

ISEE

MIDDLE LEVEL

MATH FORMULA REVIEW



$$y = mx + b$$

$$a : b = c : d$$



$$A = \pi r^2$$

$$C = 2\pi r$$



$$A = \frac{1}{2}bh$$

$$P = \frac{1}{2}100$$

$$x\% = \frac{x}{100}$$

$$50\% = \frac{1}{2}$$

$$2x + 3y = 12$$

$$x - y = 4$$



RATIOS & PROPORTIONS

$$a : b = c : d$$

$$\frac{a}{b} = \frac{c}{d}$$

PERCENTS

$$P = \frac{x}{100}$$

$$x\% = \frac{x}{100}$$

LINEAR EQUATIONS

$$y = mx + b$$

$m = \text{slope}$
 $b = \text{y-intercept}$

GEOMETRY

$$A = \frac{1}{2}bh$$

$$A = lw$$

$$P = 2(l + w)$$

USEFUL FORMULAS

- $A = \pi r^2$
- $C = 2\pi r$
- $a^2 + b^2 = c^2$
- $V = lwh$



1 Whole Numbers, Properties & Order

Number rules

Place value

A digit's value depends on its position: ones, tens, hundreds, tenths, hundredths, and so on.

Order of operations

Parentheses, exponents, multiply or divide left to right, then add or subtract left to right.

Commutative property

$$a + b = b + a \text{ and } ab = ba.$$

Associative property

$$(a + b) + c = a + (b + c) \text{ and } (ab)c = a(bc).$$

Distributive property

$$a(b + c) = ab + ac.$$

Identity properties

$$a + 0 = a \text{ and } a \cdot 1 = a.$$

Inverse properties

$$a + (-a) = 0 \text{ and } a \cdot \frac{1}{a} = 1 \text{ when } a \neq 0.$$

Tutor's Note

Order of operations keeps everyone evaluating expressions the same way. Multiplication and division are a team: do whichever appears first from left to right. Addition and subtraction work the same way.

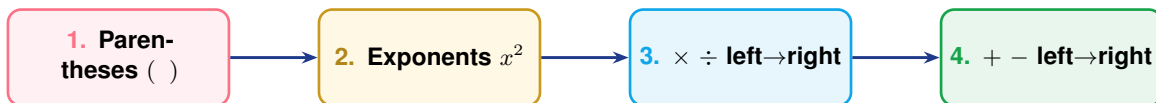
Example
 $18 - 3(2 + 4)^2 \div 6$. Parentheses: 6. Exponent: 36. Multiply/divide: $3 \cdot 36 \div 6 = 18$. Then $18 - 18 = 0$.



PEMDAS

PEMDAS does not mean all multiplication before all division. Work multiplication and division left to right.

Order of operations (PEMDAS) Work top to bottom, but multiply/divide share a step (left to right) and so do add/subtract.



2 Factors, Multiples & Number Types

Factors and number sense

Factor	A whole number that divides another whole number evenly.
Multiple	A result of multiplying a number by a whole number.
Prime number	A whole number greater than 1 with exactly two factors: 1 and itself.
Composite number	A whole number greater than 1 with more than two factors.
GCF	Greatest common factor: the largest factor shared by numbers.
LCM	Least common multiple: the smallest positive multiple shared by numbers.
GCF-LCM connection	For positive integers a, b : $\text{gcd}(a, b) \cdot \text{lcm}(a, b) = ab$.
Divisibility: 2, 5, 10	By 2 if even; by 5 if ending in 0 or 5; by 10 if ending in 0.
Divisibility: 3, 4, 6, 9	By 3 or 9 if the digit sum is divisible by 3 or 9; by 4 if the last two digits form a multiple of 4; by 6 if divisible by 2 and 3.

Tutor's Note

GCF helps you simplify and factor. LCM helps you build common denominators. Prime factorization is the cleanest way to find both.

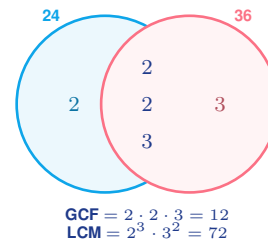
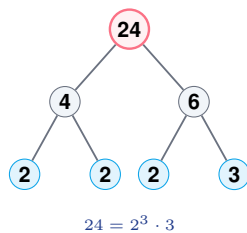
$24 = 2^3 \cdot 3$ and $36 = 2^2 \cdot 3^2$. $\text{GCF} = 2^2 \cdot 3 = 12$ and $\text{LCM} = 2^3 \cdot 3^2 = 72$.

Example



Zero is a multiple of every nonzero whole number, but it is not used as the least positive common multiple.

Prime factor trees & the GCF/LCM Venn Break each number into primes, then share them: the overlap is the GCF and the whole picture is the LCM.



3 Fractions & Mixed Numbers

Fraction operations

Equivalent fractions	$\frac{a}{b} = \frac{ak}{bk}$ when $b \neq 0$ and $k \neq 0$.
Simplify	$\frac{a}{b}$ is simplified by dividing numerator and denominator by their GCF, with $b \neq 0$.
Add/subtract	$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$, with $b, d \neq 0$.
Multiply	$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, with $b, d \neq 0$.
Divide	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$, with $b, c, d \neq 0$.
Reciprocal	The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$, with $a, b \neq 0$.
Mixed to improper	$a\frac{b}{c} = \frac{ac + b}{c}$ for positive mixed numbers; keep a negative sign outside until the end.
Part of a whole	A fraction $\frac{a}{b}$ means a parts out of b equal parts.

Tutor's Note

To add or subtract fractions, the denominators must name the same size pieces. To multiply, go straight across. To divide, keep the first fraction, change division to multiplication, and flip the second fraction.

$$\frac{2}{3} + \frac{5}{6} = \frac{4}{6} + \frac{5}{6} = \frac{9}{6} = \frac{3}{2} = 1\frac{1}{2}$$

Example

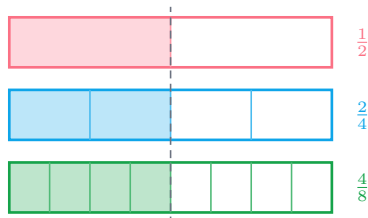


DENOMINATORS

A denominator can never be 0. Division by zero is undefined.

Fraction models: equivalent fractions

The same amount can be cut into more, smaller pieces. Each bar below is shaded *half*, so $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

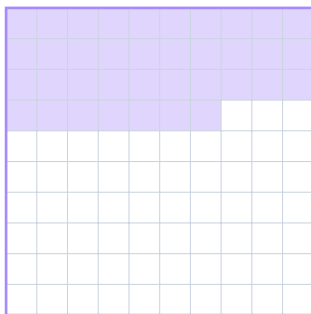


4 Decimals, Percents & Money Math

Decimal and percent formulas

Add/subtract decimals	Line up the decimal points, then add or subtract as usual.
Multiply decimals	Multiply as whole numbers, then place the point so the answer has the total number of decimal places.
Divide decimals	Move both decimals right until the divisor is whole, then divide.
Decimal to percent	Move the decimal two places right: $0.37 = 37\%$.
Percent to decimal	Move the decimal two places left: $85\% = 0.85$.
Percent to fraction	$p\% = \frac{p}{100}$, then simplify if possible.
Percent equation	part = percent · whole, using percent as a decimal.
Percent change	$\frac{\text{new} - \text{old}}{\text{old}} \cdot 100\%$, where $\text{old} \neq 0$.
Simple interest	$I = Prt$ and $A = P + I = P(1 + rt)$.
Tax / tip / markup	Total = original amount $\times (1 + r)$, where r is the rate as a decimal.
Discount / markdown	Sale price = original price $\times (1 - r)$, where r is the discount rate.

Percent means “out of 100” Shade a 10×10 grid: each little square is 1%. The same value is a fraction over 100 and a decimal.



$$37\% = 0.37 = \frac{37}{100}$$

Percent bar: part = percent \times whole



$$0.25 \times 80 = 20$$

Tutor's Note

The most common percent mistake is choosing the wrong whole. The whole is the original or total amount that the percent is based on.



Example
A jacket costs \$80 and is discounted 25%. Discount = $0.25(80) = 20$, so the sale price is \$60.



MONEY

Round money to the nearest cent only at the end unless the problem tells you to round earlier.

5 Ratios, Rates & Proportions

Proportion tools

Ratio

$a : b$, a to b , and $\frac{a}{b}$ all compare two quantities.

Rate

A ratio with different units, such as miles per hour.

Unit rate

A rate with denominator 1.

Proportion

$\frac{a}{b} = \frac{c}{d}$ means $ad = bc$, with $b, d \neq 0$.

Scale factor

scale factor = $\frac{\text{new length}}{\text{original length}}$

Similar figures

Corresponding side lengths are proportional and corresponding angles match.

Constant of proportionality

$y = kx$, where $k = \frac{y}{x}$ when $x \neq 0$.

Proportional graph

A proportional relationship graphs as a straight line through $(0, 0)$.

Unit price

unit price = $\frac{\text{total cost}}{\text{number of units}}$

Distance, rate, time

$d = rt$; so $r = \frac{d}{t}$ and $t = \frac{d}{r}$.

Percent proportion

$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$, with whole $\neq 0$.

Tutor's Note

A proportion is two equal ratios. Cross-multiplication works because multiplying both sides by the denominators clears the fractions.

Example
If 3 notebooks cost \$7.50, the unit rate is $7.50 \div 3 = \$2.50$ per notebook.

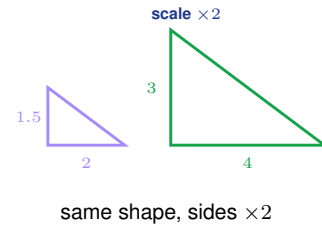
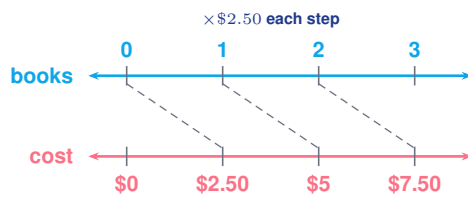


UNITS

Ratios compare. Rates compare with units. Keep units attached until the answer makes sense.



Ratio tables & similar figures Scale both quantities by the same number and the ratio stays the same.



6 Integers & Absolute Value

Signed number rules

Add same signs

Add absolute values and keep the common sign.

Add different signs

Subtract absolute values and keep the sign of the number farther from 0.

Subtract

$$a - b = a + (-b).$$

Multiply/divide signs

Same signs give positive; different signs give negative.

Zero pairs

$$a + (-a) = 0; \text{ opposites cancel in addition.}$$

Absolute value

$|a|$ is the distance from 0, so $|a| \geq 0$.

Opposites

a and $-a$ are the same distance from 0 in opposite directions.

Coordinate distance on a line

Distance between a and b is $|a - b|$.

Tutor's Note

For multiplying or dividing, count negatives: an even number of negative signs gives a positive result; an odd number gives a negative result.

$-7 + 12 = 5$ because the signs are different, so subtract $12 - 7$ and keep the sign of 12.

Example

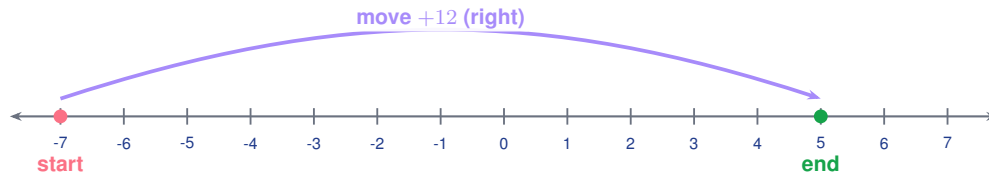


SIGNS

Do not treat subtraction like multiplication. For $5 - (-3)$, add the opposite:
 $5 + 3 = 8$.



Adding integers on a number line Start at the first number, then *move*: right to add a positive, left to add a negative.



$$-7 + 12 = 5. \quad \text{The two points are } |-7 - 5| = 12 \text{ apart.}$$

7 Expressions, Equations & Inequalities

Algebra basics

Expression	Numbers, variables, and operations without an equals sign.
Equation	A statement that two expressions are equal.
Evaluate	Substitute a number for a variable and simplify.
Substitution	Replace a variable with a given value, then use order of operations.
Combine like terms	Add/subtract coefficients of terms with the same variable part.
Distribute	$a(b + c) = ab + ac$.
One-step equation	$x + a = b \Rightarrow x = b - a$ and $ax = b \Rightarrow x = \frac{b}{a}$ when $a \neq 0$.
Two-step equation	For $ax + b = c$, subtract b first, then divide by a when $a \neq 0$.
Inequality rule	If multiplying or dividing by a negative number, flip the inequality symbol.

Tutor's Note

Solving means keeping the equation balanced while isolating the variable. Whatever you do to one side, do to the other side.

Solve $3x + 5 = 20$. Subtract 5: $3x = 15$. Divide by 3: $x = 5$.

Example



9 Coordinate Plane & Linear Patterns

Graphing basics

Ordered pair

(x, y) gives horizontal movement first, vertical movement second.

Quadrants

I: (+, +), II: (-, +), III: (-, -), IV: (+, -).

Slope

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}, \text{ with } x_2 \neq x_1.$$

Rate of change

Another name for slope: how much y changes for each 1 unit of x .

y -intercept

The point where a graph crosses the y -axis.

x -intercept

The point where a graph crosses the x -axis.

Linear pattern

$y = mx + b$, where m is slope and b is the y -intercept.

Direct variation

$y = kx$ is a proportional linear pattern through the origin.

Horizontal line

$y = c$ has slope 0.

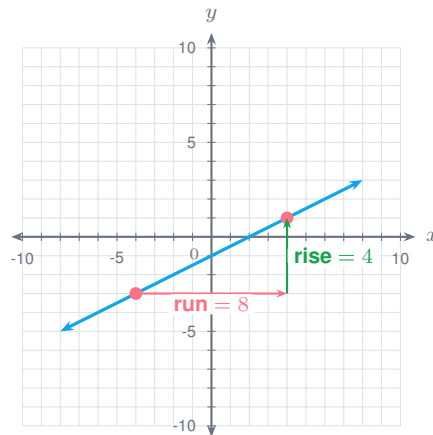
Vertical line

$x = c$ has undefined slope.

Tutor's Note

The coordinate plane is a map. The x -axis moves left and right; the y -axis moves down and up. Slope tells how much y changes for each 1 step in x .

Visual: slope is rise over run

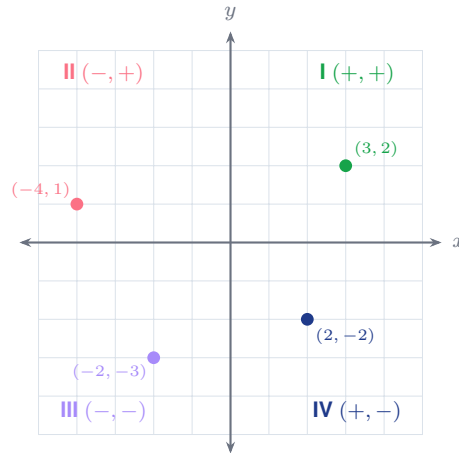




GRAPH

A vertical line fails the function test because one input has many outputs.

The coordinate plane: quadrants & plotting An ordered pair (x, y) moves right/left first, then up/down. The signs tell you the quadrant.



Graphing checkpoint Read the plane like a map: horizontal first, vertical second.

Ordered pair

$(-4, 3)$ means left 4, then up 3. The x -move comes first.

Intercepts

y -intercept: where the graph crosses the y -axis. x -intercept: where it crosses the x -axis.

Quadrant signs

I $(+, +)$, II $(-, +)$, III $(-, -)$, IV $(+, -)$.

Line patterns

$y = mx + b$ shows slope m and starting value b .

Slope

$m = \frac{\text{rise}}{\text{run}}$. Up/right is positive; down/right is negative.

Fast check

A vertical line has undefined slope because its run is 0.

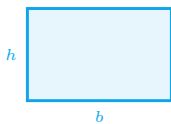


10 Geometry & Measurement

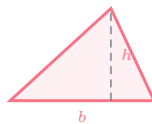
Plane geometry

Rectangle perimeter	$P = 2l + 2w.$
Rectangle area	$A = lw.$
Square perimeter / area	$P = 4s$ and $A = s^2.$
Triangle area	$A = \frac{1}{2}bh.$
Parallelogram area	$A = bh.$
Trapezoid area	$A = \frac{1}{2}(b_1 + b_2)h.$
Circle circumference	$C = 2\pi r = \pi d.$
Circle area	$A = \pi r^2.$
Pythagorean theorem	$a^2 + b^2 = c^2$ for right triangles, where c is the hypotenuse.

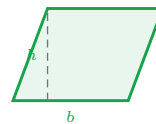
Area formulas you can see Area is the flat space inside a figure; the height is always perpendicular to the base.



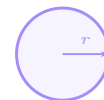
Rectangle
 $A = bh$



Triangle
 $A = \frac{1}{2}bh$



Parallelogram
 $A = bh$



Circle
 $A = \pi r^2$

Geometry checkpoint Name the measurement before choosing a formula.

Perimeter

Add side lengths around the outside. Units stay linear.

Circles

$C = 2\pi r = \pi d$ and $A = \pi r^2$. Diameter is twice the radius.

Area

Covering a flat region uses square units. Rectangle: $A = lw$.

Pythagorean check

Use $a^2 + b^2 = c^2$ only for right triangles; c is the hypotenuse.

Triangle area

$A = \frac{1}{2}bh$. The height must be perpendicular to the base.

Fast check

Area and volume answers should include square or cubic units.



Solid geometry: volume & surface area

Rectangular prism

$$V = lwh; \text{ surface area } SA = 2lw + 2lh + 2wh.$$

Cube

$$V = s^3; \text{ surface area } SA = 6s^2.$$

Cylinder

$$V = \pi r^2 h; \text{ surface area } SA = 2\pi r^2 + 2\pi r h.$$

Triangular prism

$$V = \left(\frac{1}{2}bh\right)L; \text{ } SA = \text{two triangle ends} + \text{three rectangle faces.}$$

Cone

$$V = \frac{1}{3}\pi r^2 h.$$

Pyramid

$$V = \frac{1}{3}(\text{base area})h.$$

Sphere

$$V = \frac{4}{3}\pi r^3; \text{ surface area } SA = 4\pi r^2.$$

Surface area idea

Add the areas of every outside face or curved surface; nets help organize the faces.

Tutor's Note

Area is measured in square units because it covers a flat region. Volume is measured in cubic units because it fills space. Make sure all measurements use the same unit before using a formula.

Example

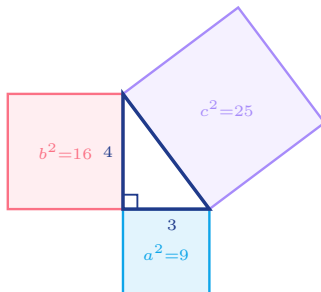
A triangle with base 10 cm and height 7 cm has area $A = \frac{1}{2}(10)(7) = 35$ square centimeters.



GEOMETRY

For the Pythagorean theorem, c is always the longest side, across from the right angle.

The Pythagorean theorem: $a^2 + b^2 = c^2$ For a right triangle, the square built on the hypotenuse equals the two squares on the legs added together.



The legs are $a = 3$ and $b = 4$, so
 $c^2 = a^2 + b^2 = 9 + 16 = 25$,
 which gives $c = \sqrt{25} = 5$.

The hypotenuse c is always the longest side, opposite the right angle.



11 Measurement & Unit Conversions

Customary (U.S.) units

Length

12 in = 1 ft; 3 ft = 1 yd; 5280 ft = 1 mi.

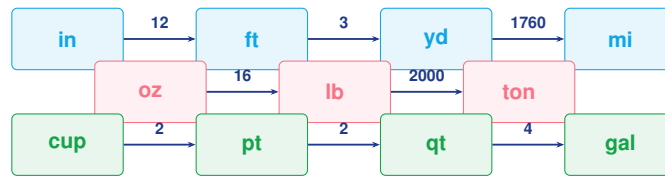
Weight

16 oz = 1 lb; 2000 lb = 1 ton.

Capacity

8 fl oz = 1 cup; 2 cups = 1 pt; 2 pt = 1 qt; 4 qt = 1 gal.

Unit conversion checkpoint Write the unit you want to cancel on the bottom.



Length path

1 yd = 3 ft and 1 ft = 12 in, so
2 yd = 72 in.

Fraction setup

$5 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}}$: feet cancel.

Weight path

1 lb = 16 oz; multiply pounds by 16 to
get ounces.

Reasonableness

Smaller units make larger numbers;
larger units make smaller numbers.

Capacity path

1 gal = 4 qt = 8 pt = 16 cups.

Fast check

Keep units attached until the final
answer. They show whether the setup
is right.

Try it 4 ft = 48 in 3 lb = 48 oz 2 gal = 8 qt = 32 cups Smaller units make the number larger.



Metric units & time

Metric length

$$10 \text{ mm} = 1 \text{ cm}; 100 \text{ cm} = 1 \text{ m}; 1000 \text{ m} = 1 \text{ km}.$$

Metric mass

$$1000 \text{ mg} = 1 \text{ g}; 1000 \text{ g} = 1 \text{ kg}.$$

Metric capacity

$$1000 \text{ mL} = 1 \text{ L}.$$

Metric prefixes

$$\text{kilo} = 1000, \text{hecto} = 100, \text{deca} = 10, \text{deci} = \frac{1}{10}, \text{centi} = \frac{1}{100}, \text{milli} = \frac{1}{1000}.$$

Time

$$60 \text{ s} = 1 \text{ min}; 60 \text{ min} = 1 \text{ hr}; 24 \text{ hr} = 1 \text{ day}; 7 \text{ days} = 1 \text{ wk}.$$

Temperature

$$F = \frac{9}{5}C + 32 \text{ and } C = \frac{5}{9}(F - 32).$$

Converting units: multiply by a fraction equal to 1 Write the conversion so the *old* unit cancels and the *new* unit is left.

$$5 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} \Rightarrow 60 \text{ in}$$

the "ft" units cancel, leaving inches

Tutor's Note

To convert, multiply by a fraction that equals 1 (the same amount written two ways). Put the unit you want to cancel on the bottom and the unit you want to keep on top.

Example

$$\text{Convert 3 yards to inches: } 3 \text{ yd} \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{12 \text{ in}}{1 \text{ ft}} = 108 \text{ in}.$$



UNITS

Metric prefixes move the decimal by powers of 10: from km to m multiply by 1000; from m to cm multiply by 100.

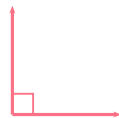
12 Angles, Lines & Polygons

Types of angles Angles are measured in degrees. A right angle is exactly 90° ; a straight angle is 180° .



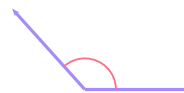
Acute

less than 90°



Right

exactly 90°



Obtuse

between 90° and 180°



Straight

exactly 180°



Angle & figure facts

Right / straight angle

A right angle is 90° ; a straight angle is 180° ; a full turn is 360° .

Complementary angles

Two angles whose measures add to 90° .

Supplementary angles

Two angles whose measures add to 180° (a straight line).

Vertical angles

When two lines cross, opposite angles are equal.

Triangle angle sum

The three interior angles of a triangle add to 180° .

Quadrilateral angle sum

The four interior angles of a quadrilateral add to 360° .

Polygon angle sum

For n sides, interior angles add to $(n - 2) \cdot 180^\circ$.

Regular polygon angle

Each interior angle is $\frac{(n-2)180^\circ}{n}$.

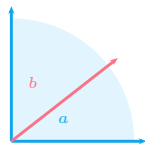
Exterior angles

One exterior angle at each vertex adds to 360° ; regular exterior angle is $\frac{360^\circ}{n}$.

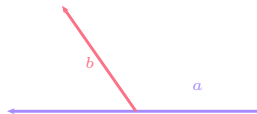
Parallel lines

Lines in a plane that never meet; a crossing line makes equal matching angles.

Angle pairs Complementary angles add to 90° , supplementary angles add to 180° , and vertical angles (across an X) are equal.



Complementary
 $a + b = 90^\circ$



Supplementary
 $a + b = 180^\circ$



Vertical
opposite angles equal

Tutor's Note

Most angle problems are just addition. If two angles form a right angle, they are complementary; if they form a straight line, they are supplementary. Set the known sum equal to the parts and solve.

If one angle of a complementary pair is 35° , the other is $90^\circ - 35^\circ = 55^\circ$.

Example



ANGLES

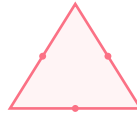
The angles of a triangle always total 180° , so two angles determine the third:
 $c = 180^\circ - a - b$.



Triangle angle sum & figure types The three angles of any triangle add to 180° ; the four angles of any quadrilateral add to 360° .



$a + b + c = 180^\circ$



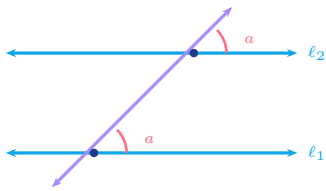
equilateral: all sides =



rectangle

4 right angles

Parallel lines cut by a transversal A line crossing two parallel lines makes equal matching angles.



Corresponding angles (same position at each crossing) are **equal**.

Alternate interior angles (opposite sides, between the lines) are **equal**.

Co-interior angles (same side, between the lines) are **supplementary** — they add to 180° .

Interior angle sums: $(n - 2) \times 180^\circ$ Every extra side adds another 180° to the total.



Triangle

3 sides = 180°



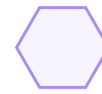
Quadrilateral

4 sides = 360°



Pentagon

5 sides = 540°



Hexagon

6 sides = 720°

Angle checkpoint Most angle questions are sum facts plus simple equations.

Pairs

$a + b = 90^\circ$ for complementary;
 $a + b = 180^\circ$ for supplementary.

Triangles

$a + b + c = 180^\circ$. Two angles determine the third.

Quadrilaterals

Four interior angles add to 360° .

Polygons

Interior sum = $(n - 2)180^\circ$. Divide by n only for regular polygons.

Parallel lines

Corresponding and alternate interior angles match; same-side interior angles add to 180° .

Fast check

A straight line is 180° and a full turn is 360° .



13 Data, Statistics & Probability

Statistics formulas

Mean	$\bar{x} = \frac{\text{sum of values}}{\text{number of values}}$
Median	The middle value after the data are ordered.
Mode	The value or values that occur most often.
Range	Maximum minus minimum.
Quartiles	Q_1 , median, Q_3 split ordered data into four equal parts.
Interquartile range	$IQR = Q_3 - Q_1$, the spread of the middle half.
Mean absolute deviation	$MAD = \frac{\sum x - \bar{x} }{n}$, the average distance from the mean.
Probability	$P(\text{event}) = \frac{\text{favorable outcomes}}{\text{total equally likely outcomes}}$
Experimental probability	$\frac{\text{times event occurs}}{\text{number of trials}}$
Complement	$P(\text{not } A) = 1 - P(A)$.
Simple counting principle	If one choice has m options and another has n options, together there are mn outcomes.

Tutor's Note

Statistics summarize data. Probability predicts chance. Both depend on reading the question carefully: are the outcomes equally likely, and what exactly counts as a success?

For data 4, 5, 6, 9, the mean is $\frac{4+5+6+9}{4} = 6$ and the range is $9 - 4 = 5$.

Example



Outliers can pull the mean more than the median. If one value is far away from the rest, compare both measures.

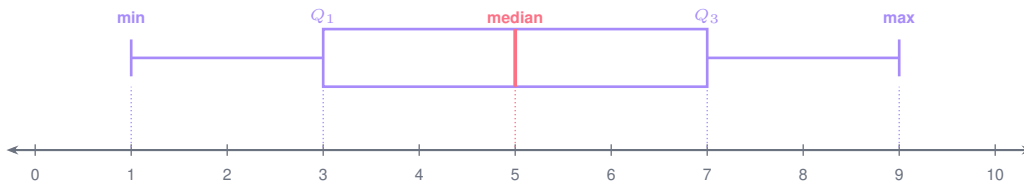


Mean, median & mode on a dot plot Stack a dot for each value, then read the center three ways. The mean balances the data; the median is the middle; the mode is the most common.



Data 2, 3, 3, 4, 8: mean $\frac{20}{5} = 4$, median 3, mode 3, range $8 - 2 = 6$.

The five-number summary & box plot Order the data, then mark the minimum, first quartile Q_1 , median, third quartile Q_3 , and maximum.



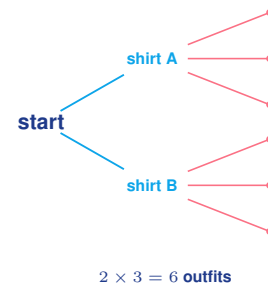
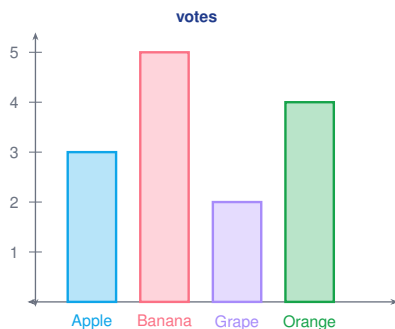
The box holds the middle half of the data; its width ($Q_3 - Q_1$) is the interquartile range (IQR).

The probability scale Every probability is a number from 0 (impossible) to 1 (certain). Half-way is an even chance.



0% to 100% — e.g. one head on a fair coin flip is $\frac{1}{2} = 50\%$.

Reading data & counting outcomes A bar graph compares categories; the counting principle multiplies the choices at each stage.



1 Foundations of Algebra

Before we build anything tall, we need a solid base. This section covers the ground rules that every later topic leans on — how numbers behave, the order we work in, and how to handle signs and fractions without fear.

◆ The Real-Number Properties

These four properties are the “rules of the game.” You already use them without thinking; algebra just gives them names.

Commutative (order)

$$a + b = b + a$$

$$a \cdot b = b \cdot a$$

Distributive (share out)

$$a(b + c) = ab + ac$$

Associative (grouping)

$$(a + b) + c = a + (b + c)$$

$$(ab)c = a(bc)$$

Identity & Inverse

$$a + 0 = a, \quad a \cdot 1 = a$$

$$a + (-a) = 0, \quad a \cdot \frac{1}{a} = 1 \quad (a \neq 0)$$

Tutor's Note

Commutative means order doesn't matter for adding or multiplying: $3 + 5$ and $5 + 3$ both give 8. **Associative** means it doesn't matter how you group them: you can add the first two numbers or the last two and land in the same place. The **distributive** property is the real workhorse of algebra — it lets you “hand out” a multiplier to everything inside parentheses, which is how we expand and factor later. Notice these work for $+$ and \times , but *not* for subtraction or division: $6 - 2 \neq 2 - 6$.

◆ Order of Operations (PEMDAS)

When an expression has several operations, we all need to agree on what to do first — otherwise the same expression could give different answers.

Do them in this order

Parentheses → **E**xponents → **M**ultiplication & **D**ivision (left → right) → **A**ddition & **S**ubtraction (left → right)

Tutor's Note

A useful phrase is “*Please Excuse My Dear Aunt Sally*.” The one trap: multiply and divide are a *team* — do whichever comes first reading left to right, not all multiplication before all division. Same goes for add and subtract.

Example
 $3 + 4 \times 2^2 - (6 - 2)$. First parentheses: $(6 - 2) = 4$. Then the exponent: $2^2 = 4$. Then multiply: $4 \times 4 = 16$. Finally left to right: $3 + 16 - 4 = 15$.



◆ Solving Linear Equations

The goal: isolate the variable

Undo operations in reverse PEMDAS order.

$$ax + b = c \implies x = \frac{c - b}{a} \quad (a \neq 0)$$

Tutor's Note

Picture a wrapped present: it was wrapped in a certain order, and you unwrap it in the *opposite* order. If x was multiplied by a and then had b added, you undo it by first subtracting b , then dividing by a . Every step you do to the left side, do to the right side too — that keeps the scale balanced.

$3x + 5 = 20$. Subtract 5: $3x = 15$. Divide by 3: $x = 5$. Check: $3(5) + 5 = 20$. ✓

Example

◆ Special Solution Cases

Sometimes the variable disappears as you solve. That's not a mistake — it's telling you something.

One solution

$$2x + 1 = x + 4 \implies x = 3$$

Infinitely many

$$2(x + 1) = 2x + 2 \text{ (always true)}$$

No solution

$$x + 2 = x + 5 \implies 2 = 5 \text{ (false)}$$

Literal equations

$$A = \ell w \implies w = \frac{A}{\ell} \quad (\ell \neq 0)$$

Tutor's Note

If you end with something *false* like $2 = 5$, no number can make it work, so there's *no solution*. If you end with something *always true* like $2 = 2$, then *every* number works. A "literal" equation just has more than one letter — you solve for one variable using the same balancing moves, treating the others like numbers.

◆ Inequalities

The one new rule

Solve exactly like an equation, **but flip the inequality sign** whenever you multiply or divide both sides by a *negative* number.

$$-2x < 6 \implies x > -3$$



Tutor's Note

Why the flip? Think about a true statement like $3 < 5$. Multiply both sides by -1 and you get -3 and -5 . But -3 is actually *greater* than -5 , so the sign has to turn around to stay true. That's the whole reason — negatives reverse order on the number line.

Visual: graphing $x > -3$



Reading the symbols

$<$ less than $>$ greater than
 \leq at most \geq at least

Compound inequalities

AND: $a < x < b$ (between)
 OR: $x < a$ or $x > b$



GRAPH

A solid dot or closed bracket means “or equal to” (\leq, \geq). An open dot means strictly less or strictly greater.

◆ Absolute-Value Equations & Inequalities

Split into two cases

$$|ax + b| = c \Rightarrow ax + b = c \text{ or } ax + b = -c \quad (c \geq 0)$$

$$|x| < c \Rightarrow -c < x < c \quad |x| > c \Rightarrow x < -c \text{ or } x > c \quad (c > 0)$$

Tutor's Note

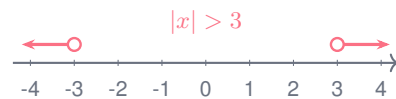
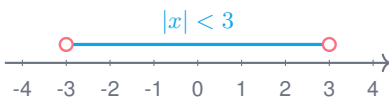
Because absolute value measures distance, $|x| = 5$ asks “what is 5 away from zero?” — and there are *two* answers, 5 and -5 . So every absolute-value equation splits into two. For “less than,” you want the points *close* to zero (a sandwich, between two values); for “greater than,” you want the points *far* from zero (two pieces heading outward).



ABS

If an absolute-value equation equals a negative number, it has no solution. For absolute-value inequalities, check $c < 0$ as a special case because distance is never negative.

Visual: close to zero vs. far from zero



◆ Translating Words into Algebra

Half of algebra is turning English into symbols. Here's your dictionary.

- sum / increased by / more than → +
- difference / less than / fewer → −
- product / of / times / twice → ×
- quotient / per / split → ÷
- is / was / equals / gives → =
- a number / unknown → x



WORDS

“5 less than a number” is $x - 5$, *not* $5 - x$. The order matters because “less than” reverses the order in the expression.

3 Functions, Lines & Slope

A function is one of the biggest ideas in all of math: a reliable machine that turns each input into exactly one output. Lines are the simplest functions, and slope is the number that describes how steep they are.

◆ What a Function Is

Definition

A **function** gives exactly one output y for each input x . We write $y = f(x)$. **Vertical Line Test:** a graph is a function if no vertical line ever hits it twice.

Tutor's Note

Think of a vending machine: press B4 and you always get the same snack. That “same input, same single output” rule is what makes something a function. The notation $f(x)$ is read “ f of x ” and just means “the output when the input is x .” It is *not* multiplication — it’s a name tag for the machine’s result.

Visual: vertical line test

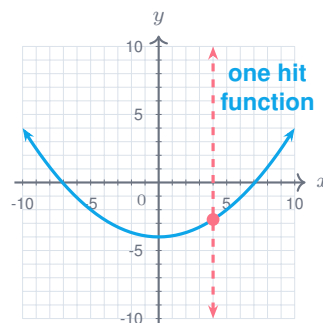
How to read it

A graph is a function when each input x has only one output y .
The dashed vertical line touches this curve once, so it passes the test.

Domain — allowed inputs.

Range — possible outputs.

Evaluate: $f(3) = 2(3) + 1 = 7$.



◆ Slope — Steepness in a Number

Slope between two points

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \quad (x_2 \neq x_1)$$

Tutor's Note

Slope answers “for every step I take to the right, how far do I go up or down?” That’s the *rise over run*. A big slope is a steep hill; a small slope is a gentle ramp. The sign tells direction: positive climbs as you read left to right, negative descends.



SLOPE

Subtract the *x*’s and the *y*’s in the *same order*. Mixing the order flips the sign and changes the slope.

Visual: rise over run

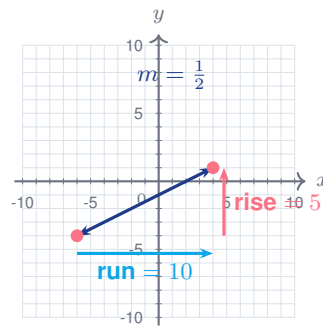
Slope is a rate

Count the horizontal change first, then the vertical change.

Here, run = 10 and rise = 5, so

$$m = \frac{5}{10} = \frac{1}{2}$$

Positive slope rises left to right. Negative slope falls left to right.



- $m > 0$: line rises (↗)
- $m < 0$: line falls (↘)
- $m = 0$: flat, horizontal $y = b$
- m undefined: vertical $x = a$

◆ Forms of a Linear Equation

The same line can be written several ways. Each form is handy for a different job.

Slope-Intercept

$$y = mx + b$$

slope m , y -intercept b

Standard

$$Ax + By = C$$

(A and B not both 0)

Point-Slope

$$y - y_1 = m(x - x_1)$$

Intercepts

x -int: set $y = 0$
 y -int: set $x = 0$



Tutor's Note

Use **slope-intercept** when you know the slope and where the line crosses the y -axis — you can graph it instantly. Use **point-slope** when you know the slope and *any* one point; it's the fastest way to write a line's equation from scratch. **Standard form** is tidy for finding both intercepts. They all describe the same line — just dressed for different occasions.

Visual: slope-intercept form

Use slope-intercept form

The line crosses the y -axis at $b = 1$.

The slope is $\frac{1}{2}$: move right 2, then up 1; or right 4, then up 2.

The arrows show the line continues forever in both directions.



Parallel & Perpendicular Lines

It's all in the slopes

Parallel: $m_1 = m_2$ **Perpendicular:** $m_1 \cdot m_2 = -1$ (negative reciprocals, for nonvertical lines)

Tutor's Note

Parallel lines never meet, so they must rise at the exact same rate — equal slopes. Perpendicular lines cross at a right angle, and it turns out their slopes are “negative reciprocals”: flip the fraction and change the sign. So a slope of 2 is perpendicular to $-\frac{1}{2}$. A quick check: multiply them and you should get -1 .



LINES

A vertical line and a horizontal line are perpendicular, but the vertical line has undefined slope, so the product rule does not apply to that pair.

Distance & Midpoint

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Tutor's Note

The distance formula is secretly the Pythagorean theorem in disguise: the horizontal gap and vertical gap are the two legs of a right triangle, and the distance is the hypotenuse. The midpoint is even friendlier — it's just the average of the two x 's and the average of the two y 's, landing you



exactly halfway.

◆ Direct & Inverse Variation

Direct: $y = kx$

Inverse: $y = \frac{k}{x}$ ($x \neq 0, k \neq 0$), or $xy = k$

Tutor's Note

Direct variation is a straight line through the origin — double the input, double the output when k is positive (think hours worked and pay earned). If k is negative, the line still passes through the origin but goes downward. **Inverse** variation is a constant-product trade-off — with positive quantities, more speed means less travel time. The constant k stays fixed and ties the two quantities together.

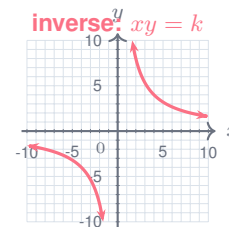
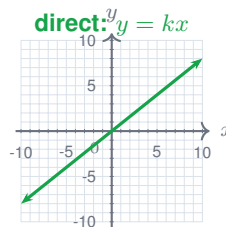
Visual: direct vs. inverse variation

Compare the shapes

Direct variation is a line through the origin: $y = kx$.

Inverse variation bends in two branches because $xy = k$ stays fixed.

Both graphs continue beyond the viewing window.



4 Exponents & Scientific Notation

An exponent is just a shorthand for repeated multiplication. Once you see that, every exponent rule below stops being something to memorize and becomes something you can figure out.

◆ The Laws of Exponents

These rules apply whenever every expression involved is defined. In this list, m and n are integers unless a fractional exponent is shown.

Product

$$a^m \cdot a^n = a^{m+n}$$

Power of a Power

$$(a^m)^n = a^{mn}$$

Quotient

$$\frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$$

Power of a Product

$$(ab)^n = a^n b^n$$



Power of a Quotient

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (b \neq 0)$$

Negative Exponent

$$a^{-n} = \frac{1}{a^n} \quad (a \neq 0)$$

Zero Exponent

$$a^0 = 1 \quad (a \neq 0)$$

Fractional Exponent

$$a^{m/n} = (\sqrt[n]{a})^m \quad (n \in \mathbb{Z}^+)$$

Real-number note: with even roots, the base under the root must be nonnegative. For example, $\sqrt{x^2} = |x|$, not always x . With odd roots, negative radicands are allowed.

Tutor's Note

Why these all make sense $a^3 \cdot a^2$ means $(a \cdot a \cdot a)(a \cdot a)$ — five a 's, so you *add* the exponents. Dividing cancels factors, so you *subtract*. A power of a power stacks groups, so you *multiply*. The strange ones follow the same logic: $a^0 = 1$ because $\frac{a^3}{a^3} = a^{3-3} = a^0$ and anything over itself is 1. A *negative* exponent just means “flip it to the other side of the fraction.” Memorize the pattern once and the rest is bookkeeping.

$$\frac{2x^3 \cdot 3x^4}{x^2} = \frac{6x^7}{x^2} = 6x^5. \quad (\text{Multiply coefficients, add then subtract exponents.})$$

Example**Scientific Notation**

A compact way to write very big or very small numbers without drowning in zeros.

Standard form

$$a \times 10^n, \quad 1 \leq |a| < 10, \quad n \in \mathbb{Z}$$

Big numbers: $n > 0$. Numbers with $0 < |x| < 1$: $n < 0$. Zero is written as 0.

Tutor's Note

The power of 10 just counts how many places the decimal point moves. Positive powers push it right (making the number bigger); negative powers pull it left (smaller). To multiply two of these, multiply the front numbers and *add* the exponents; to divide, divide the fronts and *subtract* — exactly the exponent laws from above.

$$3,400 = 3.4 \times 10^3 \quad 0.0052 = 5.2 \times 10^{-3} \\ (2 \times 10^5)(3 \times 10^2) = 6 \times 10^7.$$

Example

Tip: The front number always has exactly one digit before the decimal point — that's what makes it “standard.”

