

Different random samples can give different proportions. If two samples estimate 34% and 37%, their estimates for a population of 2,000 differ by $0.03(2000) = 60$. That difference reflects the two sample results; it does not by itself show that the population changed. Keep unrounded fractions during the calculation and round an estimated count only as directed.

PSAT Math Style Question

A random sample of 160 of a town's 6,400 households includes 56 households that use rain barrels. Based on this sample, which is the best estimate of the number of town households that use rain barrels?

A. 35

B. 56

C. $2,240$

D. $4,160$

The sample proportion is $\frac{56}{160} = 0.35$. Apply that proportion to the town's 6,400 households: $0.35(6400) = 2,240$, choice C. The 56 is the observed sample count, while 4,160 estimates households without rain barrels. A random sample supports this population estimate but does not make it an exact count.

PSAT Math Style Question

Of 240 randomly selected students at a school, 72 belong to a music group. Of those 72 students, 48 play a string instrument. The school has 1,800 students. Based on the sample, what is the best estimate of the number of students at the school who belong to a music group and play a string instrument?

A. 360

B. 540

C. $1,200$

D. $1,440$

The requested group contains 48 of the full 240-student sample, so its sample proportion is $\frac{48}{240} = 0.2$. The estimate is $0.2(1800) = 360$, choice A. Using $\frac{72}{240}$ would count every music-group member. Using $\frac{48}{72}$ with the entire school population would incorrectly treat all students as music-group members.

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Comparing and interpreting sample means

A random sample's mean can estimate a population's mean. If a random sample of 40 library visits lasts an average of 32 minutes, 32 minutes is an estimate of the mean duration of all visits in the population represented by that sample. It is not a claim that every visit lasts 32 minutes, and another random sample may have a different mean.

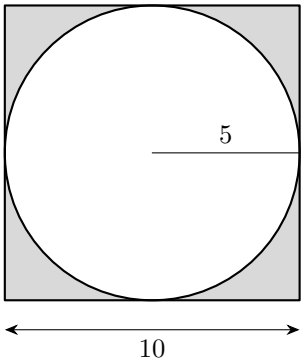
Estimate a total from a sample mean

A building manager randomly selects 50 of 1,200 apartments and records their monthly water use. The sample mean is 180 gallons per apartment. Applying that mean to all apartments gives an estimated total of $1200(180) = 216,000$ gallons. Multiplying by 50 instead gives the observed sample's total, $9,000$ gallons, which is a different quantity.

Group	Apartments	Water use
Random sample	50	$50(180) = 9,000$ gallons
Entire building	1,200	estimated $1,200(180) = 216,000$ gallons

The mean has units of gallons per apartment; multiplying by apartments produces gallons. This unit check helps distinguish a total from an average.

Compare areas with a common reference whole



A disk of radius 5 fits exactly inside a square of side 10. The square's area is 100 and the disk's area is 25π. The fraction of the square covered by the disk is $\frac{25\pi}{100} = \frac{\pi}{4}$. The shaded fraction outside the disk is $1 - \frac{\pi}{4}$. The denominator stays the square's area because the question compares both regions with that square.

If the question instead asks for the ratio of the disk's area to the shaded area, use $\frac{25\pi}{100-25\pi} = \frac{\pi}{4-\pi}$. This is a different comparison from the fraction of the whole square covered by the disk.

Recover a length from an area comparison

Two circles have areas in the ratio $9 : 25$. Their radii have ratio $3 : 5$, because area squares the radius. If the larger radius is 20, the smaller radius is $20(\frac{3}{5}) = 12$. Taking the square root recovers the length comparison; multiplying a radius directly by $\frac{9}{25}$ would apply an area ratio to a length.

For a ring, subtract the inner disk from the outer disk. If the ring's area is 45π and the outer radius is 9, then $81\pi - \pi r^2 = 45\pi$. Dividing by π gives $r^2 = 36$, so the inner radius is 6. The ring's width is $9 - 6 = 3$. It is not $\sqrt{45}$: the area is a difference of squared radii, not the square of their difference.

Keep exact values such as $\frac{\pi}{4}$ until a decimal or percentage is requested. A fraction of a whole must lie from 0 to 1; a ratio comparing two separate regions may exceed 1. That distinction provides a useful check before choosing an answer.

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PSAT Math Style Question

Two concentric circles enclose a ring of area 48π square centimeters. The larger circle has radius 8 centimeters. What is the radius, in centimeters, of the smaller circle?

A. 2

B. 4

C. $4\sqrt{3}$

D. 16

The larger disk has area 64π. Subtract the ring to find the smaller disk's area: $64\pi - 48\pi = 16\pi$. Thus $r^2 = 16$ and the positive radius is 4 centimeters, choice B. Taking the square root of 48 would use the ring's area as though it were a complete disk.

PSAT Math Style Question

A circular disk fits exactly inside a square, touching all four sides. The disk has area 49π square centimeters. What is the area, in square centimeters, of the part of the square outside the disk?

A. 49π

B. $196 - 49\pi$

C. $196 - 7\pi$

D. 196

The disk's radius is 7 centimeters because $r^2 = 49$. Its diameter, 14 centimeters, equals the square's side length. The square has area $14^2 = 196$, so the area outside the disk is $196 - 49\pi$, choice B. Using 7 as the square's side would confuse the radius with the diameter.

Inverse volume and capacity problems

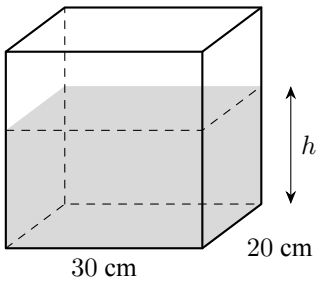
A volume can be given while a height, radius, or amount of liquid is unknown. Write the volume formula first, substitute the known quantities, and solve for the requested dimension. Keep the units visible: dividing cubic centimeters by square centimeters leaves centimeters, which is the unit of a height.

Recover the missing dimension

A rectangular tank has volume 2,400 cubic centimeters and a base 20 centimeters long by 15 centimeters wide. The base area is $20(15) = 300$ square centimeters. From $2400 = 300h$, the height is 8 centimeters. Dividing by $20 + 15$ would use a sum of lengths instead of the area of a layer.

For a cylinder, $V = \pi r^2 h$. If $V = 200\pi$ and $h = 8$, then $200\pi = 8\pi r^2$. Dividing by 8π gives $r^2 = 25$, so $r = 5$. The requested radius requires a square root. The diameter would be 10, which answers a different question.

Connect liquid volume to fill height



A rectangular tank with a 30-by-20-centimeter base contains 7.2 liters of water. Since 1 liter is 1,000 cubic centimeters, the water volume is 7,200 cubic centimeters. Its depth satisfies $7200 = 30(20)h$, giving $h = 12$ centimeters. The full tank height is unnecessary if there is enough capacity for that depth.

The fill height is proportional to volume when a tank has a constant horizontal cross-sectional area. This holds for a rectangular tank and an upright cylinder. It does not hold for a cone, whose cross-sections change with height. Half a cone's full height does not contain half its full volume.

Transfer the same volume between containers

If no liquid is lost, the volume removed from one container equals the volume added to the other. A cylinder of radius 3 filled to height 8 contains 72π cubic centimeters. Pouring that liquid into a cylinder of radius 6 gives $72\pi = \pi(6)^2 h$, so $h = 2$ centimeters. The receiving base has four times the area, so the same liquid reaches one fourth the height.

A cone or pyramid uses $V = \frac{1}{3}Bh$ when B is the full base area and h its perpendicular height. With volume 90 and base area 30, $90 = \frac{1}{3}(30)h = 10h$, so $h = 9$. Multiplying by 3 before dividing by the base area reverses that formula correctly. Slant height cannot replace perpendicular height.

Before reporting an answer, check capacity and what the question asks for. A calculated depth greater than a container's full height means the liquid will overflow. A question asking for additional liquid requires full capacity minus the amount already present, rather than the full capacity alone.

PSAT Math Style Question

A rectangular tank has internal base dimensions 40 centimeters by 30 centimeters and internal height 25 centimeters. It contains 18 liters of water. How many additional liters of water are needed to fill the tank? Use 1 liter $= 1000$ cubic centimeters.

A. 7

B. 12

C. 18

D. 30

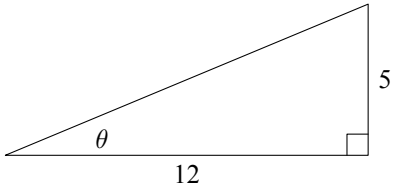
The tank's full volume is $40(30)(25) = 30,000$ cubic centimeters, or 30 liters. Subtract the 18 liters already present: $30 - 18 = 12$ additional liters, choice B. Reporting 30 would give full capacity rather than the additional amount requested.

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8. An observer stands on level ground 30 meters horizontally from the base of a vertical tower. The observer's eyes are 1.5 meters above the ground. The angle of elevation to the top is $45°$. Find the tower's height.
9. A roof rises 3 feet over a horizontal run of 4 feet. Express its slope angle using an inverse trig function.
10. From a 16-meter cliff, a boat is seen at a $30°$ depression angle. Neglect eye height. Find its horizontal distance from the cliff.
11. A cable has a 5-meter vertical rise and a 12-meter horizontal run. Find its length.
12. A ramp rises 2 meters with a $20°$ ground angle. Write expressions for its length and horizontal run.

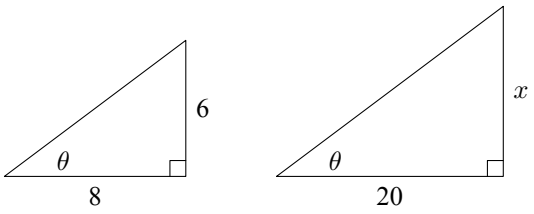
Mixed Review

13. The right triangle has the shown side lengths. What is $\sin\theta + \cos\theta$?



Not to scale

14. An acute angle satisfies $\tan\alpha = \frac{3}{4}$. What is $\cos\alpha$?
15. A 10-meter cable makes a $30°$ angle with horizontal ground. What is its vertical rise?
16. Right triangle ABC has $\angle C = 90°$, $AC = 12$, and $BC = 5$. By how much does $\sin B$ exceed $\sin A$?
17. A ramp rises 1.5 meters over a horizontal run of 8 meters. Write its angle to the horizontal using an inverse trigonometric function.
18. From level ground 24 meters from a vertical tower, the angle of elevation to its top is $45°$. The observer's eyes are 1.6 meters high. What is the tower's height?
19. The two right triangles share the same acute angle θ. What is the value of x?



Not to scale

20. Two marks lie on the same vertical line on a wall. From a point on level ground 18 meters horizontally from the foot of that line, the angles of elevation to the marks are $30°$ and $60°$. How far apart vertically are the marks?
21. For acute θ, $\sin\theta = \frac{5}{13}$. What is $\tan\theta$?

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17. Digital Tools and Test-Day Decisions

Reading stems, quantifiers, units, and requested forms

A calculation answers a test question only when it produces the quantity requested. A stem may ask for an integer, an expression, a percent increase, or the value of a different variable from the one you first find. The algebra is familiar; the reading determines which algebra to use. Fractions, inequalities, and unit conversion supply the mathematical tools in this topic.

Translate each restriction into mathematics

"At least 8" means $x \geq 8$, while "more than 8" means $x > 8$. "At most" includes its endpoint; "fewer than" excludes it. If x counts people, it must also be a whole number. An inequality such as $x \leq 7.6$ therefore permits at most 7 people. Keep the original direction of the condition when deciding whether to round up or down.

"Which could be" asks for one allowed possibility. "Which must be" asks for a claim true for every allowed case. To reject a must-be claim, one valid counterexample is enough. For example, if n is even, $n + 1$ must be odd, but n need not be positive: -2 is an allowed even integer.

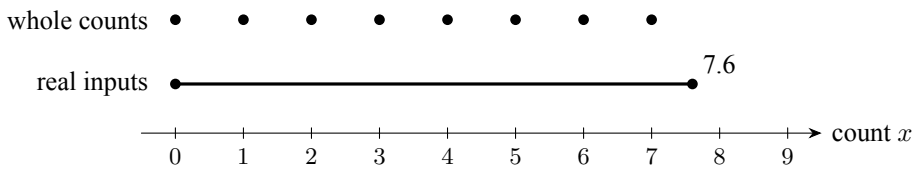
Check the requested form and unit

If $3x + 2 = 17$, subtracting 2 gives $3x = 15$ and dividing by 3 gives $x = 5$. When the question asks for $2x$, that 5 is an intermediate result. Substitute into the requested expression: $2x = 2(5) = 10$. Similarly, a ratio of $\frac{3}{5}$ can mean 60%, but a 60% increase means the new amount is 160% of the old amount. Write the reference quantity in a percent calculation: increase divided by original amount.

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Units can change the numerical answer even when the calculation is otherwise sound. A speed of 12 feet per second is $12(60) = 720$ feet per minute. If a diagram supplies a diameter but a formula uses radius, halve the supplied length before squaring. For areas, convert both dimensions or square the linear conversion factor. One square yard contains $3 \times 3 = 9$ square feet.

Before choosing an answer, reread the final sentence and check the quantity, unit, restrictions, and form. This check should compare your result with the actual stem; it should not substitute a guess about what a problem usually asks.



The lower marked segment shows $0 \le x \le 7.6$ for real inputs. If x is a count, only the whole-number dots are permitted, and the greatest is 7.

PSAT Math Style Question

A club charges a 9-dollar fee plus 4 dollars per visit. A member can spend at most 40 dollars. What is the greatest number of whole visits the member can afford?

A. 7

B. 8

C. 9

D. 10

If v is the number of visits, $9 + 4v \le 40$. Subtracting 9 and dividing by 4 gives $v \le 7.75$. Visits are whole numbers, so 7 is the greatest allowed count, choice A. Eight visits would cost 41 dollars, exceeding the budget.

Using diagrams and calculator results responsibly

Use a diagram's labels and geometric marks to establish measurements and relationships. Use a calculator to carry out the expression those relationships require. Apparent size can be misleading, and a calculator will faithfully evaluate an incorrectly entered expression. Reading labels, keeping units, and estimating size connect these tools with the mathematics.

Use stated geometry, not visual appearance

A right-angle mark establishes a 90-degree angle. A pair of matching tick marks establishes equal lengths. A segment that merely looks vertical or bisected does not establish either fact. If a rectangle has side labels 5 and 12, its diagonal is $\sqrt{5^2 + 12^2} = 13$, even if the illustration looks nearly square. Where a graph is the source of data, use its numbered axis scale and the precision indicated in the question rather than assuming each grid step represents 1.

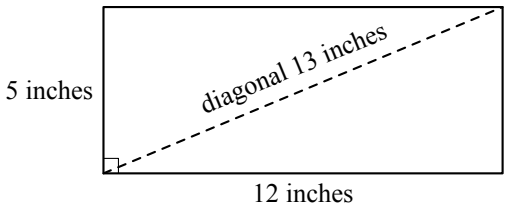
Redraw a crowded figure and label known lengths, angles, or coordinates. Preserve relationships rather than artistic proportion. On a coordinate plane, horizontal change and vertical change can give a distance through the Pythagorean theorem. A point's height alone is not its distance from the origin.

Enter the entire intended expression

Parentheses keep a numerator or denominator together. For $\frac{8+4}{2+1}$, evaluate $(8 + 4) \div (2 + 1)$ to obtain 4. Entering $8 + 4 \div 2 + 1$ instead gives 11 by order of operations. For negative bases, $(-3)^2 = 9$ while $-3^2 = -9$ under the usual algebraic convention. Check how the displayed expression groups the negative sign.

Angle mode must match the input: $\sin 30^\circ = \frac{1}{2}$, whereas $\sin 30$ in radian mode is a different value. Keep unrounded intermediate values when a final answer requires rounding. For $\sqrt{50}$, the exact form is $5\sqrt{2}$; a decimal approximation is useful only if that is the requested form.

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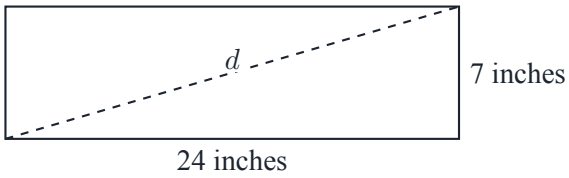


Estimate before accepting a display. Multiplying 0.19 by 48 should give a value near $0.2(50) = 10$, so 912 indicates a decimal error. Substitute an algebraic result into the original relationship whenever practical. Agreement checks the equation you actually needed to solve, including its restrictions.

The diagonal joins opposite corners, so it is the hypotenuse of the marked right triangle. The side labels determine its length through $5^2 + 12^2 = 13^2$.

PSAT Math Style Question

A rectangular sign is 7 inches wide and 24 inches long. What is the length, in inches, of a diagonal of the sign?



A. 17

B. 25

C. 31

D. 168

The diagonal is the hypotenuse of a right triangle whose legs are the rectangle's width and length. The Pythagorean theorem gives $d = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$ inches, choice B. Adding the side lengths gives the distance along two edges rather than the diagonal.

Choosing algebra, substitution, estimation, or answer testing

Several valid methods may solve the same question. The efficient choice depends on the structure: a simple equation may be easiest to rearrange, while supplied numerical choices may be easiest to test. Algebra, substitution, and estimation remain mathematical methods, each with conditions that keep its conclusion valid.

Choose a method that preserves the question

For $4x+7 = 31$, subtracting and dividing gives $x = 6$ directly. Testing four choices would repeat more work. For a complicated expression asked at $x = 2$, substitution may avoid an unnecessary symbolic expansion. Substitute into every occurrence of the variable and retain parentheses around negative values.

To compare candidate expressions, choosing a simple permitted value can eliminate wrong choices. If the expression contains $\frac{1}{x-1}$, then $x = 1$ is not permitted. A single matching value does not prove two expressions equivalent for all inputs: x^2 and x agree at 0 and 1 but differ at 2. After eliminating choices, establish the remaining equivalence algebraically when needed.

Use answer testing and estimation with a check

When the stem asks for an unknown number and supplies numerical candidates, substitute a candidate into the original condition. If total cost is $8 + 3n$, a proposed $n = 9$ gives 35 dollars. For an increasing relationship, a result too large directs you toward smaller candidates. This ordering shortcut requires monotonic behavior on the relevant domain: the output must keep increasing, or keep decreasing, as the input increases. It cannot be assumed for every quadratic or absolute-value expression.

Estimation can reject implausible choices without determining the exact answer. If $\frac{19.8}{4.1}$ is requested to the nearest whole number, it is near $\frac{20}{4} = 5$. More carefully, it lies between 4.5 and 5.5, so it rounds to 5. When choices are close together, exact calculation or tighter bounds are necessary.

Compare methods by the information already supplied, the number of steps, and the ease of checking. Use a sketch for geometry, a table for a short repeated pattern, and equations for relationships. Whatever method you choose, verify its result against the original restrictions and requested quantity.

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PSAT Math Style Question

The expressions $(x+2)(x+k)$ and x^2+7x+m are equivalent for all real x, where k and m are constants. What is the value of m?

A. 5 **B.** 7

C. 9 **D.** 10

Expand the product to get $x^2+(k+2)x+2k$. Equivalent polynomials have matching coefficients, so $k+2=7$ and $k=5$. The constant term is then $m=2k=10$, choice D. Finding k is an intermediate step; the requested value is the constant m.

Using Desmos to solve equations and systems

When two expressions must be equal, graph each as a separate function of x. An intersection has the same output for both expressions, so its x-coordinate solves the equation. For a system in x and y, the full intersection point solves both equations. Read the quantity requested before recording the result.

For $2x+1=x^2-2$, graph $y=2x+1$ and $y=x^2-2$. Select an intersection to read its coordinates. The displayed pairs are $(-1,-1)$ and $(3,7)$, so the intersection inputs are -1 and 3. Algebra confirms this: $x^2-2x-3=(x-3)(x+1)=0$. Adjust the window if an intersection is off screen. A rounded display can suggest a value, but an exact fraction or algebraic check may be needed.

Graphing a system is useful when you want intersection coordinates. It is less useful for a question about an identity for every real x, where two curves can look identical at the current zoom even when they differ.

PSAT Math Style Question

The system is $y=3x+2$ and $y=14-x$. What is the y-coordinate of the intersection of the graphs of these equations?

A. 3 **B.** 8

C. 11 **D.** 14

At an intersection the outputs agree: $3x+2=14-x$. Thus $4x=12$ and $x=3$. Substituting gives $y=3(3)+2=11$, choice C. The input 3 is not the requested coordinate.

Calculator tables, regression, and model checks

A table helps you compare inputs and outputs without repeatedly retyping an expression. Enter the function once, then inspect the outputs at the needed inputs. For a linear model, consecutive output changes divided by input changes should agree. A calculator's regression feature can estimate a line for a scatterplot, but the fitted line is a model rather than an exact rule for every observed point.

For $f(x)=x^2-3x$, enter $f(x)$ and use a table with inputs 0, 1, 2, 3, and 4. The outputs are 0, -2, -2, 0, and 4. A table lets you check a requested input directly. For measured data, put the input values in a column x_1 and the corresponding outputs in y_1. A linear regression uses $y_1 \sim mx_1+b$, with a tilde instead of an equals sign, to estimate the slope and intercept. Practice these controls in the testing calculator before test day.

Read a model's variables and units first. If $p=2.5h+40$ predicts a score from practice hours, the slope is 2.5 score points per hour and the intercept is the predicted score at zero hours. A prediction far beyond the observed input range may be unreliable. A model cannot establish causation by itself.

When checking an equivalent expression, testing two or three values can reject a wrong choice quickly. It cannot prove an identity. Avoid inputs that accidentally hide a difference, such as zero when both expressions contain a factor of x.

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PSAT Math Style Question

The model $C(t) = 100 - 6t$ gives a battery's remaining charge as a percentage of its initial charge, where t is the number of hours since the battery began to discharge. According to the model, what does 6 represent?

A. The remaining charge decreases by 6% of its current value each hour.

B. The remaining charge increases by 6 percentage points each hour.

C. The initial charge is 6% of its starting value.

D. The remaining charge decreases by 6 percentage points each hour.

Increasing t by 1 changes the output by -6, so the remaining charge decreases by 6 percentage points each hour, choice D. For example, the model predicts 100% at $t = 0$ and 94% at $t = 1$. This is a fixed decrease measured relative to the initial charge, not a repeated loss of 6% of the charge currently remaining. The constant 100 represents the initial percentage.

Entering a student-produced response

A student-produced response is a number. Solve for the requested quantity, then choose a supported representation. Enter `3/4` or `.75` for three-fourths. Do not enter an equation, a unit, or a percent sign. If a question asks for a percentage and your result is 25 percent, enter 25, unless the question asks for the decimal form.

Convert mixed numbers to improper fractions or decimals: $2\frac{1}{3}$ becomes $\frac{7}{3}$ or a sufficiently precise decimal. The response field does not accept mixed-number notation. Do not enter `21/3`: it represents 7, rather than two and one-third. For repeating decimals, use as much precision as the official field permits. For $\frac{2}{3}$, `.6666` and `.6667` are acceptable examples. Use the exact fraction when it fits. The official examples also accept `0.666` and `0.667` for $\frac{2}{3}$, but reject `0.66`, `.66`, `0.67`, and `.67`. Check the supported precision as well as the value.

The official field accepts up to five characters for a positive number and six for a negative number, including its sign. Check the current Bluebook directions for accepted formats. The companion applies the five-character positive and six-character negative limits and accepts supported equivalent fractions and decimals. Use official Bluebook practice to become familiar with the exam's own response field.

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PSAT Math Style Question

A solution to the equation $4x + 3 = 10$ is $x = k$. What is the value of k?

Subtract 3 from both sides to obtain $4x = 7$, then divide by 4 to get $x = \frac{7}{4} = 1.75$. Enter either `7/4` or `1.75` in the response field. A mixed-number string such as `1 3/4` is unsuitable because the entry should represent one number using the permitted fraction or decimal format. Substitution checks the value: $4(1.75) + 3 = 10$.

Pacing, marking, and checking within a module

A 35-minute module has 22 questions, leaving about 95 seconds per question on average. That is a planning average, not a time limit on each item. Short questions can create room for a longer one. Check the timer at a few planned points so you can adjust before the final minute.

If a question is taking too long, record your best available answer, mark it for review, and continue. Return while the current module is still open. Once that module ends, you cannot revisit it on the official exam. There is no penalty for an incorrect answer, so leaving an answer blank gains nothing.

When checking, focus on the requested expression, signs, units, and graph scale. Repeating all the arithmetic may miss the same setup error. Substitute a candidate into the original equation when appropriate. If a denominator or square root imposes restrictions, check those before accepting a candidate.

PSAT Math Style Question

A machine must process 22 identical batches within 35 minutes. It has completed 14 batches after 20 minutes. What average time, in seconds per batch, is available for the remaining batches if the machine is to finish within the scheduled time?

A. 60

B. 75

C. 112.5

D. 150

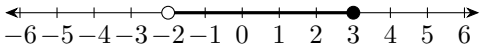
The remaining time is $35-20=15$ minutes, and $22-14=8$ batches remain. Convert 15 minutes to $15(60)=900$ seconds, then divide by 8 to obtain 112.5 seconds per batch, choice C. This average uses the remaining work and remaining time. Dividing the original 35 minutes by all 22 batches would describe the full schedule instead.

Chapter 17 Practice

Answers: page 261.

Reading stems, quantifiers, units, and requested forms

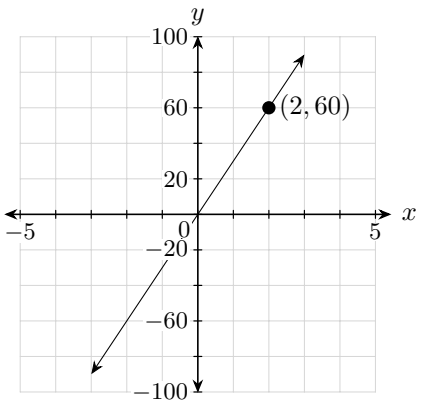
1. If $5x-3=22$, find $3x+1$.
2. A price rises from 40 dollars to 50 dollars. Find the percent increase.
3. The interval marks allowed values of x. How many integer values of $2x$ are possible when x itself must be an integer?



4. Convert 15 feet per second to feet per minute.
5. A circle has diameter 10 inches. Find its area.
6. If x is positive, must $x^2>x$? Explain with a valid value.

Using diagrams and calculator results responsibly

7. Evaluate $\frac{6+9}{2+3}$.
8. Find $(-4)^2$ and -4^2.
9. Find $\sin 30^\circ$ exactly.
10. The graph's vertical scale is marked in units. What is the slope of the line through the origin and $(2,60)$?



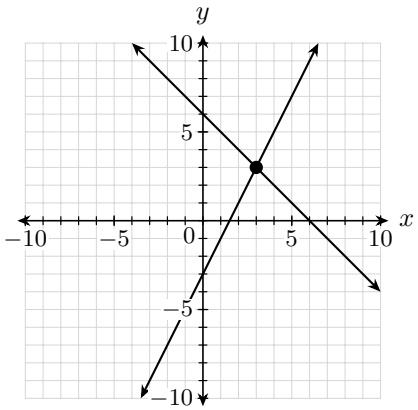
11. Find the distance from $(0,0)$ to $(6,8)$.
12. Estimate $0.31(198)$ before finding its exact value.

Choosing algebra, substitution, estimation, or answer testing

13. Without first finding x, determine $21x+15$ when $7x+5=47$.
14. A rectangle has sides $a+b$ and $a-b$, with $a=13$ and $b=5$. What is its area?
15. Which of $n=4,5,6$ satisfies $9+4n=29$?
16. Use $x=2$ to show that x^2 and $2x$ are not proven identical by one matching value. Find a counterexample.
17. Round $\frac{39.6}{8.1}$ to the nearest integer and justify your answer.
18. A candidate solution of $\frac{x+2}{x-3}=2$ is $x=8$. Check it.

Using Desmos to solve equations and systems

19. The graph shows $y=2x-3$ and $y=-x+6$. What is the x-coordinate of their intersection?



20. Find the intersection points of $y=x^2$ and $y=4$.
21. Why can't a graphing-window screenshot prove two expressions equal for every real input?

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Calculator tables, regression, and model checks

22. The calculator table lists $f(x) = x^2 - 3x$. What is the smallest listed input for which $f(x) > 0$?

x	0	1	2	3	4
$f(x)$	0	-2	-2	0	4

23. A fitted line is $y = 4x + 18$. What output does it predict at $x = 7$?
24. A data set covers ages 12 through 17. Is a model prediction at age 45 an interpolation?

Entering a student-produced response

25. A rectangle has area 15 square units and width 6 units. Enter its length as an exact decimal and as an exact fraction.
26. A question asks what percent of 80 is 20. What number should you enter?
27. Write a fraction for the repeating decimal $0.\overline{3}$.

Pacing, marking, and checking within a module

28. After 24 minutes of a 35-minute module, 6 questions remain unanswered. You reserve 2 minutes for review. What is the average time available, in seconds, per remaining question?
29. Can you return to Module 1 after starting Module 2?
30. When checking a proposed solution of $\frac{1}{x-2} = 3$, which input must remain excluded?

Practice Tests

PART II

Before you begin. Each test contains 44 questions in two modules. Allow 35 minutes for Module 1 and 35 minutes for Module 2. You may review answers within the current module until its time ends. After beginning Module 2, do not return to Module 1. These fixed forms provide timed practice; they do not reproduce adaptive routing or official score calibration.

Shared directions. Choose one answer for each multiple-choice question. Write a numerical response in its blank box without units. A permitted non-CAS calculator may be used throughout. Figures are drawn to scale unless indicated otherwise.

Online practice. All 50 tests are interactive online. Tests 1–10 are also printed here, and Tests 11–50 are the 40 additional online tests. Scan or tap the QR code in a printed test banner to take that same test online.

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Practice Answer Sheet

Test: _____ Date: _____ Calculator: _____ Module 1 time: _____ Module 2 time: _____

Write your choice letter or numerical answer on the line beside each question number.

Module 1

1. _____	9. _____	17. _____
2. _____	10. _____	18. _____
3. _____	11. _____	19. _____
4. _____	12. _____	20. _____
5. _____	13. _____	21. _____
6. _____	14. _____	22. _____
7. _____	15. _____	
8. _____	16. _____	

Module 2

1. _____	9. _____	17. _____
2. _____	10. _____	18. _____
3. _____	11. _____	19. _____
4. _____	12. _____	20. _____
5. _____	13. _____	21. _____
6. _____	14. _____	22. _____
7. _____	15. _____	
8. _____	16. _____	

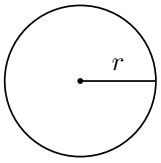
After the test: Module 1: _____ out of 22 Module 2: _____ out of 22 Total: _____ out of 44

Record one skill to review: _____

Math Reference Sheet

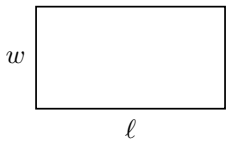
In Bluebook, a reference sheet is available during both Math modules. Keep this page handy during your practice tests.

Circle



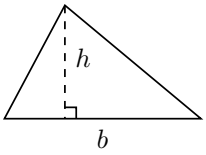
$A = \pi r^2 \quad C = 2\pi r$

Rectangle



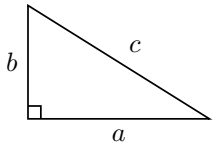
$A = \ell w$

Triangle



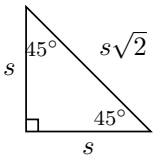
$A = \frac{1}{2}bh$

Right triangle



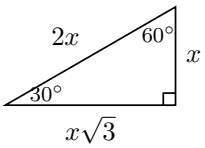
$c^2 = a^2 + b^2$

45°–45°–90° triangle



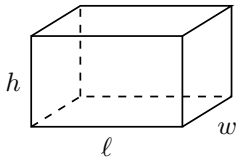
Legs: s, s; hypotenuse: $s\sqrt{2}$

30°–60°–90° triangle



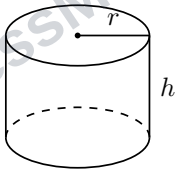
Sides: x, $x\sqrt{3}$, $2x$

Rectangular prism



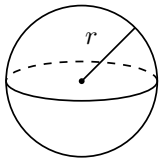
$V = \ell w h$

Cylinder



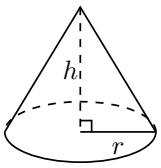
$V = \pi r^2 h$

Sphere



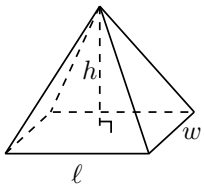
$V = \frac{4}{3}\pi r^3$

Cone



$V = \frac{1}{3}\pi r^2 h$

Rectangular pyramid



$V = \frac{1}{3}\ell w h$

Angle facts

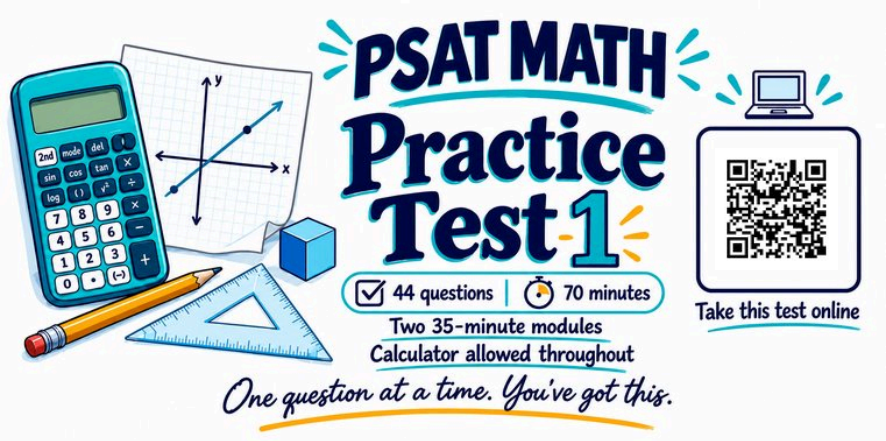
One full circle:
$360° = 2\pi$ radians

Triangle angle sum:
$180°$

A: area; C: circumference; V: volume; r: radius; h: perpendicular height.

Formula coverage follows the College Board PSAT/NMSQT practice reference. Diagrams and layout are original.

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Math | Module 1

22 questions | 35 minutes | Calculator allowed

Answer every question. Work only within this module until its 35-minute timer ends. Use the shared directions and reference sheet.

1

What value of x satisfies $3x = 15$?

2

At an archive, 15 percent of 220 documents have been digitized. How many documents have been digitized?

A) 15

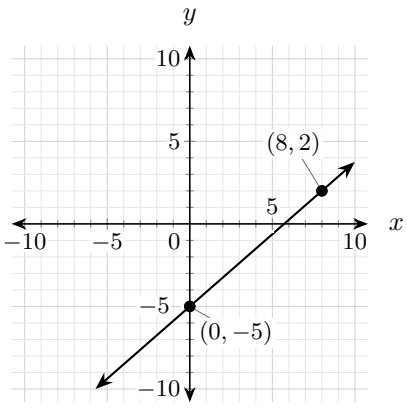
B) 22

C) 33

D) 187

3

The graph shows line r. Line s is perpendicular to line r. What is the slope of line s?



A) $-\frac{8}{7}$

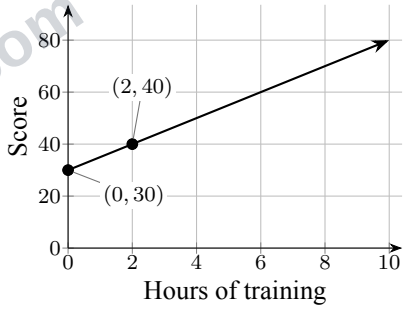
B) $-\frac{7}{8}$

C) $\frac{7}{8}$

D) $\frac{8}{7}$

4

The graph shows a linear model relating an athlete's score to hours of training. According to the model, what score is predicted after 8 hours of training?



5

Measurements of fish are used to fit a model for their length y, in centimeters, at an age of x weeks. The model is $y = \frac{3}{5}x + 7$. What value of y does the model predict when $x = 0$?

A) 0

B) $\frac{3}{5}$

C) 7

D) $\frac{38}{5}$

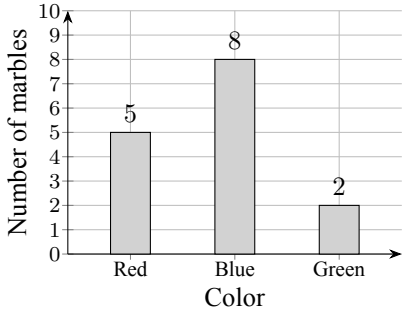
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6

A boat rental charges a fixed fee of \$13 plus \$3 per hour. The total charge is \$25, with no other charges. How many hours were charged?

7

The bar graph shows the number of marbles of each color in a bag. One marble is selected at random. What is the probability that it is blue?



A) $\frac{2}{15}$
B) $\frac{1}{3}$
C) $\frac{7}{15}$
D) $\frac{8}{15}$

8

Which expression is equivalent to $(x+8)(x-9)$?
A) $x^2-17x-72$
B) x^2-x+72
C) x^2+x-72
D) x^2-x-72

9

The system $x+y=21$ and $x-y=15$ has solution (x,y). What is the value of x?
A) 3
B) 18
C) 21
D) 36

10

Which expression is equivalent to $x^2+13x+30$?
A) $(x+3)(x+10)$
B) $(x-10)(x-3)$
C) $(x+1)(x+30)$
D) $(x-3)(x+10)$

11

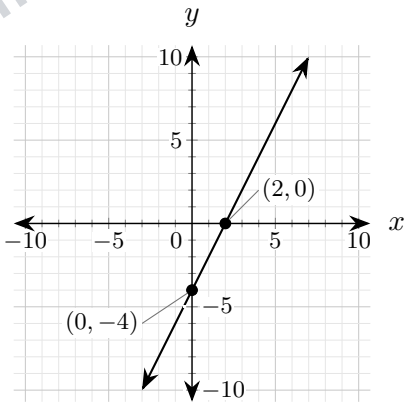
The line $3x+5y=20$ is graphed in the xy-plane. What is the y-coordinate of its y-intercept?
A) -4
B) 4
C) $\frac{20}{3}$
D) 20

12

The function p is defined by $p(x)=9x(5x+4)$. Which nonzero value of x satisfies $p(x)=0$?
A) $-\frac{5}{4}$
B) $-\frac{4}{5}$
C) 0
D) $\frac{4}{5}$

13

The graph of a linear function f is shown. What is the rate of change of f?



A) -4
B) -2
C) $\frac{1}{2}$
D) 2

14

The function f is defined by $f(x)=\sqrt{567x}$. What value of k satisfies $f(k)=63$?
A) $\frac{1}{9}$
B) $\frac{1}{7}$
C) 7
D) 49

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15

The cylindrical container shown has volume $k\pi$ cubic centimeters. What is k?

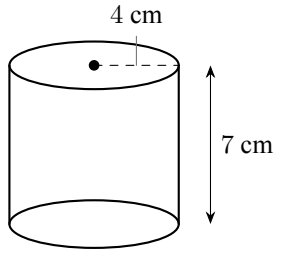


Figure not drawn to scale

A) 28
B) 56
C) 112
D) 448

16

For every real number x, $2^{x+1} + 2^{x+1} = 2^k$, where k depends on x. Which expression equals k?

A) $2x + 1$
B) $2x + 2$
C) $x + 1$
D) $x + 2$

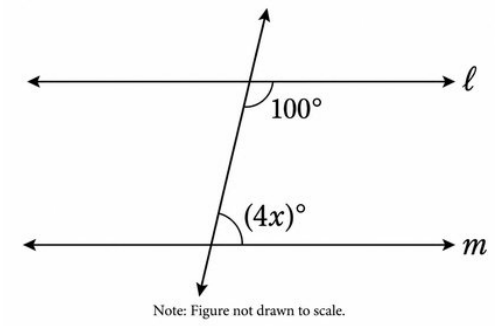
17

The point $(5, 7)$ lies on the line $ax + y = 47$. What is the value of a?

A) 8
B) $\frac{47}{5}$
C) $\frac{54}{5}$
D) 40

18

In the figure, lines ℓ and m are parallel. What is the value of x?



Note: Figure not drawn to scale.

A) 20
B) 25
C) 45
D) 80

19

The system $3x + 2y = 17$ and $x - y = 4$ has solution (x, y). What is y?

A) 1
B) 4
C) 5
D) $\frac{17}{2}$

20

Two similar rectangular prisms have corresponding edge lengths in the ratio $3 : 4$. The smaller prism has volume 54 cubic units. What is the volume, in cubic units, of the larger prism?

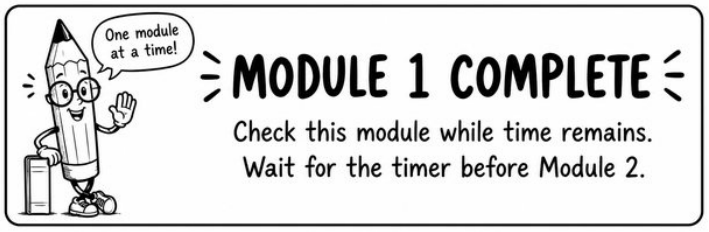
21

The two solutions of $5x^2 - 9x - 3 = 0$ are r and s. What is the value of $(2r + 1)(2s + 1)$?

A) $-\frac{12}{5}$
B) $\frac{6}{5}$
C) $\frac{11}{5}$
D) 7

22

For $x > 16$, what value of x satisfies $x + \frac{352}{x} = 38$?



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Math | Module 2

22 questions | 35 minutes | Calculator allowed
Answer every question. Stop when this module's 35-minute timer ends.

1

How many grams are equivalent to 4 kilograms? (1 kilogram = 1,000 grams)

A) $\frac{1}{250}$
B) 400
C) 4000
D) 40,000

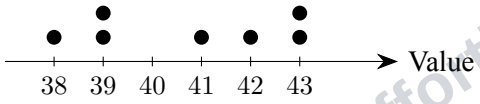
2

The amount of water W, in liters, remaining in a tank t minutes after draining begins is modeled by $W = 316 - 16t$. What does 316 represent?

A) The amount of water, in liters, in the tank when draining begins
B) The amount of water, in liters, remaining after 1 minute
C) The number of minutes needed to empty the tank
D) The amount of water, in liters, removed each minute

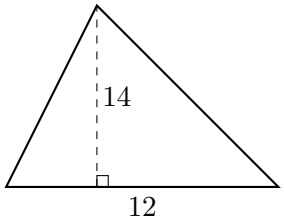
3

The dot plot shows all the values in a data set. What is the median?



4

The triangle shown has base and perpendicular height measured in centimeters. What is its area, in square centimeters?



Note: Figure not drawn to scale.

A) 42
B) 52
C) 84
D) 168

5

A random sample of 40 households in a town uses an average of 18 gallons of water per household for gardening each week. The town has 600 households. Based on the sample, what is the best estimate of the total number of gallons all 600 households in the town use for gardening each week?

6

A culture starts with 70 cells and its cell count is multiplied by 3 every hour. According to this model, how many cells are in the culture after 4 hours?

A) 280
B) 840
C) 1890
D) 5670

7

In right triangle ABC, angle C is a right angle and the side lengths are $BC = 18$, $AC = 24$, and $AB = 30$. What is $\cos B$?

8

What is the sum of the two solutions of $(x - 8)^2 = 81$?

A) -1
B) 16
C) 17
D) 18

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9

A rental company charges a fixed fee plus an hourly fee. The table shows the total charge, in dollars, for renting equipment for t hours.

t	Charge (dollars)
3	57
7	69

According to this relationship, what is the output, in dollars, when $t = 0$?

A) 3
B) 9
C) 48
D) 57

10

In right triangle ABC, angle C is a right angle, $AC = 11$, and $\tan A = 5$. What is the length of BC?

A) $\frac{11}{5}$
B) 16
C) 55
D) $11\sqrt{26}$

11

The function f is defined by $f(x) = (x - 8)^3$. What value of x satisfies $f(x) = 64$?

12

The equation $a(x - 3) + 2x = 7x + b$ is true for every real number x, where a and b are constants. What is b?

A) -21
B) -15
C) 5
D) 15

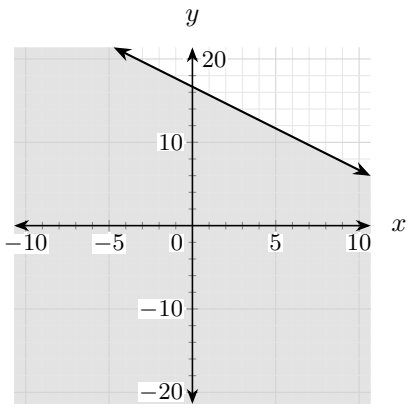
13

Which expression is equivalent to $4x^2 - 100$?

A) $(2x - 10)^2$
B) $(2x + 10)^2$
C) $4(x - 10)(x + 10)$
D) $(2x - 10)(2x + 10)$

14

The shaded region, including its boundary, represents the solutions to an inequality. If x and y are integers in this region and $x = 5$, what is the greatest possible value of y?

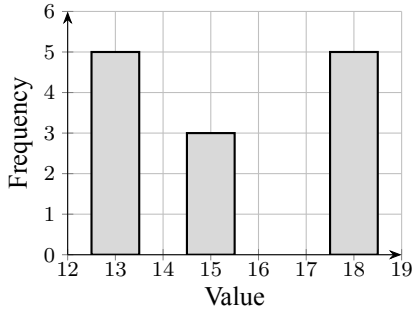


Boundary: $3x + 3y = 50$

A) 10
B) 11
C) 12
D) 16

15

The frequency graph shows a data set of integer values. Each bar is centered at its corresponding value. What is the mean of the data set?



A) 15
B) $\frac{46}{3}$
C) $\frac{200}{13}$
D) 18

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16

A theater sold 26 tickets for one performance. Adult tickets cost \$9 each and child tickets cost \$6 each. Ticket revenue was \$198. How many adult tickets were sold?

A) 7
B) 12
C) 14
D) 22

17

A store increases a price of \$650 by 25 percent and then decreases the increased price by 20 percent. What is the final price, in dollars?

A) 520.00
B) 650.00
C) 682.50
D) 812.50

18

The system $3x + 6y = 14$ and $kx + 12y = 34$ has no solution. What is the value of k?

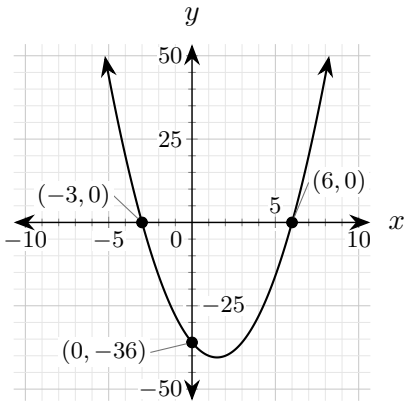
A) $\frac{3}{2}$
B) 3
C) 6
D) 12

19

At a store, 5 pens and 3 notebooks cost \$19, while 2 pens and 3 notebooks cost \$13. Each pen has the same price, and each notebook has the same price. What is the price, in dollars, of one notebook?

20

The graph shows a quadratic function f. If $f(x) = ax^2 + bx + c$, what is b?



A) -36
B) -6
C) 2
D) 6

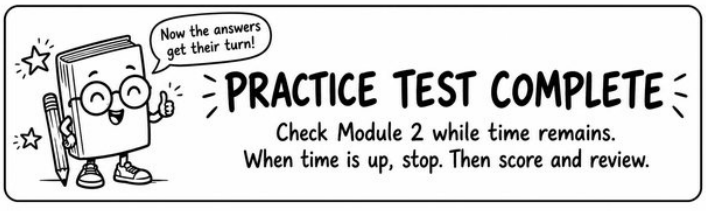
21

The equation $x^2 + kx + 289 = 0$ has exactly one distinct real solution. If $k > 0$, what is the value of k?

22

The functions f and g are defined by $f(x) = 3(x - 1)^2 + 8$ and $g(x) = f(x) - 4x$. What is the minimum value of $g(x)$?

A) $\frac{5}{3}$
B) $\frac{8}{3}$
C) 4
D) 8



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PART III

Answers and Explanations

Test 1

Module 1

1. 5	**2.** C	**3.** A	**4.** 70	**5.** C	**6.** 4	**7.** D	**8.** D
9. B	**10.** A	**11.** B	**12.** B	**13.** D	**14.** C	**15.** C	**16.** D
17. A	**18.** A	**19.** A	**20.** 128	**21.** C	**22.** 22		

Module 2

1. C	**2.** A	**3.** 41	**4.** C	**5.** 10800	**6.** D	**7.** $\frac{3}{5}$	**8.** B
9. C	**10.** C	**11.** 12	**12.** B	**13.** D	**14.** B	**15.** C	**16.** C
17. B	**18.** C	**19.** 3	**20.** B	**21.** 34	**22.** B		

Test 2

Module 1

1. B	**2.** 96	**3.** C	**4.** C	**5.** A	**6.** A	**7.** B	**8.** B
9. A	**10.** 240	**11.** 5	**12.** C	**13.** B	**14.** A	**15.** -4	**16.** C
17. D	**18.** D	**19.** C	**20.** C	**21.** 23	**22.** B		

Module 2

1. 7.5	**2.** A	**3.** D	**4.** 11	**5.** B	**6.** 6	**7.** D	**8.** C
9. 14	**10.** A	**11.** B	**12.** B	**13.** D	**14.** C	**15.** C	**16.** C
17. C	**18.** 6	**19.** A	**20.** 10	**21.** C	**22.** D		

Test 3

Module 1

1. C	**2.** B	**3.** C	**4.** B	**5.** C	**6.** C	**7.** 260	**8.** D
9. D	**10.** A	**11.** B	**12.** 2	**13.** 4	**14.** C	**15.** 20	**16.** D
17. C	**18.** B	**19.** A	**20.** B	**21.** 12	**22.** C		

Module 2

1. 625	**2.** B	**3.** B	**4.** D	**5.** 40	**6.** 1	**7.** C	**8.** A
9. A	**10.** C	**11.** 11	**12.** 27	**13.** 104	**14.** A	**15.** B	**16.** B
17. C	**18.** A	**19.** D	**20.** B	**21.** D	**22.** B		

Test 4

Module 1

1. 9	**2.** D	**3.** A	**4.** B	**5.** B	**6.** A	**7.** -5	**8.** B
9. D	**10.** B	**11.** $\frac{21}{5}$	**12.** D	**13.** C	**14.** D	**15.** 13500	**16.** D
17. D	**18.** A	**19.** C	**20.** B	**21.** 49	**22.** D		

Module 2

1. B	**2.** A	**3.** D	**4.** C	**5.** A	**6.** $\frac{3}{5}$	**7.** C	**8.** C
9. B	**10.** A	**11.** C	**12.** A	**13.** A	**14.** C	**15.** 50	**16.** 47
17. 10	**18.** B	**19.** C	**20.** 20	**21.** -16	**22.** D		

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Test 1: Explanations

Module 1

1. 5 Divide both sides by the coefficient 3. This gives $x = \frac{15}{3} = 5$. *Review:* Linear equations, identities, and contradictions, page 37

2. **C** Convert 15 percent to the fraction $\frac{15}{100}$, then multiply by the total: $(\frac{15}{100})(220) = 33$. This counts the digitized documents. *Review:* Percent of a Quantity and Reverse Percentages, page 98

3. **A** The two labeled points give line r a slope of $\frac{7}{8}$. Perpendicular nonvertical lines have slopes with product -1. The negative reciprocal of $\frac{7}{8}$ is $-\frac{8}{7}$. *Review:* Parallel and perpendicular lines, page 43

4. 70 The graph includes $(0, 30)$ and $(2, 40)$. The score increases by $\frac{40-30}{2} = 5$ points per hour, starting at 30. For 8 hours, the prediction is $5(8) + 30 = 70$. *Review:* Linear models and interpreting constraints, page 44

5. **C** Substitute zero for the input: $y = \frac{3}{5}(0) + 7 = 7$. This is the model's predicted output at zero input. It does not by itself establish that an actual observation at $x = 0$ would have this value. *Review:* Linear regression, residuals, and interpolation limits, page 118

6. 4 Subtract the fixed fee from the total: $25 - 13 = 12$ dollars. Divide the remaining charge by 3 dollars per hour, giving $\frac{12}{3} = 4$ hours. *Review:* Linear equations, identities, and contradictions, page 37

7. **D** There are $5 + 8 + 2 = 15$ marbles altogether. Of these, 8 are blue. Divide the number of blue marbles by the total: $P(\text{blue}) = \frac{8}{15}$. *Review:* Sample spaces, complements, and equally likely outcomes, page 125

8. **D** Multiplication gives $x \cdot x = x^2$, $x \cdot (-9) = -9x$, $(8) \cdot x = 8x$, and $(8)(-9) = -72$. The variable terms combine to $8x + (-9x) = -x$. The expanded expression is $x^2 - x - 72$. *Review:* Polynomial operations and structure, page 61

9. **B** The terms $+y$ and $-y$ cancel when the equations are added. This leaves $2x = 21 + 15 = 36$. Dividing by 2 gives $x = 18$; the sum 36 is twice the requested coordinate. *Review:* Linear systems by substitution and elimination, page 47

10. **A** For $(x + m)(x + n)$, the sum $m + n$ gives the coefficient of x, and the product mn gives the constant. Here the numbers are 10 and 3: their sum is $10 + 3 = 13$, and their product is $(10)(3) = 30$. The factored expression is $(x + 3)(x + 10)$. *Review:* Factoring: common factors, grouping, and special products, page 62

11. **B** At the y-intercept, $x = 0$, so $5y = 20$ and $y = 4$. *Review:* Equations of lines from graphs, points, and conditions, page 42

12. **B** A product is zero when at least one factor is zero. The factor x gives $x = 0$, which the question excludes. The other factor gives $5x + 4 = 0$, so the required nonzero solution is $x = \frac{-4}{5}$. *Review:* Quadratic equations by factoring and square roots, page 63

13. **D** Using the marked points, the vertical change is $0 + 4 = 4$ and the horizontal change is $2 - 0 = 2$. The slope is $\frac{4}{2} = 2$. *Review:* Slope, intercepts, and rate of change, page 41

14. **C** The equation is $\sqrt{567k} = 63$. Squaring both sides gives $567k = 3969$, and dividing by 567 gives $k = 7$. Substitution confirms that $\sqrt{567(7)} = \sqrt{3969} = 63$. Square the output before dividing by 567. *Review:* Radical equations and verification, page 73

15. **C** The volume is $\pi r^2 h = \pi(4)^2(7) = 112\pi$. This gives $k = 112$. The question asks for the coefficient of π, so give the number without a unit or π. *Review:* Prisms and cylinders: volume and surface area, page 151

16. **D** The two terms are equal, so their sum is $2 \cdot 2^{x+1}$. Multiplying powers with base 2 adds the exponents: $2^1 \cdot 2^{x+1} = 2^{x+2}$. Therefore, $k = x + 2$. Adding the two original exponents would incorrectly treat the sum as a product. *Review:* Exponential equations and common bases, page 91

17. **A** A point on a line satisfies its equation. Use $x = 5$ and $y = 7$ in $ax + y = 47$ to obtain $5a + 7 = 47$. Thus $5a = 40$ and $a = 8$. *Review:* Equations of lines from graphs, points, and conditions, page 42

18. **A** Same-side interior angles between parallel lines are supplementary. This gives $100 + 4x = 180$, so $4x = 80$ and $x = 20$. *Review:* Angle relationships, parallel lines, and transversals, page 131

19. **A** The second equation gives $x = y + 4$. Substitute into the first: $3(y + 4) + 2y = 17$, so $5y + 12 = 17$ and $y = 1$. Then $x = 5$, confirming both original equations. The question asks for y, not for the larger coordinate x. *Review:* Linear systems by substitution and elimination, page 47

20. 128 For similar solids, volume scales by the cube of the corresponding edge-length factor. This gives $V = 54(\frac{4}{3})^3 = 128$. *Review:* Scaling lengths, areas, and volumes, page 153

21. **C** Divide the quadratic by 5 and compare it with $(x - r)(x - s) = x^2 - (r + s)x + rs$. This gives $r + s = \frac{9}{5}$ and $rs = \frac{-3}{5}$. Expanding the requested product gives $4rs + 2(r + s) + 1 = \frac{-12}{5} + \frac{18}{5} + 1 = \frac{11}{5}$, so the roots themselves are not needed. *Review:* Sums and products of quadratic roots, page 68

22. 22 The restriction $x > 16$ also ensures $x \neq 0$, so multiply by x to obtain $x^2 - 38x + 352 = 0$. Factoring gives $(x - 16)(x - 22) = 0$. The root 16 does not satisfy the strict inequality, while 22 does. Therefore the allowed solution is 22. *Review:* Rational equations and extraneous solutions, page 72

Module 2

1. **C** Each kilogram contains 1,000 grams, so $4(1000) = 4000$ grams. *Review:* Unit Rates, Dimensional Analysis, and Unit Conversion, page 96

2. **A** At $t = 0$, $W = 316$. 316 is the initial amount of water, in liters. *Review:* Linear models and interpreting constraints, page 44

3. 41 In order, the seven observations are $38, 39, 39, 41, 42, 43, 43$. The fourth value is 41; three observations come before it and three after it. *Review:* Mean, median, mode, and weighted mean, page 107

4. **C** The right-angle mark shows that the dashed segment is perpendicular to the base, so the height is 14 cm. With base 12 cm, the area is $A = \frac{1}{2}bh = \frac{1}{2}(12)(14) = 84$ square centimeters. *Review:* Triangle area, altitude, and coordinate methods, page 141

5. 10800 The sample average estimates the average for all households in the town. Multiply that rate by the population size: $18(600) = 10{,}800$ gallons per week. The sample size of 40 describes how the average was obtained; it is not the number of households whose total is requested. *Review:* Comparing and interpreting sample means, page 123

6. **D** The hourly factors multiply to $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$. Multiply the starting count by this factor for the full period: $70(81) = 5670$ cells. *Review:* Exponential growth and decay models, page 90

7. $\frac{3}{5}$ For angle B, the adjacent leg is 18 and the hypotenuse is 30. Cosine is adjacent divided by hypotenuse, so $\cos B = \frac{BC}{AB} = \frac{18}{30} = \frac{3}{5}$. *Review:* Sine, cosine, and tangent in right triangles, page 163

8. **B** Taking square roots gives $x - 8 = \pm 9$. The two solutions are -1 and 17, whose sum is 16. *Review:* Quadratic equations by factoring and square roots, page 63

9. **C** From the two rows, the output increases by $69 - 57 = 12$ over an input increase of $7 - 3 = 4$. The rate is therefore $\frac{12}{4} = 3$ dollars per hour. Work backward from the first row: $57 - 3(3) = 48$. This is the starting amount before the input increases. *Review:* Linear models and interpreting constraints, page 44

10. **C** For angle A, $\tan A = \frac{BC}{AC}$. So $BC = 5(11) = 55$. *Review:* Sine, cosine, and tangent in right triangles, page 163

11. 12 Take the real cube root of both sides. This gives $x - 8 = 4$ and $x = 12$. *Review:* Polynomial operations and structure, page 61

12. **B** Combining the x-terms on the left gives $(a + 2)x - 3a$. Matching the coefficient of x requires $a + 2 = 7$, so $a = 5$. The constant term is then $b = -3(5) = -15$. *Review:* Linear equations, identities, and contradictions, page 37

13. **D** This is a difference of squares: $(2x)^2 - 10^2 = (2x - 10)(2x + 10)$. The opposite middle terms cancel when these factors are multiplied, leaving the original difference of squares for every input. *Review:* Factoring: common factors, grouping, and special products, page 62

14. **B** The solid boundary and shading represent $3x + 3y \leq 50$.

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Substituting $x = 5$ gives $3y \le 35$. This gives $y \le \frac{35}{3}$, so the greatest allowed integer is 11. At $y = 11$, the left side is 48, within the bound 50. The next integer, $y = 12$, gives 51, which is too large. *Review:* Linear and compound inequalities, page 51

15. C The total is $13 \cdot 5 + 15 \cdot 3 + 18 \cdot 5 = 200$, and the count is $5 + 3 + 5 = 13$. Dividing the total by the count gives a mean of $\frac{200}{13}$. *Review:* Histograms and Distribution Shape, page 110

16. C Let a be the adult count. There are $26 - a$ child tickets. The revenue equation is $9a + 6(26 - a) = 198$. Distribute to get $9a + 156 - 6a = 198$, then combine the adult terms: $3a + 156 = 198$. Subtracting 156 gives $3a = 42$, so $a = 14$. This leaves 12 child tickets. *Review:* Linear systems by substitution and elimination, page 47

17. B An increase of 25 percent uses multiplier $\frac{100+25}{100} = \frac{5}{4}$. The raised price is $650.00(\frac{5}{4}) = 812.50$ dollars. A decrease of 20 percent uses multiplier $\frac{100-20}{100} = \frac{4}{5}$. Apply this multiplier to the raised price: $812.50(\frac{4}{5}) = 650$ dollars. *Review:* Percent Change, Successive Changes, Interest, and Growth Factors, page 99

18. C The y-coefficient is multiplied by 2, so $k = 2(3) = 6$. Multiplying the first equation by 2 gives $6x + 12y = 28$. With $k = 6$, the second requires the same left side to equal 34. It cannot equal both different constants. *Review:* Systems with no solution or infinitely many solutions, page 48

19. 3 Subtract the second purchase total from the first. The notebooks cancel, leaving 3 pens costing $19 - 13 = 6$ dollars, so each pen costs 2. In the second purchase, the pens cost $2(2) = 4$ dollars. The three notebooks therefore cost $13 - 4 = 9$ dollars, or $\frac{9}{3} = 3$ dollars each. *Review:* Linear systems by substitution and elimination, page 47

20. B The zeros give $f(x) = A(x + 3)(x - 6)$. At input 0, the y-intercept requires $-36 = A(3)(-6) = -18A$. Divide by -18 to get $A = 2$. Now expand: $f(x) = 2x^2 - 6x - 36$. The coefficient multiplying the x-term is $b = -6$. *Review:* Quadratic graphs, vertex, zeros, and optimization, page 67

21. 34 Here the quadratic formula is $x = \frac{-k \pm \sqrt{k^2 - 4(289)}}{2}$. The two signs give the same real root only when the square-root term is zero. Set $k^2 - 4(289) = 0$, giving $k^2 = 1156$. Taking both square roots gives $k = \pm 34$. Since the question requires $k > 0$, choose 34. As a check, this choice of k rewrites the equation as $(x + 17)^2 = 0$. Its only distinct solution is $x = -17$. The question asks for k, so report 34. *Review:* Discriminants and the Number of Real Solutions, page 65

22. B Expanding $f(x)$ and subtracting $4x$ gives $g(x) = 3x^2 - 10x + 11$. Completing the square gives $g(x) = 3(x - \frac{5}{3})^2 + \frac{8}{3}$. The squared term is nonnegative and equals zero at $x = \frac{5}{3}$, so the smallest output is $\frac{8}{3}$. The value $\frac{5}{3}$ is the input where that minimum occurs. *Review:* Quadratic graphs, vertex, zeros, and optimization, page 67

Test 2: Explanations

Module 1

1. B The points cluster near a straight line, and y generally increases as x increases. The association is strong, linear, and positive. *Review:* Scatterplots and correlation, page 117

2. 96 The increase is 20 percent of Saturday's 80 tickets: $0.20(80) = 16$. Add this increase to the original count, giving $80 + 16 = 96$ tickets on Sunday. The question asks for the new total, so the increase of 16 is only an intermediate step. *Review:* Percent of a Quantity and Reverse Percentages, page 98

3. C The line $y = 3x - 9$ crosses the y-axis at $(0, -9)$. A slope of 3 means that increasing x by 5 raises y by 15, so the line also passes through $(5, 6)$. This is Graph C. Graph B has the same intercept but slope $\frac{1}{3}$, so it rises too slowly. *Review:* Linear models and interpreting constraints, page 44

4. C Substituting $x + 1$ gives $p(x + 1) = x^2 - 64$. Subtract the entire expression $p(x) = x^2 - 2x - 63$: $x^2 - 64 - (x^2 - 2x - 63) = 2x - 1$. The quadratic terms cancel, while subtracting the negative $2x$ produces a positive $2x$. *Review:* Polynomial operations and structure, page 61

5. A The fixed fee is paid once, so extending the rental does not add another 14 dollars. The hourly charge is 4 dollars, and d additional hours therefore add $4d$ dollars. Equivalently, $(14 + 4(h + d)) - (14 + 4h) = 4d$. *Review:* Linear models and interpreting constraints, page 44

6. A The leg squares are $63^2 = 3969$ and $84^2 = 7056$. Add them: $c^2 = 3969 + 7056 = 11025$. Taking the positive square root gives $c = \sqrt{11025} = 105$. *Review:* Pythagorean theorem and distance, page 138

7. B Slope measures the change in y divided by the corresponding change in x. From $(1, 5)$ to $(4, -1)$, those changes are -6 and 3. Therefore the slope is $\frac{-1-5}{4-1} = \frac{-6}{3} = -2$. The negative sign agrees with the decreasing outputs. *Review:* Slope, intercepts, and rate of change, page 41

8. B At $x = -13$, the factor $x + 13$ is zero, so $f(-13) = 0$. Thus, $(-13, 0)$ is on the graph. The vertical intercept is $(0, 104)$, and the other horizontal intercept is $(-8, 0)$; the signs of the factors matter. *Review:* Quadratic equations by factoring and square roots, page 63

9. A The side opposite angle A is $BC = 25$, and the hypotenuse is $AB = 65$. Sine is opposite divided by hypotenuse, so $\sin A = \frac{25}{65} = \frac{5}{13}$. The side $AC = 60$ is adjacent to angle A and is used for cosine, rather than sine. *Review:* Sine, cosine, and tangent in right triangles, page 163

10. 240 Before any cells are removed, two hourly doublings give $30(2)^2 = 120$ cells. Removing half leaves $\frac{120}{2} = 60$ cells. There are then two more hourly doublings, so the final count is $60(2)^2 = 240$ cells. The removal changes the count at the end of the first interval; it does not change the hourly growth factor. *Review:* Exponential growth and decay models, page 90

11. 5 An exterior angle equals the sum of the two remote interior angles. This gives $2x + 25 + 2x + 14 = 59$, giving $4x = 20$ and $x = 5$. *Review:* Triangle angle sums, exterior angles, and side constraints, page 136

12. C Subtract x from both sides: $2x + 6 = 26$. Subtract 6 from both sides: $2x = 26 - (6) = 20$. Divide by 2 to get $x = 10$. *Review:* Linear equations, identities, and contradictions, page 37

13. B Each year, 85 percent remains. Its fractional multiplier is $\frac{85}{100} = 0.85$. The starting value is 16000 dollars, so after t years the model is $V(t) = 16000(0.85)^t$. After one year, the model gives 13600 dollars. The loss is $16000 - 13600 = 2400$ dollars, exactly 15 percent of the initial value. *Review:* Exponential growth and decay models, page 90

14. A Both sets have mean 57. The distances from the mean 57 are $1, 0, 1$ in A and $5, 0, 5$ in B. B has the larger deviations, so it has the larger standard deviation. *Review:* Standard deviation and changes to a data set, page 109

15. -4 Multiply both sides by 2 to clear the denominator: $7x + 12 = 2(-8) = -16$. Subtract 12 from both sides: $7x = -16 - 12 = -28$. Divide by 7 to get $x = \frac{-28}{7} = -4$. *Review:* Linear equations, identities, and contradictions, page 37

16. C The table contains $26+38+38+19 = 121$ students. Exactly one course means chemistry only (38 students) or physics only (38 students). The 26 students taking both do not qualify. There are $38 + 38 = 76$ qualifying students, giving probability $\frac{76}{121}$. *Review:* Combining groups and recovering missing data, page 111

17. D The boundary is $5x + 3y = 33$, and the shading is on or below it. At $(6, 1)$, $5(6) + 3(1) = 33$, so the point is on the included boundary. The other choices give 36, which is above the boundary and outside the shaded region. *Review:* Graphing inequalities and feasible regions, page 50

18. D This is a difference of squares: $(4x)^2 - 8^2 = (4x - 8)(4x + 8)$. The opposite middle terms cancel when these factors are multiplied, leaving the original difference of squares for every input. *Review:* Factoring: common factors, grouping, and special products, page 62

19. C Let n be the number of posters. The first service is cheaper when $48 + 2n < 12 + 5n$, or $36 < 3n$. Thus $n > 12$. Equality at 12 means the costs are the same, so the least whole number giving a lower cost is 13. *Review:* Linear models and interpreting constraints, page 44

20. C Write $2n+3p = 13$ and $4n+p = 11$. Doubling the first equation and subtracting the second gives $5p = 15$, so $p = 3$. Then $4n + 3 = 11$ gives $n = 2$. One of each costs $2+3 = 5$ dollars. *Review:* Linear systems by substitution and elimination, page 47

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Chapter 14 Answers

Explanations: page 272

1. 72 **2.** 63 square units **3.** 14 **4.** 32 **5.** 64 square units **6.** 6 square feet **7.** 99 **8.** 68 **9.** $100 - 4\pi$ **10.** $48 + \frac{9\pi}{2}$ **11.** $22 + 3\pi$ **12.** 42 units **13.** 60 **14.** 54 square units **15.** Volume: 126 cubic units. Total surface area: 176 square units. **16.** 90π cubic units **17.** 5 **18.** 28π **19.** 80 cubic units **20.** 100π **21.** 13 **22.** 64π square units **23.** Volume: 18π cubic units. Total surface area: 27π square units. **24.** 360 **25.** $\frac{100}{3}$ **26.** $\frac{1}{9}$ and $\frac{1}{27}$ **27.** $7:4$ **28.** Length $5:2$; surface area $25:4$ **29.** 19% **30.** The volume triples. **31.** 12,000 cm^2 **32.** 3,000 cm^3 **33.** $6.15 \le L < 6.25$ **34.** $[38, 42)$ square units **35.** $[19.75, 20.25)$ kilometers **36.** 8 bottles **37.** 42 **38.** 72π **39.** B. 252 cm^3 **40.** 972π **41.** B. $9:1$ **42.** 36 L **43.** 648 cm^3 **44.** 360 **45.** 360

Chapter 15 Answers

Explanations: page 273

1. 16π **2.** 25π **3.** 8 **4.** 7 **5.** 14π feet **6.** Circumference factor: 3. Area factor: 9. **7.** 22 cm **8.** $1 - \frac{\pi}{4}$ **9.** 15 **10.** 5 cm **11.** 6 **12.** $1:1$ **13.** 15 cm **14.** 6 cm **15.** 9 m **16.** 11 L **17.** 4 cm **18.** 12 cm **19.** 18 **20.** 4 **21.** 144π **22.** 44% **23.** 20π m

Chapter 16 Answers

Explanations: page 273

1. $\frac{7}{5}$ **2.** $\frac{15}{17}$ **3.** 9 units **4.** $\frac{2}{\sqrt{53}}$ **5.** $\frac{7}{24}$ **6.** $45°$ **7.** $10\sqrt{3} - 10$ ft **8.** 31.5 meters **9.** $\tan^{-1}\left(\frac{3}{4}\right) \approx 36.9°$ **10.** $16\sqrt{3}$ meters **11.** 13 meters **12.** Length $\frac{2}{\sin 20°}$ meters; horizontal run $\frac{2}{\tan 20°}$ meters **13.** $\frac{17}{13}$ **14.** $\frac{4}{5}$ **15.** 5 m **16.** $\frac{7}{13}$ **17.** $\tan^{-1}(\frac{3}{16})$ **18.** 25.6 m **19.** 15 **20.** $12\sqrt{3}$ m **21.** $\frac{5}{12}$

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Chapter 17 Answers

Explanations: page 274

1. 16 **2.** 25% **3.** 5 **4.** 900 feet per minute **5.** 25π square inches **6.** No. For $x = \frac{1}{2}$, $x^2 = \frac{1}{4} < x$. **7.** 3 **8.** $(-4)^2 = 16$; $-4^2 = -16$ **9.** $\frac{1}{2}$ **10.** 30 **11.** 10 **12.** Estimate: about 60; exact value: 61.38 **13.** 141 **14.** 144 **15.** $n = 5$ **16.** At $x = 2$, both equal 4; at $x = 3$, $x^2 = 9 \neq 6 = 2x$. **17.** 5, since $4.5 < \frac{39.6}{8.1} < 5.5$ **18.** Valid: $\frac{8+2}{8-3} = 2$, and $8 \neq 3$. **19.** 3 **20.** $(-2, 4)$ and $(2, 4)$. **21.** A finite window and limited resolution cannot prove a full-domain identity. **22.** 4 **23.** 46. **24.** No; it is extrapolation. **25.** 2.5 or $\frac{5}{2}$ **26.** 25. **27.** $\frac{1}{3}$. **28.** 90 **29.** No. **30.** $x = 2$.

Chapter Practice Explanations

Find your chapter and question number below. Compare the reasoning with your own work, then cover the explanation and try the question again. Short answer keys begin on page 257.

Chapter 1. Numerical Foundations

1. The graph includes -4 and excludes 3. The integers also satisfying $x^2 < 10$ are $-3, -2, -1, 0, 1, 2$, so there are 6. The value -4 is allowed by the graph but fails the squared condition.

2. The allowed integers are $-3, -2, -1, 0, 1, 2, 3$. The lower boundary is included and the upper boundary is excluded, giving 7 readings.

3. Since $5^2 < 31 < 6^2$, the positive root lies between 5 and 6. The negative root lies between -6 and -5, with the order reversed on the number line.

4. Start with $(-2, 5]$ and remove the entire interval $[1, 3)$. The value 1 is removed, while 3 is retained, leaving $(-2, 1) \cup [3, 5]$.

5. The upper limit lies between 5 and 6 because $25 < 31 < 36$. Therefore 5 is accepted, but 6 is not. The lower limit does not exclude 5.

6. Moving right adds to the coordinate. The result is $-\frac{3}{2} + \frac{7}{4} = -\frac{6}{4} + \frac{7}{4} = \frac{1}{4}$, so the move crosses zero.

7. The marked coordinates are -4 and 2, a distance of 6 units. From 2, traveling twice that distance left gives $2 - 2(6) = -10$. The final point is outside the displayed interval, which does not restrict the robot's travel.

8. The deposits add $3(14) = 42$ dollars and the withdrawals remove $2(9) = 18$ dollars. The final balance is $-18 + 42 - 18 = 6$ dollars.

9. The net change is $5(4) - 7(3) = 20 - 21 = -1$ point. Adding this change to the starting score gives $12 - 1 = 11$.

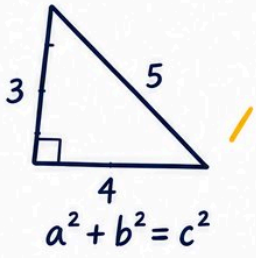
10. Division and multiplication have equal priority and proceed from left to right. First $36 \div (-3) = -12$; then $(-12)(-2) = 24$. Grouping the last two numbers changes the expression.

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