

ACT Math Practice Test 1

50 Minutes — 45 Questions

Choose the best answer for each question. A permitted calculator may be used. Figures are not necessarily drawn to scale.

- 1) A gym charges a one-time enrollment fee of \$75 plus \$40 per month for membership. Which equation represents the total cost C for m months of membership?
- ☐ A. $C = 75m + 40$ ☐ C. $C = 115m$
☐ B. $C = 40m + 75$ ☐ D. $C = 75m - 40$
- 2) The first four terms of a geometric sequence are 5, 10, 20, 40, $\dots$. What is the 7th term?
- ☐ F. 160 ☐ H. 320
☐ G. 240 ☐ J. 640
- 3) What value of x satisfies $5x - 2 = 3x + 10$?
- ☐ A. 4 ☐ C. 8
☐ B. 6 ☐ D. 5
- 4) Marcus rolls a fair six-sided die twice in a board game. What is the probability that both rolls result in an even number?
- ☐ F. $\frac{1}{4}$ ☐ H. $\frac{1}{2}$
☐ G. $\frac{1}{3}$ ☐ J. $\frac{2}{3}$
- 5) Add the matrices $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 0 \\ -2 & 1 \end{bmatrix}$. What is $A + B$?
- ☐ A. $\begin{bmatrix} 7 & -1 \\ 1 & 5 \end{bmatrix}$ ☐ C. $\begin{bmatrix} 3 & -1 \\ 1 & 5 \end{bmatrix}$
☐ B. $\begin{bmatrix} 7 & -1 \\ 5 & 3 \end{bmatrix}$ ☐ D. $\begin{bmatrix} 10 & -1 \\ 1 & 4 \end{bmatrix}$
- 6) \$3,000 is invested at 3% interest compounded quarterly. What is the investment worth after 2 years, rounded to the nearest dollar?
- ☐ F. \$3,180 ☐ H. \$3,190
☐ G. \$3,185 ☐ J. \$3,186
- 7) Which ordered pair is NOT a solution to the equation $y = 3x - 5$?
- ☐ A. (4, 7) ☐ C. (3, 5)
☐ B. (2, 1) ☐ D. (0, -5)
- 8) A line of best fit for data relating hours studied, x , to test scores, y , is $y = 8x + 52$. What does the y -intercept represent?
- ☐ F. 8; the test score increases by 8 for each hour ☐ H. 8; the hours needed to pass the test
☐ G. 60; the test score when no hours are studied ☐ J. 52; the test score when no hours are studied

- 9) For $f(x) = 2x^2 - 3x - 4$, what is the value of $f(-2)$?
- ☐ A. 2 ☐ C. -6
- ☐ B. 10 ☐ D. 6
- 10) If $f(x) = 2x - 3$ and $g(x) = x + 4$, what is the value of $f(g(2))$?
- ☐ F. 7 ☐ H. 11
- ☐ G. 5 ☐ J. 9
- 11) Which statement about $2\sqrt{2}$ and π is true?
- ☐ A. $2\sqrt{2} > \pi$ ☐ C. $2\sqrt{2} < \pi$
- ☐ B. $2\sqrt{2} = \pi$ ☐ D. $\pi - 2\sqrt{2} > \frac{1}{2}$
- 12) The positive solution of $(x - 3)(x + 4) = 18$ is what value?
- ☐ F. 5 ☐ H. 6
- ☐ G. 3 ☐ J. -6
- 13) A chord of a circle is 16 units long and is 6 units from the center of the circle. What is the radius of the circle?
- ☐ A. 8 units ☐ C. 10 units
- ☐ B. 6 units ☐ D. 14 units
- 14) The solution to the system $2x + 3y = 19$ and $x - y = 2$ is (x, y) . What is the value of $x + y$?
- ☐ F. 6 ☐ H. 12
- ☐ G. 8 ☐ J. 10
- 15) A sector of a circle has a central angle θ satisfying $45^\circ < \theta \leq 135^\circ$. If the circle has radius 4 meters, which inequality describes the area A_{sector} of the sector, in square meters?
- ☐ A. $45 < A_{\text{sector}} \leq 135$ ☐ C. $A_{\text{sector}} < 2\pi$
- ☐ B. $A_{\text{sector}} \geq 6\pi$ ☐ D. $2\pi < A_{\text{sector}} \leq 6\pi$
- 16) A car travels 60 miles at a constant speed of s miles per hour, then travels another 60 miles at $s + 15$ miles per hour. The total travel time is $\frac{10}{3}$ hours. What is s ?
- ☐ F. 36 ☐ H. 30
- ☐ G. 45 ☐ J. 25
- 17) If $f(x) = 2x^2 - 3$, which expression is equivalent to $f(a + 1) - f(a)$?
- ☐ A. $4a^2 + 2$ ☐ C. $2a^2 + 4a - 1$
- ☐ B. $4a - 2$ ☐ D. $4a + 2$
- 18) An insect population is modeled by $P(t) = 18(1.3)^t$, where t is months. By what percent has the population increased after 2 months?
- ☐ F. 69% ☐ H. 30%
- ☐ G. 60% ☐ J. 169%
- 19) How many integer values of x satisfy $|2x - 5| < 7$?
- ☐ A. 7 ☐ C. 6
- ☐ B. 5 ☐ D. 8

- ☐ F. 1
 ☐ H. 4
- ☐ G. 3
 ☐ J. -5
- 21) An isosceles trapezoid has parallel sides of lengths 10 and 22 units. Each of its other sides is 10 units long. What is the area of the trapezoid?
- ☐ A. 120 square units
 ☐ C. 144 square units
- ☐ B. 160 square units
 ☐ D. 128 square units
- 22) A code consists of 3 letters chosen from A , B , C , and D , followed by 2 digits chosen from 0 through 4. No letter may repeat, and the digits must be different. How many codes have A or B as the first letter and an even final digit?
- ☐ F. 240
 ☐ H. 144
- ☐ G. 360
 ☐ J. 288
- 23) An exterior angle of a triangle measures 132° . The two remote interior angles measure $(3x + 6)^\circ$ and $(5x - 2)^\circ$. What is the measure of the larger remote interior angle?
- ☐ A. 54°
☐ C. 102°
- ☐ B. 72°
☐ D. 78°
- 24) For $x > 0$, which expression is equivalent to $\frac{(4x^6)^{1/2}}{2x^{-1}}$?
- ☐ F. x^2
☐ H. x^3
- ☐ G. x^4
☐ J. $2x^4$
- 25) A class of 18 students has an average score of 76, and a class of 12 students has an average score of 84. One score in the combined group was recorded 24 points too high. What is the corrected average for all 30 students?
- ☐ A. 78.4
 ☐ C. 80
- ☐ B. 79.2
 ☐ D. 77.6
- 26) For $f(x) = x^2 + kx + 25$, for what values of k does the parabola not intersect the x -axis?
- ☐ F. $k < -10$ or $k > 10$
☐ H. All real values of k
- ☐ G. $k = -10$ or $k = 10$
☐ J. $-10 < k < 10$

- 27) A solid consists of a cylinder with a hemisphere on top. Both have radius 3 centimeters, and the solid's total height is 11 centimeters. What is the solid's volume?
- ☐ A. 90π cubic centimeters ☐ C. 108π cubic centimeters
- ☐ B. 99π cubic centimeters ☐ D. 126π cubic centimeters
- 28) A delivery service charges a fixed fee plus the same amount for each mile. A 3-mile delivery costs \$18, and an 8-mile delivery costs \$33. If a customer has \$45 to spend, what is the greatest whole number of miles the customer can order?
- ☐ F. 10 ☐ H. 15
- ☐ G. 12 ☐ J. 18
- 29) A committee of 3 is selected from 8 people. Two of the people are siblings, and they will not serve together. How many different committees can be formed?
- ☐ A. 50 ☐ C. 52
- ☐ B. 48 ☐ D. 56
- 30) A square garden has area $16x^2 - 40x + 25$ square units. If $x > \frac{5}{4}$, which expression represents the garden's perimeter?
- ☐ F. $4x - 5$ ☐ H. $4x + 5$
- ☐ G. $16x + 20$ ☐ J. $16x - 20$
- 31) For $f(x) = x^2 - 4x + 3$, which describes all values of x for which $f(x) > 0$?
- ☐ A. $1 < x < 3$ ☐ C. $x \leq 1$ or $x \geq 3$
- ☐ B. $x < 1$ or $x > 3$ ☐ D. $x > 1$
- 32) A rectangular screen has a diagonal of 25 centimeters. Its length is 5 centimeters greater than its width. What is the area of the screen?
- ☐ F. 250 square centimeters ☐ H. 275 square centimeters
- ☐ G. 240 square centimeters ☐ J. 300 square centimeters
- 33) Which interval contains $(\sqrt{5} - \sqrt{2})^2$?
- ☐ A. $0.4 < (\sqrt{5} - \sqrt{2})^2 < 0.6$ ☐ C. $0.6 < (\sqrt{5} - \sqrt{2})^2 < 0.8$
- ☐ B. $0.8 < (\sqrt{5} - \sqrt{2})^2 < 1.0$ ☐ D. $1.0 < (\sqrt{5} - \sqrt{2})^2 < 1.2$
- 34) Real numbers x and y satisfy $x + y = 9$ and $x^2 + y^2 = 53$. What is the value of $x^3 + y^3$?
- ☐ F. 351 ☐ H. 441
- ☐ G. 405 ☐ J. 729
- 35) A bag contains 5 green tokens and 4 yellow tokens. Two tokens are drawn at random without replacement. Given that at least one of the two tokens is green, what is the probability that both are green?
- ☐ A. $\frac{5}{18}$ ☐ C. $\frac{5}{9}$
- ☐ B. $\frac{1}{3}$ ☐ D. $\frac{2}{3}$
- 36) Pump A fills a tank in 6 hours, and Pump B fills it in 8 hours. Both pumps run together for 2 hours, after which Pump B is turned off and Pump A runs alone. How long after Pump B is turned off will Pump A need to finish filling the tank?
- ☐ F. $\frac{5}{12}$ hour ☐ H. 3 hours
- ☐ G. 2 hours ☐ J. $2\frac{1}{2}$ hours

- 37) A theater sells 240 tickets at \$50 each. For every \$2 decrease in ticket price, it expects to sell 15 more tickets. If the price can be decreased only in whole \$2 steps, what is the greatest possible revenue?
- ☐ A. \$12,600 ☐ C. \$12,570
- ☐ B. \$12,540 ☐ D. \$12,480
- 38) In a right triangle, the altitude from the right angle meets the hypotenuse and divides it into segments of lengths 9 and 16. What is the area of the triangle?
- ☐ F. 125 square units ☐ H. 144 square units
- ☐ G. 150 square units ☐ J. 180 square units
- 39) Twelve liters of a 20% solution are mixed with a 50% solution to make a 30% solution. Then 4 liters of the mixture are removed and replaced with 4 liters of water. What is the concentration of the resulting mixture?
- ☐ A. 20% ☐ C. 25%
- ☐ B. $23\frac{1}{3}\%$ ☐ D. $26\frac{2}{3}\%$
- 40) Let r be a nonzero rational number and let s be an irrational number. Which expression could be rational?
- ☐ F. $r + s$ ☐ H. $\frac{r}{s}$
- ☐ G. rs ☐ J. s^2
- 41) For which values of k does the system $(k + 1)x + 2y = 5$ and $4x + (k - 1)y = 7$ have no solution?
- ☐ A. $k = -3$ and $k = 3$ ☐ C. $k = 3$ only
- ☐ B. $k = -3$ only ☐ D. There are no such values of k
- 42) How many real solutions does $|x^2 - 4| = x$ have?
- ☐ F. 0 ☐ H. 1
- ☐ G. 3 ☐ J. 2
- 43) Two chords intersect inside a circle at point E . Their segment lengths are $AE = x + 1$, $EB = x + 17$, $CE = 2x + 1$, and $ED = x + 7$. What is the positive value of x ?
- ☐ A. 5 ☐ C. 6
- ☐ B. 4 ☐ D. 8
- 44) An event occurs independently on each of 5 trials with probability $\frac{1}{3}$. What is the probability that it occurs at least 3 times?
- ☐ F. $\frac{11}{27}$ ☐ H. $\frac{17}{81}$
- ☐ G. $\frac{5}{81}$ ☐ J. $\frac{19}{81}$
- 45) A jacket's price is reduced by \$15, then the reduced price is discounted by 20%. After a 6% sales tax is applied, the customer pays \$169.60. What was the jacket's original price?
- ☐ A. \$215 ☐ C. \$225
- ☐ B. \$200 ☐ D. \$230

Answer Key and Explanations

1. B Standard: ACT Math: Integrating Essential Skills

Think of the \$75 as the starting cost: it is charged once, no matter how many months the membership lasts. Each of the m months adds another \$40, so the monthly part is $40m$. Put the one-time fee and monthly charges together to get $C = 40m + 75$.

2. H Standard: ACT Math: Functions

Look at how one term changes into the next: each term is twice the one before it. From the first term to the seventh there are six doublings, so the seventh term is $5 \cdot 2^6 = 5 \cdot 64 = 320$. The first term counts as term 1, which is why we double six times, not seven.

3. B Standard: ACT Math: Algebra

Subtract $3x$ from both sides to collect the variable terms: $2x - 2 = 10$. Add 2 to get $2x = 12$, then divide by 2 to obtain $x = 6$. Substitution gives $5(6) - 2 = 28$ and $3(6) + 10 = 28$, so the value checks in the original equation.

4. F Standard: ACT Math: Statistics & Probability

There are three even faces on a fair die, so one roll has probability $3/6 = 1/2$ of being even. The two rolls are independent, which means the chance of getting an even number on the second roll is still $1/2$ after the first roll. Multiply the probabilities: $1/2 \times 1/2 = 1/4$. Thus the probability that both rolls are even is $1/4$ (choice F).

5. A Standard: ACT Math: Number & Quantity

Add entries in matching positions: $2 + 5 = 7$, $-1 + 0 = -1$, $3 + (-2) = 1$, and $4 + 1 = 5$. Therefore, $A + B = \begin{bmatrix} 7 & -1 \\ 1 & 5 \end{bmatrix}$. Matrix

addition works entry by entry, just like adding two number tables of the same size.

6. G Standard: ACT Math: Integrating Essential Skills

Quarterly compounding means the 3% annual rate is divided into four periods, so each quarter earns $0.03/4 = 0.0075$, or 0.75%. In two years there are $4 \cdot 2 = 8$ quarters. Starting with \$3,000, the balance is $3000(1.0075)^8 \approx \$3,184.80$, which rounds to \$3,185 to the nearest dollar (choice G).

7. C Standard: ACT Math: Algebra

Substitute each ordered pair into $y = 3x - 5$. For A, $3(4) - 5 = 7$, and for B, $3(2) - 5 = 1$, so both satisfy the equation. For C, $3(3) - 5 = 4$, not 5, so $(3, 5)$ does not satisfy it. For D, $3(0) - 5 = -5$, so that pair works as well. Therefore, choice C is the pair that is not a solution.

8. J Standard: ACT Math: Statistics & Probability

The y-intercept is the constant term in the equation $y = mx + b$, which is 52. It represents the predicted value of y when $x = 0$, meaning the test score when zero hours are studied.

9. B Standard: ACT Math: Algebra

Substitute -2 everywhere x appears: $f(-2) = 2(-2)^2 - 3(-2) - 4$. The square is positive, so this becomes $2(4) + 6 - 4 = 8 + 6 - 4 = 10$. Keeping the negative sign inside the squared term is the step that prevents the common sign error.

10. J Standard: ACT Math: Functions

Work from the inside of the composition outward. First, $g(2) = 2 + 4 = 6$; that output becomes the input to f . Then $f(6) = 2(6) - 3 = 9$. Substituting 2 directly into f would skip the inner function, so keep the order g first, then f .

11. C Standard: ACT Math: Number & Quantity

Use familiar approximations to compare the numbers: $\sqrt{2} \approx 1.414$, so $2\sqrt{2} \approx 2.828$, while $\pi \approx 3.142$. Since 2.828 is less than 3.142, choice C is true. Their difference is about 0.314, so choice D is false; the numbers are close, but they are not equal.

12. F Standard: ACT Math: Algebra

Multiply the two binomials to get $x^2 + x - 12 = 18$, so $x^2 + x - 30 = 0$. This factors as $(x + 6)(x - 5) = 0$, giving $x = -6$ or $x = 5$. The question asks for the positive solution, so the answer is 5.

13. C Standard: ACT Math: Geometry

Draw a radius to one endpoint of the chord and the perpendicular from the center to the chord. The perpendicular bisects the chord, so the right triangle has legs of 6 units and $16/2 = 8$ units. By the Pythagorean theorem, the radius is $\sqrt{6^2 + 8^2} = \sqrt{100} = 10$ units.

14. G Standard: ACT Math: Algebra

From $x - y = 2$, write $x = y + 2$. Substitute this into the other equation: $2(y + 2) + 3y = 19$, so $5y + 4 = 19$ and $y = 3$. Then $x = 5$, giving $x + y = 8$. Checking $(5, 3)$ in the first equation gives $10 + 9 = 19$.

15. D Standard: ACT Math: Geometry

A full circle with radius 4 has area $\pi(4)^2 = 16\pi$. The sector occupies $\theta/360$ of that area, so its area is $A_{\text{sector}} = (\theta/360)(16\pi)$. At the lower angle of 45° , this is 2π , but the angle is strictly greater than 45° , so the area must be greater than 2π . At 135° , the area is 6π , and that endpoint is allowed. Therefore $2\pi < A_{\text{sector}} \leq 6\pi$, choice D.

16. H Standard: ACT Math: Integrating Essential Skills

For each part of the trip, time equals distance divided by speed, so the travel times are $60/s$ and $60/(s + 15)$. Set their sum equal to the total time: $\frac{60}{s} + \frac{60}{s+15} = \frac{10}{3}$. A speed must be positive, so multiplying by $3s(s + 15)$ is valid. This gives $180(2s + 15) = 10s(s + 15)$, or $s^2 - 21s - 270 = 0$. Factor to get $(s - 30)(s + 9) = 0$. The negative solution cannot represent a speed, leaving $s = 30$ miles per hour. Check: the two speeds are 30 and 45 miles per hour, so the travel times are 2 hours and $\frac{4}{3}$ hours, totaling $\frac{10}{3}$ hours.

17. D Standard: ACT Math: Functions

Evaluate the function at each input before subtracting: $f(a+1) = 2(a+1)^2 - 3$ and $f(a) = 2a^2 - 3$. Their difference is $2(a^2 + 2a + 1) - 3 - (2a^2 - 3)$. The $2a^2$ terms and the constant terms cancel, leaving $4a + 2$.

18. F Standard: ACT Math: Integrating Essential Skills

After two months the population is multiplied by $1.3^2 = 1.69$. That factor means the new population is 169% of its starting size, so the increase is $169\% - 100\% = 69\%$. The two monthly increases compound, so adding $30\% + 30\%$ would miss the growth on the first month's increase.

19. C Standard: ACT Math: Algebra

An absolute value less than 7 means the expression inside is between -7 and 7 : $-7 < 2x - 5 < 7$. Add 5 throughout and divide by 2 to get $-1 < x < 6$. The integers in that interval are 0, 1, 2, 3, 4, 5, so there are 6 values.

20. G Standard: ACT Math: Functions

Two clear points on the drawn line are $(1, 2)$ and $(4, 5)$, so its slope is $3/3 = 1$. A perpendicular line has slope -1 . Using the given point $(4, -1)$ in point-slope form gives $y + 1 = -(x - 4)$, or $y = -x + 3$. Its y -intercept is 3, choice G.

21. D Standard: ACT Math: Geometry

Drop perpendiculars from the endpoints of the shorter base to the longer base. Because the trapezoid is isosceles, the extra $22 - 10 = 12$ units of the longer base is split evenly, leaving a horizontal leg of 6 units in each right triangle. Each slanted side is 10, so the height is $\sqrt{10^2 - 6^2} = 8$. The area is the average of the bases times the height: $((10 + 22)/2)(8) = 128$ square units.

22. H Standard: ACT Math: Statistics & Probability

There are 2 choices for the first letter, then 3 choices for the second and 2 for the third, giving $2 \cdot 3 \cdot 2 = 12$ letter strings. For the digits, the final digit must be one of 0, 2, 4 (3 choices); the first digit can then be any of the other 4 digits. That gives $12 \cdot 3 \cdot 4 = 144$ codes, choice H.

23. D Standard: ACT Math: Geometry

An exterior angle equals the sum of the two remote interior angles, so $(3x + 6) + (5x - 2) = 132$. Combining terms gives $8x + 4 = 132$, so $x = 16$. The remote angles are then $3(16) + 6 = 54^\circ$ and $5(16) - 2 = 78^\circ$; the larger is 78° . The remaining interior angle is $180^\circ - 132^\circ = 48^\circ$, which confirms the triangle's angles add to 180° .

24. G Standard: ACT Math: Number & Quantity

Because $x > 0$, $(4x^6)^{1/2} = 2x^3$. Dividing by $2x^{-1}$ is the same as multiplying by its reciprocal, $x/2$, so $\frac{2x^3}{2x^{-1}} = x^3 \cdot x = x^4$. The positive-domain condition matters: without it, simplifying the square root would require an absolute value.

25. A Standard: ACT Math: Statistics & Probability

The two classes contribute $18(76) = 1,368$ points and $12(84) = 1,008$ points, for a recorded total of 2,376. Since that total includes one score that is 24 points too high, the corrected total is $2,376 - 24 = 2,352$. Divide by the 30 students: $2,352/30 = 78.4$. Weighting by class size matters here because the groups do not contain the same number of students.

26. J Standard: ACT Math: Functions

A parabola does not intersect the x -axis when its related quadratic equation has no real solutions, so the discriminant must be negative. Here the discriminant is $k^2 - 4(1)(25) = k^2 - 100$. The inequality $k^2 - 100 < 0$ gives $k^2 < 100$, so $-10 < k < 10$.

27. A Standard: ACT Math: Geometry

The hemisphere adds 3 centimeters to the height, so the cylinder is $11 - 3 = 8$ centimeters tall. The cylinder's volume is $\pi(3)^2(8) = 72\pi$. A hemisphere is half a sphere, so its volume is $\frac{1}{2}(\frac{4}{3}\pi(3)^3) = 18\pi$. Add the two nonoverlapping parts: $72\pi + 18\pi = 90\pi$ cubic centimeters.

28. G Standard: ACT Math: Integrating Essential Skills

The cost rises by $33 - 18 = 15$ dollars over $8 - 3 = 5$ additional miles, so the rate is $15/5 = \$3$ per mile. Using the 3-mile delivery, the fixed fee is $18 - 3(3) = \$9$. With a \$45 budget, solve $9 + 3m \leq 45$: subtracting 9 gives $3m \leq 36$, so $m \leq 12$. The greatest whole number of miles is 12.

29. A Standard: ACT Math: Statistics & Probability

First count every possible group of three: there are $\binom{8}{3} = 56$. Now remove the groups that include both siblings. If both are already on the committee, the third person can be any one of the other 6 people, so exactly 6 groups are disallowed. That leaves $56 - 6 = 50$ committees in which the siblings do not serve together.

30. J Standard: ACT Math: Integrating Essential Skills

The area factors as $(4x - 5)^2$, since expanding that square gives $16x^2 - 40x + 25$. A side length is the positive square root of the area, and $x > 5/4$ ensures $4x - 5 > 0$, so each side is $4x - 5$. A square has four equal sides; its perimeter is $4(4x - 5) = 16x - 20$.

31. B Standard: ACT Math: Functions

Factor the quadratic: $f(x) = (x - 1)(x - 3)$. The zeros divide the number line into three intervals. Between 1 and 3, one factor is positive and the other is negative, so the product is below zero. To the left of 1 and to the right of 3, both factors have the same sign, so their product is positive. Since the question asks for $f(x) > 0$, the zeros themselves are excluded.

32. J Standard: ACT Math: Geometry

Let the width be w centimeters, so the length is $w + 5$. The diagonal and sides form a right triangle, so $w^2 + (w + 5)^2 = 25^2$. Expanding gives $2w^2 + 10w - 600 = 0$, or $w^2 + 5w - 300 = 0 = (w + 20)(w - 15)$. A width must be positive, so $w = 15$ and the length is 20. The area is $15(20) = 300$ square centimeters.

33. C Standard: ACT Math: Number & Quantity

Expand before estimating: $(\sqrt{5} - \sqrt{2})^2 = 5 + 2 - 2\sqrt{10} = 7 - 2\sqrt{10}$. Since $\sqrt{10} \approx 3.162$, the expression is about $7 - 6.324 = 0.676$. That value lies between 0.6 and 0.8, so choice C is the interval that contains it.

34. F Standard: ACT Math: Algebra

First use $(x + y)^2 = x^2 + 2xy + y^2$: $81 = 53 + 2xy$, so $xy = 14$. Now apply $x^3 + y^3 = (x + y)^3 - 3xy(x + y)$. Substituting the known sum and product gives $9^3 - 3(14)(9) = 729 - 378 = 351$.

35. B Standard: ACT Math: Statistics & Probability

Count unordered pairs, since every two-token pair is equally likely. There are $\binom{9}{2} = 36$ pairs total, and $\binom{5}{2} = 10$ are both green. The condition removes the $\binom{4}{2} = 6$ all-yellow pairs, leaving $36 - 6 = 30$ eligible pairs. Of those, 10 have both tokens green, so the conditional probability is $10/30 = 1/3$.

36. J Standard: ACT Math: Integrating Essential Skills

In one hour, the pumps together fill $1/6 + 1/8 = 7/24$ of the tank. In two hours they fill $2(7/24) = 7/12$, leaving $1 - 7/12 = 5/12$ of the tank. Pump A alone fills $1/6$ of the tank each hour, so its time for the remaining portion is $(5/12)/(1/6) = (5/12)(6) = 5/2$ hours, or $2\frac{1}{2}$ hours.

37. A Standard: ACT Math: Functions

Let k be the number of \$2 price reductions. The theater then sells $240 + 15k$ tickets at $\$50 - \$2k$ each, so its revenue is $R(k) = (240 + 15k)(50 - 2k) = -30k^2 + 270k + 12,000$. This downward-opening parabola has its vertex at $k = -270/(2(-30)) = 4.5$. Since k must be an integer, check the two integer values surrounding the vertex, $k = 4$ and $k = 5$: at $k = 4$, revenue is $300 \cdot \$42 = \$12,600$, and at $k = 5$, it is $315 \cdot \$40 = \$12,600$. That is the greatest possible revenue.

38. G Standard: ACT Math: Geometry

The altitude to the hypotenuse is the geometric mean of the two segments it creates, so its length h satisfies $h^2 = 9 \cdot 16 = 144$ and $h = 12$. The full hypotenuse is $9 + 16 = 25$. Using the hypotenuse as the base and the altitude as the height, the triangle's area is $\frac{1}{2}(25)(12) = 150$ square units.

39. B Standard: ACT Math: Integrating Essential Skills

First find how much of the 50% solution is added. If that amount is x liters, then $0.20(12) + 0.50x = 0.30(12 + x)$, which gives $x = 6$. The resulting 18-liter mixture contains $0.30(18) = 5.4$ liters of the dissolved substance. Removing 4 liters of well-mixed solution removes $0.30(4) = 1.2$ liters of the substance, leaving 4.2 liters in 14 liters of solution. Adding 4 liters of water brings the volume back to 18 liters without changing the 4.2 liters of substance, so the new concentration is $4.2/18 = 7/30 = 23\frac{1}{3}\%$.

40. J Standard: ACT Math: Number & Quantity

The sum of a rational number and an irrational number is irrational, so $r + s$ cannot be rational. The product rs is irrational because r is a nonzero rational number; if rs were rational, dividing by r would make s rational. The quotient r/s also cannot be rational: if it were, then $s = r/(r/s)$ would be rational. But s^2 can be rational. For example, if $s = \sqrt{2}$, then $s^2 = 2$. Thus choice J is the one that could be rational.

41. A Standard: ACT Math: Algebra

Eliminate x by multiplying the first equation by 4 and the second by $k + 1$, then subtracting. The result is $(k^2 - 9)y = 7k - 13$. When k is neither 3 nor -3 , this determines y , and the original equations then determine x . At $k = 3$, the two left sides are both $4x + 2y$, but the equations require that expression to equal both 5 and 7, so there is no solution. At $k = -3$, the left sides are proportional while the constants are not: multiplying the first equation by -2 gives $4x - 4y = -10$, whereas the second requires $4x - 4y = 7$. Thus the system is inconsistent for exactly $k = -3$ and $k = 3$.

42. J Standard: ACT Math: Functions

Because an absolute value is nonnegative, the right side requires $x \geq 0$. Split by the expression inside the absolute value. If $x^2 - 4 \geq 0$, then $x \geq 2$ and the equation becomes $x^2 - 4 = x$; its positive root, $\frac{1+\sqrt{17}}{2}$, is greater than 2 and works. If $x^2 - 4 < 0$, then $0 \leq x < 2$ and the equation becomes $4 - x^2 = x$; its positive root, $\frac{-1+\sqrt{17}}{2}$, lies between 0 and 2 and works. The other root from each quadratic fails its interval condition, so there are exactly two solutions.

43. A Standard: ACT Math: Geometry

For intersecting chords, the products of the two segment lengths are equal: $AE \cdot EB = CE \cdot ED$. Substitution gives $(x + 1)(x + 17) = (2x + 1)(x + 7)$. Expanding and collecting terms yields $x^2 - 3x - 10 = 0$, or $(x - 5)(x + 2) = 0$. The roots are 5 and -2 , but $x = -2$ would make segment lengths negative. Therefore the positive value is $x = 5$; the segment products are then $6 \cdot 22 = 11 \cdot 12 = 132$.

44. H Standard: ACT Math: Statistics & Probability

The event can occur exactly 3, 4, or 5 times. For exactly 3 successes, choose which three trials succeed and multiply by the probability of that pattern: $\binom{5}{3}(1/3)^3(2/3)^2 = 40/243$. The corresponding probabilities for exactly 4 and exactly 5 successes are $\binom{5}{4}(1/3)^4(2/3) = 10/243$ and $(1/3)^5 = 1/243$. Adding these disjoint cases gives $51/243 = 17/81$.

45. A Standard: ACT Math: Integrating Essential Skills

Work backward through the price changes. Removing the 6% tax means the pre-tax discounted price was $\$169.60 \div 1.06 = \160 . A 20% discount leaves 80% of the markdown price, so that price was $\$160 \div 0.80 = \200 . The fixed \$15 reduction came first, which means the original price was $\$200 + \$15 = \$215$. Checking forward gives $215 - 15 = 200$, then $0.80(200) = 160$, and $1.06(160) = 169.60$.