

$$r = \frac{d}{t}$$

$$t = \frac{d}{r}$$



# ASTB MATH

## FORMULA REVIEW



### Rate Problems

$$d = rt$$

$$r = \frac{d}{t}$$

$$t = \frac{d}{r}$$

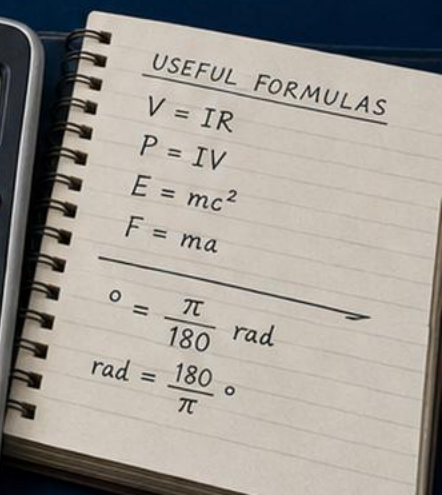
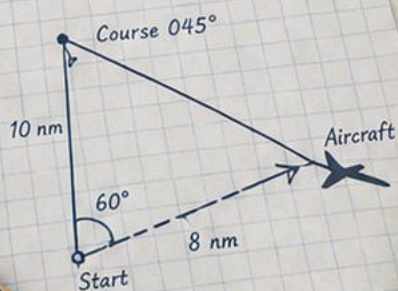
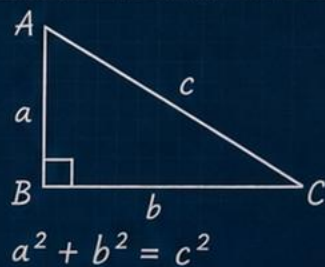
An aircraft flies 360 miles in 2.4 hours. What is its average rate (speed)?

$$r = \frac{360}{2.4} = 150 \text{ mph}$$

### Geometry

$$A = \pi r^2$$

$$C = 2\pi r$$



FLIGHT READY

RATE PROBLEMS

# 1 Whole Numbers, Properties & Order

## Number rules

### Place value

A digit's value depends on its position: ones, tens, hundreds, tenths, hundredths, and so on.

### Order of operations

Parentheses, exponents, multiply or divide left to right, then add or subtract left to right.

### Commutative property

$$a + b = b + a \text{ and } ab = ba.$$

### Associative property

$$(a + b) + c = a + (b + c) \text{ and } (ab)c = a(bc).$$

### Distributive property

$$a(b + c) = ab + ac.$$

### Identity properties

$$a + 0 = a \text{ and } a \cdot 1 = a.$$

### Inverse properties

$$a + (-a) = 0 \text{ and } a \cdot \frac{1}{a} = 1 \text{ when } a \neq 0.$$

## Tutor's Note

Order of operations keeps everyone evaluating expressions the same way. Multiplication and division are a team: do whichever appears first from left to right. Addition and subtraction work the same way.

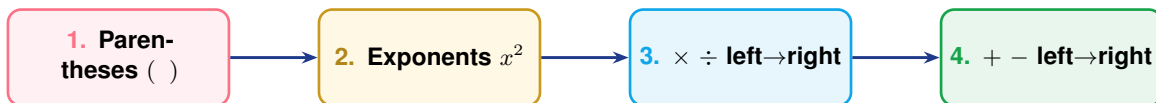
**Example**  
 $18 - 3(2 + 4)^2 \div 6$ . Parentheses: 6. Exponent: 36. Multiply/divide:  $3 \cdot 36 \div 6 = 18$ . Then  $18 - 18 = 0$ .



PEMDAS

PEMDAS does not mean all multiplication before all division. Work multiplication and division left to right.

**Order of operations (PEMDAS)** Work top to bottom, but multiply/divide share a step (left to right) and so do add/subtract.



## 2 Factors, Multiples & Number Types

### Factors and number sense

<b>Factor</b>	A whole number that divides another whole number evenly.
<b>Multiple</b>	A result of multiplying a number by a whole number.
<b>Prime number</b>	A whole number greater than 1 with exactly two factors: 1 and itself.
<b>Composite number</b>	A whole number greater than 1 with more than two factors.
<b>GCF</b>	Greatest common factor: the largest factor shared by numbers.
<b>LCM</b>	Least common multiple: the smallest positive multiple shared by numbers.
<b>GCF-LCM connection</b>	For positive integers $a, b$ : $\text{gcd}(a, b) \cdot \text{lcm}(a, b) = ab$ .
<b>Divisibility:</b> 2, 5, 10	By 2 if even; by 5 if ending in 0 or 5; by 10 if ending in 0.
<b>Divisibility:</b> 3, 4, 6, 9	By 3 or 9 if the digit sum is divisible by 3 or 9; by 4 if the last two digits form a multiple of 4; by 6 if divisible by 2 and 3.

#### Tutor's Note

GCF helps you simplify and factor. LCM helps you build common denominators. Prime factorization is the cleanest way to find both.

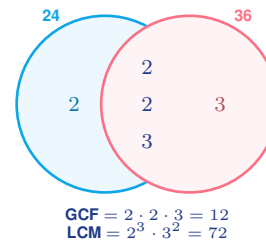
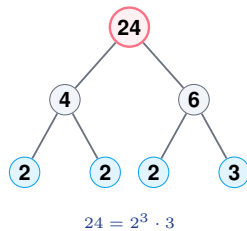
$24 = 2^3 \cdot 3$  and  $36 = 2^2 \cdot 3^2$ .  $\text{GCF} = 2^2 \cdot 3 = 12$  and  $\text{LCM} = 2^3 \cdot 3^2 = 72$ .

**Example**



Zero is a multiple of every nonzero whole number, but it is not used as the least positive common multiple.

**Prime factor trees & the GCF/LCM Venn** Break each number into primes, then share them: the overlap is the GCF and the whole picture is the LCM.



### 3 Fractions & Mixed Numbers

#### Fraction operations

<b>Equivalent fractions</b>	$\frac{a}{b} = \frac{ak}{bk}$ when $b \neq 0$ and $k \neq 0$ .
<b>Simplify</b>	$\frac{a}{b}$ is simplified by dividing numerator and denominator by their GCF, with $b \neq 0$ .
<b>Add/subtract</b>	$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$ , with $b, d \neq 0$ .
<b>Multiply</b>	$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ , with $b, d \neq 0$ .
<b>Divide</b>	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$ , with $b, c, d \neq 0$ .
<b>Reciprocal</b>	The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$ , with $a, b \neq 0$ .
<b>Mixed to improper</b>	$a\frac{b}{c} = \frac{ac + b}{c}$ for positive mixed numbers; keep a negative sign outside until the end.
<b>Part of a whole</b>	A fraction $\frac{a}{b}$ means $a$ parts out of $b$ equal parts.

#### Tutor's Note

To add or subtract fractions, the denominators must name the same size pieces. To multiply, go straight across. To divide, keep the first fraction, change division to multiplication, and flip the second fraction.

$$\frac{2}{3} + \frac{5}{6} = \frac{4}{6} + \frac{5}{6} = \frac{9}{6} = \frac{3}{2} = 1\frac{1}{2}$$

**Example**

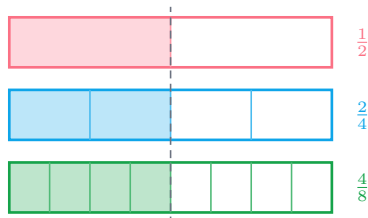


#### DENOMINATORS

A denominator can never be 0. Division by zero is undefined.

#### Fraction models: equivalent fractions

The same amount can be cut into more, smaller pieces. Each bar below is shaded *half*, so  $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$ .

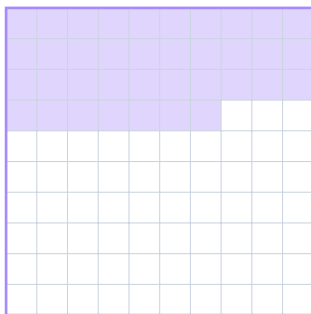


## 4 Decimals, Percents & Money Math

### Decimal and percent formulas

<b>Add/subtract decimals</b>	Line up the decimal points, then add or subtract as usual.
<b>Multiply decimals</b>	Multiply as whole numbers, then place the point so the answer has the total number of decimal places.
<b>Divide decimals</b>	Move both decimals right until the divisor is whole, then divide.
<b>Decimal to percent</b>	Move the decimal two places right: $0.37 = 37\%$ .
<b>Percent to decimal</b>	Move the decimal two places left: $85\% = 0.85$ .
<b>Percent to fraction</b>	$p\% = \frac{p}{100}$ , then simplify if possible.
<b>Percent equation</b>	part = percent · whole, using percent as a decimal.
<b>Percent change</b>	$\frac{\text{new} - \text{old}}{\text{old}} \cdot 100\%$ , where $\text{old} \neq 0$ .
<b>Simple interest</b>	$I = Prt$ and $A = P + I = P(1 + rt)$ .
<b>Tax / tip / markup</b>	Total = original amount $\times (1 + r)$ , where $r$ is the rate as a decimal.
<b>Discount / markdown</b>	Sale price = original price $\times (1 - r)$ , where $r$ is the discount rate.

**Percent means “out of 100”** Shade a  $10 \times 10$  grid: each little square is 1%. The same value is a fraction over 100 and a decimal.



$$37\% = 0.37 = \frac{37}{100}$$

**Percent bar: part = percent  $\times$  whole**



$$0.25 \times 80 = 20$$

#### Tutor's Note

The most common percent mistake is choosing the wrong whole. The whole is the original or total amount that the percent is based on.



A jacket costs \$80 and is discounted 25%. Discount =  $0.25(80) = 20$ , so the sale price is \$60. **Example**



MONEY

Round money to the nearest cent only at the end unless the problem tells you to round earlier.

## 5 Ratios, Rates & Proportions

### Proportion tools

**Ratio**

$a : b$ ,  $a$  to  $b$ , and  $\frac{a}{b}$  all compare two quantities.

**Rate**

A ratio with different units, such as miles per hour.

**Unit rate**

A rate with denominator 1.

**Proportion**

$\frac{a}{b} = \frac{c}{d}$  means  $ad = bc$ , with  $b, d \neq 0$ .

**Scale factor**

scale factor =  $\frac{\text{new length}}{\text{original length}}$

**Similar figures**

Corresponding side lengths are proportional and corresponding angles match.

**Constant of proportionality**

$y = kx$ , where  $k = \frac{y}{x}$  when  $x \neq 0$ .

**Proportional graph**

A proportional relationship graphs as a straight line through  $(0, 0)$ .

**Unit price**

unit price =  $\frac{\text{total cost}}{\text{number of units}}$

**Distance, rate, time**

$d = rt$ ; so  $r = \frac{d}{t}$  and  $t = \frac{d}{r}$ .

**Percent proportion**

$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$ , with whole  $\neq 0$ .

### Tutor's Note

A proportion is two equal ratios. Cross-multiplication works because multiplying both sides by the denominators clears the fractions.

If 3 notebooks cost \$7.50, the unit rate is  $7.50 \div 3 = \$2.50$  per notebook. **Example**

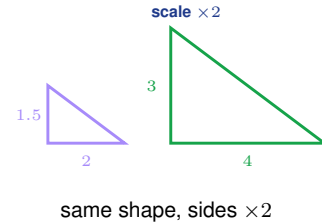
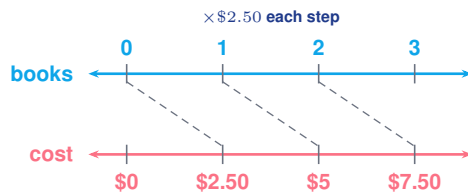


UNITS

Ratios compare. Rates compare with units. Keep units attached until the answer makes sense.



**Ratio tables & similar figures** Scale both quantities by the same number and the ratio stays the same.



**6 Integers & Absolute Value**

**Signed number rules**

**Add same signs**

Add absolute values and keep the common sign.

**Add different signs**

Subtract absolute values and keep the sign of the number farther from 0.

**Subtract**

$$a - b = a + (-b).$$

**Multiply/divide signs**

Same signs give positive; different signs give negative.

**Zero pairs**

$$a + (-a) = 0; \text{ opposites cancel in addition.}$$

**Absolute value**

$|a|$  is the distance from 0, so  $|a| \geq 0$ .

**Opposites**

$a$  and  $-a$  are the same distance from 0 in opposite directions.

**Coordinate distance on a line**

Distance between  $a$  and  $b$  is  $|a - b|$ .

**Tutor's Note**

For multiplying or dividing, count negatives: an even number of negative signs gives a positive result; an odd number gives a negative result.

$-7 + 12 = 5$  because the signs are different, so subtract  $12 - 7$  and keep the sign of 12.

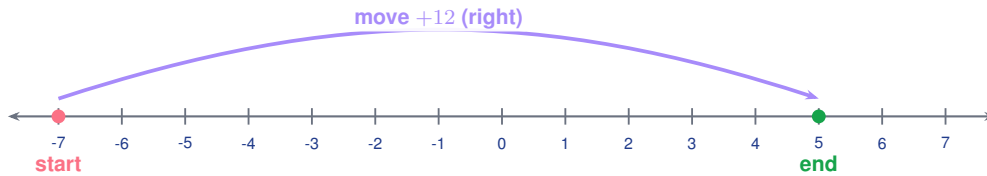
**Example**



Do not treat subtraction like multiplication. For  $5 - (-3)$ , add the opposite:  
 $5 + 3 = 8$ .



**Adding integers on a number line** Start at the first number, then *move*: right to add a positive, left to add a negative.



$-7 + 12 = 5$ . The two points are  $|-7 - 5| = 12$  apart.

## 7 Expressions, Equations & Inequalities

### Algebra basics

<b>Expression</b>	Numbers, variables, and operations without an equals sign.
<b>Equation</b>	A statement that two expressions are equal.
<b>Evaluate</b>	Substitute a number for a variable and simplify.
<b>Substitution</b>	Replace a variable with a given value, then use order of operations.
<b>Combine like terms</b>	Add/subtract coefficients of terms with the same variable part.
<b>Distribute</b>	$a(b + c) = ab + ac$ .
<b>One-step equation</b>	$x + a = b \Rightarrow x = b - a$ and $ax = b \Rightarrow x = \frac{b}{a}$ when $a \neq 0$ .
<b>Two-step equation</b>	For $ax + b = c$ , subtract $b$ first, then divide by $a$ when $a \neq 0$ .
<b>Inequality rule</b>	If multiplying or dividing by a negative number, flip the inequality symbol.

### Tutor's Note

Solving means keeping the equation balanced while isolating the variable. Whatever you do to one side, do to the other side.

Solve  $3x + 5 = 20$ . Subtract 5:  $3x = 15$ . Divide by 3:  $x = 5$ .

**Example**



## 9 Coordinate Plane & Linear Patterns

### Graphing basics

#### Ordered pair

$(x, y)$  gives horizontal movement first, vertical movement second.

#### Quadrants

I: (+, +), II: (-, +), III: (-, -), IV: (+, -).

#### Slope

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}, \text{ with } x_2 \neq x_1.$$

#### Rate of change

Another name for slope: how much  $y$  changes for each 1 unit of  $x$ .

#### $y$ -intercept

The point where a graph crosses the  $y$ -axis.

#### $x$ -intercept

The point where a graph crosses the  $x$ -axis.

#### Linear pattern

$y = mx + b$ , where  $m$  is slope and  $b$  is the  $y$ -intercept.

#### Direct variation

$y = kx$  is a proportional linear pattern through the origin.

#### Horizontal line

$y = c$  has slope 0.

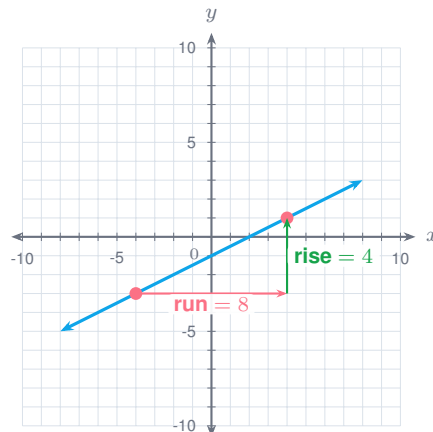
#### Vertical line

$x = c$  has undefined slope.

### Tutor's Note

The coordinate plane is a map. The  $x$ -axis moves left and right; the  $y$ -axis moves down and up. Slope tells how much  $y$  changes for each 1 step in  $x$ .

### Visual: slope is rise over run





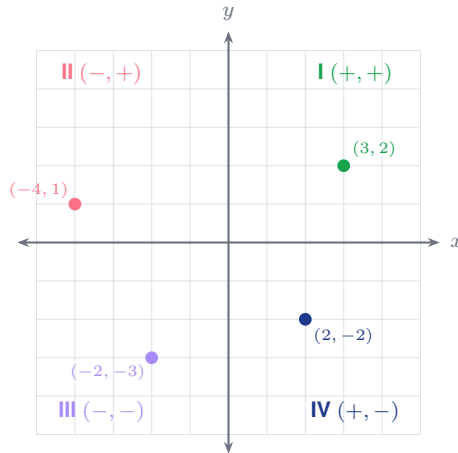
GRAPH

A vertical line fails the function test because one input has many outputs.

**The coordinate plane: quadrants & plotting**

An ordered pair  $(x, y)$  moves right/left first, then up/down.

The signs tell you the quadrant.



**Graphing checkpoint**

Read the plane like a map: horizontal first, vertical second.

**Ordered pair**

$(-4, 3)$  means left 4, then up 3. The  $x$ -move comes first.

**Intercepts**

$y$ -intercept: where the graph crosses the  $y$ -axis.  $x$ -intercept: where it crosses the  $x$ -axis.

**Quadrant signs**

I  $(+, +)$ , II  $(-, +)$ , III  $(-, -)$ , IV  $(+, -)$ .

**Line patterns**

$y = mx + b$  shows slope  $m$  and starting value  $b$ .

**Slope**

$m = \frac{\text{rise}}{\text{run}}$ . Up/right is positive; down/right is negative.

**Fast check**

A vertical line has undefined slope because its run is 0.

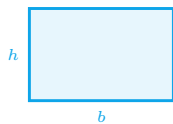


## 10 Geometry & Measurement

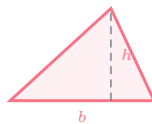
### Plane geometry

<b>Rectangle perimeter</b>	$P = 2l + 2w.$
<b>Rectangle area</b>	$A = lw.$
<b>Square perimeter / area</b>	$P = 4s$ and $A = s^2.$
<b>Triangle area</b>	$A = \frac{1}{2}bh.$
<b>Parallelogram area</b>	$A = bh.$
<b>Trapezoid area</b>	$A = \frac{1}{2}(b_1 + b_2)h.$
<b>Circle circumference</b>	$C = 2\pi r = \pi d.$
<b>Circle area</b>	$A = \pi r^2.$
<b>Pythagorean theorem</b>	$a^2 + b^2 = c^2$ for right triangles, where $c$ is the hypotenuse.

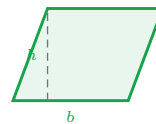
**Area formulas you can see** Area is the flat space inside a figure; the height is always perpendicular to the base.



**Rectangle**  
 $A = bh$



**Triangle**  
 $A = \frac{1}{2}bh$



**Parallelogram**  
 $A = bh$



**Circle**  
 $A = \pi r^2$

**Geometry checkpoint** Name the measurement before choosing a formula.

**Perimeter**

Add side lengths around the outside. Units stay linear.

**Circles**

$C = 2\pi r = \pi d$  and  $A = \pi r^2$ . Diameter is twice the radius.

**Area**

Covering a flat region uses square units. Rectangle:  $A = lw$ .

**Pythagorean check**

Use  $a^2 + b^2 = c^2$  only for right triangles;  $c$  is the hypotenuse.

**Triangle area**

$A = \frac{1}{2}bh$ . The height must be perpendicular to the base.

**Fast check**

Area and volume answers should include square or cubic units.



**Solid geometry: volume & surface area**

**Rectangular prism**

$$V = lwh; \text{ surface area } SA = 2lw + 2lh + 2wh.$$

**Cube**

$$V = s^3; \text{ surface area } SA = 6s^2.$$

**Cylinder**

$$V = \pi r^2 h; \text{ surface area } SA = 2\pi r^2 + 2\pi r h.$$

**Triangular prism**

$$V = \left(\frac{1}{2}bh\right)L; \text{ } SA = \text{two triangle ends} + \text{three rectangle faces.}$$

**Cone**

$$V = \frac{1}{3}\pi r^2 h.$$

**Pyramid**

$$V = \frac{1}{3}(\text{base area})h.$$

**Sphere**

$$V = \frac{4}{3}\pi r^3; \text{ surface area } SA = 4\pi r^2.$$

**Surface area idea**

Add the areas of every outside face or curved surface; nets help organize the faces.

**Tutor's Note**

Area is measured in square units because it covers a flat region. Volume is measured in cubic units because it fills space. Make sure all measurements use the same unit before using a formula.

**Example**

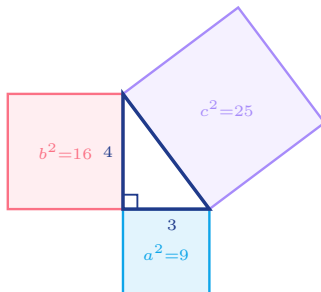
A triangle with base 10 cm and height 7 cm has area  $A = \frac{1}{2}(10)(7) = 35$  square centimeters.



GEOMETRY

For the Pythagorean theorem,  $c$  is always the longest side, across from the right angle.

**The Pythagorean theorem:**  $a^2 + b^2 = c^2$  For a right triangle, the square built on the hypotenuse equals the two squares on the legs added together.



The legs are  $a = 3$  and  $b = 4$ , so  
 $c^2 = a^2 + b^2 = 9 + 16 = 25$ ,  
 which gives  $c = \sqrt{25} = 5$ .

The hypotenuse  $c$  is always the longest side, opposite the right angle.



## 11 Measurement & Unit Conversions

### Customary (U.S.) units

**Length**

12 in = 1 ft; 3 ft = 1 yd; 5280 ft = 1 mi.

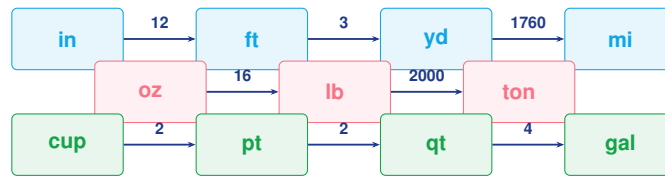
**Weight**

16 oz = 1 lb; 2000 lb = 1 ton.

**Capacity**

8 fl oz = 1 cup; 2 cups = 1 pt; 2 pt = 1 qt; 4 qt = 1 gal.

**Unit conversion checkpoint** Write the unit you want to cancel on the bottom.



**Length path**

1 yd = 3 ft and 1 ft = 12 in, so  
2 yd = 72 in.

**Fraction setup**

$5 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}}$ : feet cancel.

**Weight path**

1 lb = 16 oz; multiply pounds by 16 to  
get ounces.

**Reasonableness**

Smaller units make larger numbers;  
larger units make smaller numbers.

**Capacity path**

1 gal = 4 qt = 8 pt = 16 cups.

**Fast check**

Keep units attached until the final  
answer. They show whether the setup  
is right.

**Try it** 4 ft = 48 in   3 lb = 48 oz   2 gal = 8 qt = 32 cups   Smaller units make the number larger.



**Metric units & time**

**Metric length**

$$10 \text{ mm} = 1 \text{ cm}; 100 \text{ cm} = 1 \text{ m}; 1000 \text{ m} = 1 \text{ km}.$$

**Metric mass**

$$1000 \text{ mg} = 1 \text{ g}; 1000 \text{ g} = 1 \text{ kg}.$$

**Metric capacity**

$$1000 \text{ mL} = 1 \text{ L}.$$

**Metric prefixes**

$$\text{kilo} = 1000, \text{hecto} = 100, \text{deca} = 10, \text{deci} = \frac{1}{10}, \text{centi} = \frac{1}{100}, \text{milli} = \frac{1}{1000}.$$

**Time**

$$60 \text{ s} = 1 \text{ min}; 60 \text{ min} = 1 \text{ hr}; 24 \text{ hr} = 1 \text{ day}; 7 \text{ days} = 1 \text{ wk}.$$

**Temperature**

$$F = \frac{9}{5}C + 32 \text{ and } C = \frac{5}{9}(F - 32).$$

**Converting units: multiply by a fraction equal to 1** Write the conversion so the *old* unit cancels and the *new* unit is left.

$$5 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} \Rightarrow 60 \text{ in}$$

the "ft" units cancel, leaving inches

**Tutor's Note**

To convert, multiply by a fraction that equals 1 (the same amount written two ways). Put the unit you want to cancel on the bottom and the unit you want to keep on top.

**Example**

$$\text{Convert 3 yards to inches: } 3 \text{ yd} \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{12 \text{ in}}{1 \text{ ft}} = 108 \text{ in}.$$



UNITS

Metric prefixes move the decimal by powers of 10: from km to m multiply by 1000; from m to cm multiply by 100.

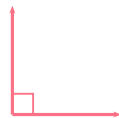
**12 Angles, Lines & Polygons**

**Types of angles** Angles are measured in degrees. A right angle is exactly  $90^\circ$ ; a straight angle is  $180^\circ$ .



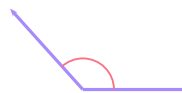
**Acute**

less than  $90^\circ$



**Right**

exactly  $90^\circ$



**Obtuse**

between  $90^\circ$  and  $180^\circ$



**Straight**

exactly  $180^\circ$



**Angle & figure facts**

**Right / straight angle**

A right angle is  $90^\circ$ ; a straight angle is  $180^\circ$ ; a full turn is  $360^\circ$ .

**Complementary angles**

Two angles whose measures add to  $90^\circ$ .

**Supplementary angles**

Two angles whose measures add to  $180^\circ$  (a straight line).

**Vertical angles**

When two lines cross, opposite angles are equal.

**Triangle angle sum**

The three interior angles of a triangle add to  $180^\circ$ .

**Quadrilateral angle sum**

The four interior angles of a quadrilateral add to  $360^\circ$ .

**Polygon angle sum**

For  $n$  sides, interior angles add to  $(n - 2) \cdot 180^\circ$ .

**Regular polygon angle**

Each interior angle is  $\frac{(n-2)180^\circ}{n}$ .

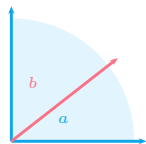
**Exterior angles**

One exterior angle at each vertex adds to  $360^\circ$ ; regular exterior angle is  $\frac{360^\circ}{n}$ .

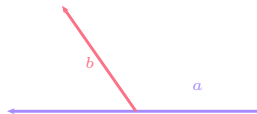
**Parallel lines**

Lines in a plane that never meet; a crossing line makes equal matching angles.

**Angle pairs** Complementary angles add to  $90^\circ$ , supplementary angles add to  $180^\circ$ , and vertical angles (across an X) are equal.



**Complementary**  
 $a + b = 90^\circ$



**Supplementary**  
 $a + b = 180^\circ$



**Vertical**  
opposite angles equal

**Tutor's Note**

Most angle problems are just addition. If two angles form a right angle, they are complementary; if they form a straight line, they are supplementary. Set the known sum equal to the parts and solve.

If one angle of a complementary pair is  $35^\circ$ , the other is  $90^\circ - 35^\circ = 55^\circ$ .

**Example**



ANGLES

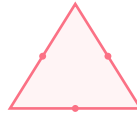
The angles of a triangle always total  $180^\circ$ , so two angles determine the third:  
 $c = 180^\circ - a - b$ .



**Triangle angle sum & figure types** The three angles of any triangle add to  $180^\circ$ ; the four angles of any quadrilateral add to  $360^\circ$ .



$a + b + c = 180^\circ$



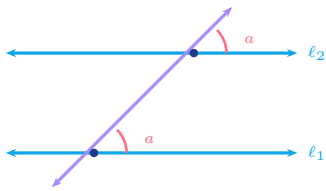
equilateral: all sides =



rectangle

4 right angles

**Parallel lines cut by a transversal** A line crossing two parallel lines makes equal matching angles.



**Corresponding angles** (same position at each crossing) are **equal**.

**Alternate interior angles** (opposite sides, between the lines) are **equal**.

**Co-interior angles** (same side, between the lines) are **supplementary** — they add to  $180^\circ$ .

**Interior angle sums:**  $(n - 2) \times 180^\circ$  Every extra side adds another  $180^\circ$  to the total.



Triangle

3 sides =  $180^\circ$



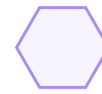
Quadrilateral

4 sides =  $360^\circ$



Pentagon

5 sides =  $540^\circ$



Hexagon

6 sides =  $720^\circ$

**Angle checkpoint** Most angle questions are sum facts plus simple equations.

**Pairs**

$a + b = 90^\circ$  for complementary;  
 $a + b = 180^\circ$  for supplementary.

**Triangles**

$a + b + c = 180^\circ$ . Two angles determine the third.

**Quadrilaterals**

Four interior angles add to  $360^\circ$ .

**Polygons**

Interior sum =  $(n - 2)180^\circ$ . Divide by  $n$  only for regular polygons.

**Parallel lines**

Corresponding and alternate interior angles match; same-side interior angles add to  $180^\circ$ .

**Fast check**

A straight line is  $180^\circ$  and a full turn is  $360^\circ$ .



## 13 Data, Statistics & Probability

### Statistics formulas

<b>Mean</b>	$\bar{x} = \frac{\text{sum of values}}{\text{number of values}}$
<b>Median</b>	The middle value after the data are ordered.
<b>Mode</b>	The value or values that occur most often.
<b>Range</b>	Maximum minus minimum.
<b>Quartiles</b>	$Q_1$ , median, $Q_3$ split ordered data into four equal parts.
<b>Interquartile range</b>	$IQR = Q_3 - Q_1$ , the spread of the middle half.
<b>Mean absolute deviation</b>	$MAD = \frac{\sum  x - \bar{x} }{n}$ , the average distance from the mean.
<b>Probability</b>	$P(\text{event}) = \frac{\text{favorable outcomes}}{\text{total equally likely outcomes}}$
<b>Experimental probability</b>	$\frac{\text{times event occurs}}{\text{number of trials}}$
<b>Complement</b>	$P(\text{not } A) = 1 - P(A)$ .
<b>Simple counting principle</b>	If one choice has $m$ options and another has $n$ options, together there are $mn$ outcomes.

### Tutor's Note

Statistics summarize data. Probability predicts chance. Both depend on reading the question carefully: are the outcomes equally likely, and what exactly counts as a success?

For data 4, 5, 6, 9, the mean is  $\frac{4+5+6+9}{4} = 6$  and the range is  $9 - 4 = 5$ .

**Example**



Outliers can pull the mean more than the median. If one value is far away from the rest, compare both measures.

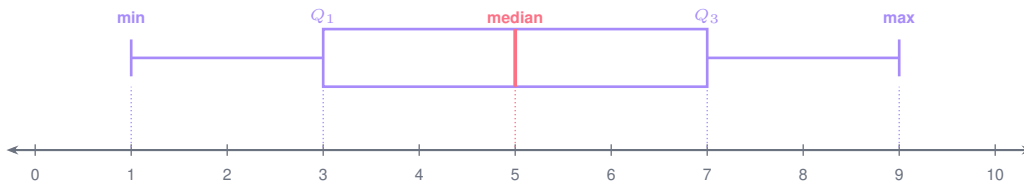


**Mean, median & mode on a dot plot** Stack a dot for each value, then read the center three ways. The mean balances the data; the median is the middle; the mode is the most common.



Data 2, 3, 3, 4, 8: mean  $\frac{20}{5} = 4$ , median 3, mode 3, range  $8 - 2 = 6$ .

**The five-number summary & box plot** Order the data, then mark the minimum, first quartile  $Q_1$ , median, third quartile  $Q_3$ , and maximum.



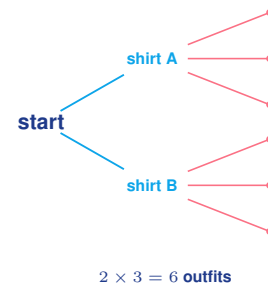
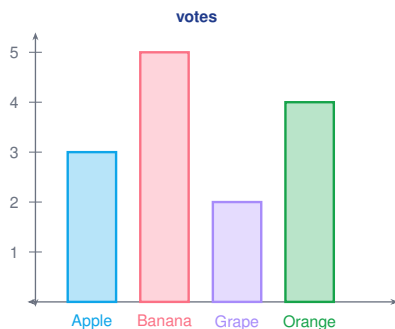
The box holds the middle half of the data; its width ( $Q_3 - Q_1$ ) is the interquartile range (IQR).

**The probability scale** Every probability is a number from 0 (impossible) to 1 (certain). Half-way is an even chance.



0% to 100% — e.g. one head on a fair coin flip is  $\frac{1}{2} = 50\%$ .

**Reading data & counting outcomes** A bar graph compares categories; the counting principle multiplies the choices at each stage.



## 1 Foundations of Algebra

Before we build anything tall, we need a solid base. This section covers the ground rules that every later topic leans on — how numbers behave, the order we work in, and how to handle signs and fractions without fear.

### ◆ The Real-Number Properties

These four properties are the “rules of the game.” You already use them without thinking; algebra just gives them names.

#### Commutative (order)

$$a + b = b + a$$

$$a \cdot b = b \cdot a$$

#### Distributive (share out)

$$a(b + c) = ab + ac$$

#### Associative (grouping)

$$(a + b) + c = a + (b + c)$$

$$(ab)c = a(bc)$$

#### Identity & Inverse

$$a + 0 = a, \quad a \cdot 1 = a$$

$$a + (-a) = 0, \quad a \cdot \frac{1}{a} = 1 \quad (a \neq 0)$$

#### Tutor's Note

**Commutative** means order doesn't matter for adding or multiplying:  $3 + 5$  and  $5 + 3$  both give 8. **Associative** means it doesn't matter how you group them: you can add the first two numbers or the last two and land in the same place. The **distributive** property is the real workhorse of algebra — it lets you “hand out” a multiplier to everything inside parentheses, which is how we expand and factor later. Notice these work for  $+$  and  $\times$ , but *not* for subtraction or division:  $6 - 2 \neq 2 - 6$ .

### ◆ Order of Operations (PEMDAS)

When an expression has several operations, we all need to agree on what to do first — otherwise the same expression could give different answers.

#### Do them in this order

**P**arentheses → **E**xponents → **M**ultiplication & **D**ivision (left → right) → **A**ddition & **S**ubtraction (left → right)

#### Tutor's Note

A useful phrase is “*Please Excuse My Dear Aunt Sally*.” The one trap: multiply and divide are a *team* — do whichever comes first reading left to right, not all multiplication before all division. Same goes for add and subtract.

**Example**  
 $3 + 4 \times 2^2 - (6 - 2)$ . First parentheses:  $(6 - 2) = 4$ . Then the exponent:  $2^2 = 4$ . Then multiply:  $4 \times 4 = 16$ . Finally left to right:  $3 + 16 - 4 = 15$ .



## ◆ Solving Linear Equations

### The goal: isolate the variable

Undo operations in reverse PEMDAS order.

$$ax + b = c \implies x = \frac{c - b}{a} \quad (a \neq 0)$$

### Tutor's Note

Picture a wrapped present: it was wrapped in a certain order, and you unwrap it in the *opposite* order. If  $x$  was multiplied by  $a$  and then had  $b$  added, you undo it by first subtracting  $b$ , then dividing by  $a$ . Every step you do to the left side, do to the right side too — that keeps the scale balanced.

$3x + 5 = 20$ . Subtract 5:  $3x = 15$ . Divide by 3:  $x = 5$ . Check:  $3(5) + 5 = 20$ . ✓

**Example**

## ◆ Special Solution Cases

Sometimes the variable disappears as you solve. That's not a mistake — it's telling you something.

### One solution

$$2x + 1 = x + 4 \implies x = 3$$

### Infinitely many

$$2(x + 1) = 2x + 2 \text{ (always true)}$$

### No solution

$$x + 2 = x + 5 \implies 2 = 5 \text{ (false)}$$

### Literal equations

$$A = \ell w \implies w = \frac{A}{\ell} \quad (\ell \neq 0)$$

### Tutor's Note

If you end with something *false* like  $2 = 5$ , no number can make it work, so there's *no solution*. If you end with something *always true* like  $2 = 2$ , then *every* number works. A "literal" equation just has more than one letter — you solve for one variable using the same balancing moves, treating the others like numbers.

## ◆ Inequalities

### The one new rule

Solve exactly like an equation, **but flip the inequality sign** whenever you multiply or divide both sides by a *negative* number.

$$-2x < 6 \implies x > -3$$



**Tutor's Note**

Why the flip? Think about a true statement like  $3 < 5$ . Multiply both sides by  $-1$  and you get  $-3$  and  $-5$ . But  $-3$  is actually *greater* than  $-5$ , so the sign has to turn around to stay true. That's the whole reason — negatives reverse order on the number line.

**Visual: graphing  $x > -3$**



**Reading the symbols**

$<$  less than     $>$  greater than  
 $\leq$  at most     $\geq$  at least

**Compound inequalities**

AND:  $a < x < b$  (between)  
 OR:  $x < a$  or  $x > b$



**GRAPH**

A solid dot or closed bracket means “or equal to” ( $\leq, \geq$ ). An open dot means strictly less or strictly greater.

**◆ Absolute-Value Equations & Inequalities**

**Split into two cases**

$$|ax + b| = c \Rightarrow ax + b = c \text{ or } ax + b = -c \quad (c \geq 0)$$

$$|x| < c \Rightarrow -c < x < c \quad |x| > c \Rightarrow x < -c \text{ or } x > c \quad (c > 0)$$

**Tutor's Note**

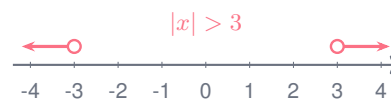
Because absolute value measures distance,  $|x| = 5$  asks “what is 5 away from zero?” — and there are *two* answers, 5 and  $-5$ . So every absolute-value equation splits into two. For “less than,” you want the points *close* to zero (a sandwich, between two values); for “greater than,” you want the points *far* from zero (two pieces heading outward).



**ABS**

If an absolute-value equation equals a negative number, it has no solution. For absolute-value inequalities, check  $c < 0$  as a special case because distance is never negative.

**Visual: close to zero vs. far from zero**



### ◆ Translating Words into Algebra

Half of algebra is turning English into symbols. Here's your dictionary.

- sum / increased by / more than → +
- difference / less than / fewer → −
- product / of / times / twice → ×
- quotient / per / split → ÷
- is / was / equals / gives → =
- a number / unknown →  $x$



**WORDS**

“5 less than a number” is  $x - 5$ , *not*  $5 - x$ . The order matters because “less than” reverses the order in the expression.

## 3 Functions, Lines & Slope

A function is one of the biggest ideas in all of math: a reliable machine that turns each input into exactly one output. Lines are the simplest functions, and slope is the number that describes how steep they are.

### ◆ What a Function Is

**Definition**

A **function** gives exactly one output  $y$  for each input  $x$ . We write  $y = f(x)$ . **Vertical Line Test:** a graph is a function if no vertical line ever hits it twice.

**Tutor's Note**

Think of a vending machine: press B4 and you always get the same snack. That “same input, same single output” rule is what makes something a function. The notation  $f(x)$  is read “ $f$  of  $x$ ” and just means “the output when the input is  $x$ .” It is *not* multiplication — it’s a name tag for the machine’s result.

**Visual: vertical line test**

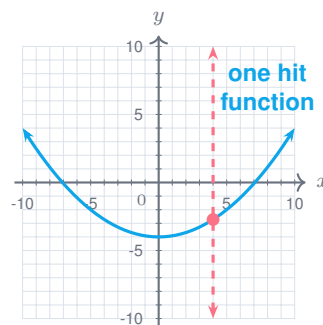
**How to read it**

A graph is a function when each input  $x$  has only one output  $y$ .  
The dashed vertical line touches this curve once, so it passes the test.

**Domain** — allowed inputs.

**Range** — possible outputs.

**Evaluate:**  $f(3) = 2(3) + 1 = 7$ .



### ◆ Slope — Steepness in a Number

#### Slope between two points

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \quad (x_2 \neq x_1)$$

#### Tutor's Note

Slope answers “for every step I take to the right, how far do I go up or down?” That’s the *rise over run*. A big slope is a steep hill; a small slope is a gentle ramp. The sign tells direction: positive climbs as you read left to right, negative descends.



#### SLOPE

Subtract the *x*’s and the *y*’s in the *same order*. Mixing the order flips the sign and changes the slope.

#### Visual: rise over run

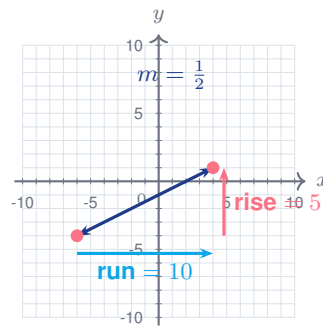
##### Slope is a rate

Count the horizontal change first, then the vertical change.

Here, run = 10 and rise = 5, so

$$m = \frac{5}{10} = \frac{1}{2}$$

Positive slope rises left to right. Negative slope falls left to right.



- $m > 0$ : line rises (↗)
- $m < 0$ : line falls (↘)
- $m = 0$ : flat, horizontal  $y = b$
- $m$  undefined: vertical  $x = a$

### ◆ Forms of a Linear Equation

The same line can be written several ways. Each form is handy for a different job.

#### Slope-Intercept

$$y = mx + b$$

slope  $m$ ,  $y$ -intercept  $b$

#### Standard

$$Ax + By = C$$

( $A$  and  $B$  not both 0)

#### Point-Slope

$$y - y_1 = m(x - x_1)$$

#### Intercepts

$x$ -int: set  $y = 0$   
 $y$ -int: set  $x = 0$



**Tutor's Note**

Use **slope-intercept** when you know the slope and where the line crosses the  $y$ -axis — you can graph it instantly. Use **point-slope** when you know the slope and *any* one point; it's the fastest way to write a line's equation from scratch. **Standard form** is tidy for finding both intercepts. They all describe the same line — just dressed for different occasions.

**Visual: slope-intercept form**

**Use slope-intercept form**

The line crosses the  $y$ -axis at  $b = 1$ .

The slope is  $\frac{1}{2}$ : move right 2, then up 1; or right 4, then up 2.

The arrows show the line continues forever in both directions.



**Parallel & Perpendicular Lines**

**It's all in the slopes**

**Parallel:**  $m_1 = m_2$       **Perpendicular:**  $m_1 \cdot m_2 = -1$  (negative reciprocals, for nonvertical lines)

**Tutor's Note**

Parallel lines never meet, so they must rise at the exact same rate — equal slopes. Perpendicular lines cross at a right angle, and it turns out their slopes are “negative reciprocals”: flip the fraction and change the sign. So a slope of 2 is perpendicular to  $-\frac{1}{2}$ . A quick check: multiply them and you should get  $-1$ .



**LINES**

A vertical line and a horizontal line are perpendicular, but the vertical line has undefined slope, so the product rule does not apply to that pair.

**Distance & Midpoint**

**Distance Formula**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Midpoint Formula**

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**Tutor's Note**

The distance formula is secretly the Pythagorean theorem in disguise: the horizontal gap and vertical gap are the two legs of a right triangle, and the distance is the hypotenuse. The midpoint is even friendlier — it's just the average of the two  $x$ 's and the average of the two  $y$ 's, landing you



exactly halfway.

### ◆ Direct & Inverse Variation

**Direct:**  $y = kx$

**Inverse:**  $y = \frac{k}{x}$  ( $x \neq 0, k \neq 0$ ), or  $xy = k$

#### Tutor's Note

**Direct** variation is a straight line through the origin — double the input, double the output when  $k$  is positive (think hours worked and pay earned). If  $k$  is negative, the line still passes through the origin but goes downward. **Inverse** variation is a constant-product trade-off — with positive quantities, more speed means less travel time. The constant  $k$  stays fixed and ties the two quantities together.

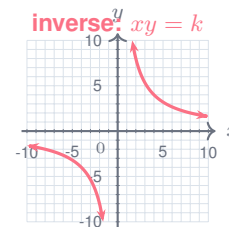
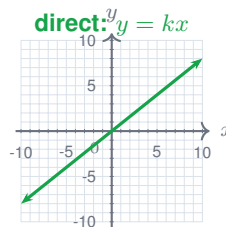
#### Visual: direct vs. inverse variation

##### Compare the shapes

Direct variation is a line through the origin:  $y = kx$ .

Inverse variation bends in two branches because  $xy = k$  stays fixed.

Both graphs continue beyond the viewing window.



## 4 Exponents & Scientific Notation

An exponent is just a shorthand for repeated multiplication. Once you see that, every exponent rule below stops being something to memorize and becomes something you can figure out.

### ◆ The Laws of Exponents

These rules apply whenever every expression involved is defined. In this list,  $m$  and  $n$  are integers unless a fractional exponent is shown.

#### Product

$$a^m \cdot a^n = a^{m+n}$$

#### Power of a Power

$$(a^m)^n = a^{mn}$$

#### Quotient

$$\frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$$

#### Power of a Product

$$(ab)^n = a^n b^n$$



**Power of a Quotient**

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (b \neq 0)$$

**Negative Exponent**

$$a^{-n} = \frac{1}{a^n} \quad (a \neq 0)$$

**Zero Exponent**

$$a^0 = 1 \quad (a \neq 0)$$

**Fractional Exponent**

$$a^{m/n} = (\sqrt[n]{a})^m \quad (n \in \mathbb{Z}^+)$$

**Real-number note:** with even roots, the base under the root must be nonnegative. For example,  $\sqrt{x^2} = |x|$ , not always  $x$ . With odd roots, negative radicands are allowed.

**Tutor's Note**

Why these all make sense  $a^3 \cdot a^2$  means  $(a \cdot a \cdot a)(a \cdot a)$  — five  $a$ 's, so you *add* the exponents. Dividing cancels factors, so you *subtract*. A power of a power stacks groups, so you *multiply*. The strange ones follow the same logic:  $a^0 = 1$  because  $\frac{a^3}{a^3} = a^{3-3} = a^0$  and anything over itself is 1. A *negative* exponent just means “flip it to the other side of the fraction.” Memorize the pattern once and the rest is bookkeeping.

$$\frac{2x^3 \cdot 3x^4}{x^2} = \frac{6x^7}{x^2} = 6x^5. \quad (\text{Multiply coefficients, add then subtract exponents.})$$

**Example****Scientific Notation**

A compact way to write very big or very small numbers without drowning in zeros.

**Standard form**

$$a \times 10^n, \quad 1 \leq |a| < 10, \quad n \in \mathbb{Z}$$

Big numbers:  $n > 0$ . Numbers with  $0 < |x| < 1$ :  $n < 0$ . Zero is written as 0.

**Tutor's Note**

The power of 10 just counts how many places the decimal point moves. Positive powers push it right (making the number bigger); negative powers pull it left (smaller). To multiply two of these, multiply the front numbers and *add* the exponents; to divide, divide the fronts and *subtract* — exactly the exponent laws from above.

$$3,400 = 3.4 \times 10^3 \quad 0.0052 = 5.2 \times 10^{-3} \\ (2 \times 10^5)(3 \times 10^2) = 6 \times 10^7.$$

**Example**

**Tip:** The front number always has exactly one digit before the decimal point — that's what makes it “standard.”



## 5 Polynomials & Factoring

A polynomial is just a sum of terms like  $3x^2$ ,  $-5x$ , and 7. Multiplying them out and then factoring them back apart are two sides of the same coin — and factoring is the key that unlocks quadratics in the next section.

### ◆ Polynomial Vocabulary

**Degree** — the highest exponent in a nonzero polynomial.

**Leading coefficient** — the number on the highest-degree term.

#### Named by number of terms

- Monomial — 1 term
- Binomial — 2 terms
- Trinomial — 3 terms

#### Tutor's Note

“Like terms” are terms with the exact same variable part —  $3x^2$  and  $5x^2$  are like terms, but  $3x^2$  and  $3x$  are not. You can only combine like terms, the same way you can add apples to apples but not apples to oranges.

### ◆ Adding, Subtracting & Multiplying

#### Add / Subtract

Combine like terms.

$$(3x^2 + 2x) - (x^2 - 5x) = 2x^2 + 7x$$

#### Multiply — FOIL for two binomials

$$(a + b)(c + d) = ac + ad + bc + bd$$

First, Outer, Inner, Last.

#### Tutor's Note

When subtracting, distribute the minus sign to *every* term in the second parentheses — that sign flip is the #1 source of errors here. For multiplying, FOIL is really just the distributive property done twice: every term in the first group shakes hands with every term in the second.



#### SIGNS

$-(x^2 - 5x)$  becomes  $-x^2 + 5x$ . A minus sign outside parentheses distributes to every term inside.



### ◆ Special Products (memorize these — they save time)

#### Square of a Sum

$$(a + b)^2 = a^2 + 2ab + b^2$$

#### Difference of Squares

$$(a + b)(a - b) = a^2 - b^2$$

#### Square of a Difference

$$(a - b)^2 = a^2 - 2ab + b^2$$

#### Sum/Diff of Cubes

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

#### Tutor's Note

These are just FOIL results worth remembering. Notice the “difference of squares” magic: the middle terms cancel, leaving only  $a^2 - b^2$ . And watch the common slip —  $(a + b)^2$  is *not*  $a^2 + b^2$ ; you must include that  $2ab$  middle term.

### ◆ Factoring — a Reliable Game Plan

Factoring reverses multiplication: it rewrites a polynomial as a product. Always work through these steps in order.

#### The factoring checklist

1. **GCF first.** Pull out the greatest common factor from every term.
2. **Count the terms:**
  - 2 terms → try difference of squares or sum/difference of cubes
  - 3 terms → trinomial factoring
  - 4 terms → factor by grouping
3. **Check** by multiplying your answer back out.

#### Tutor's Note

Factoring feels like a puzzle at first, but the checklist removes the guesswork. Step one is non-negotiable: always look for a common factor before anything else — it makes every later step easier. Then let the number of terms point you to the right tool.

### ◆ Factoring Trinomials

**When  $a = 1$ :** find two numbers that *multiply* to  $c$  and *add* to  $b$ .

$$x^2 + bx + c = (x + p)(x + q), \quad p + q = b, \quad pq = c$$

**When  $a \neq 1$ :** use the *ac-method* — split the middle term, then group.

**Example**  
 $x^2 + 7x + 12$ : we need two numbers that multiply to 12 and add to 7 — that's 3 and 4. So  
 $x^2 + 7x + 12 = (x + 3)(x + 4)$ .



**Tutor's Note**

This “multiply to  $c$ , add to  $b$ ” trick works because when you FOIL  $(x+p)(x+q)$  you get  $x^2+(p+q)x+pq$ . So the outside number  $c$  is the product and the middle number  $b$  is the sum. List the factor pairs of  $c$  and find the pair that adds up right.

## 6 Quadratic Functions & Equations

A quadratic has an  $x^2$  in it, and its graph is a graceful U-shaped curve called a parabola. This is the showpiece of Algebra 1 — and once you have factoring and the quadratic formula, you can solve any of them.

### ◆ Standard & Vertex Forms

**Standard Form**

$$y = ax^2 + bx + c \quad (a \neq 0)$$

**Vertex Form**

$$y = a(x - h)^2 + k$$

vertex at  $(h, k)$

**Tutor's Note**

Both forms describe the same parabola. **Standard form** shows the  $y$ -intercept at a glance (it's  $c$ ). **Vertex form** hands you the most important point — the very top or bottom of the U — directly as  $(h, k)$ . The number  $a$  controls the shape: large  $a$  makes a narrow U, small  $a$  a wide one, and a negative  $a$  flips it upside down.

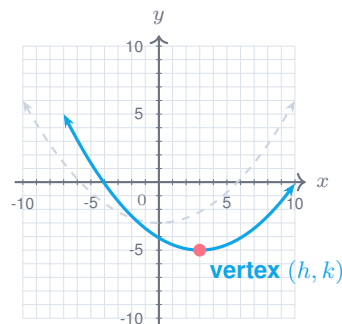
### Visual: vertex form shifts the parabola

**Vertex form**

$y = a(x - h)^2 + k$  puts the vertex at  $(h, k)$ .

The highlighted parabola has vertex  $(3, -5)$ , so the graph shifts right and down.

The arrows show each arm keeps rising outside the window.



### Reading the Graph (the Parabola)

#### Key features

- Opens **up** if  $a > 0$ , **down** if  $a < 0$ .
- Axis of symmetry:  $x = -\frac{b}{2a}$
- Vertex: plug that  $x$  back in to get  $y$ .
- $y$ -intercept:  $(0, c)$

#### Tutor's Note

Every parabola is perfectly symmetric, like a mirror folded down the middle. That mirror line is the axis of symmetry, found by  $x = -\frac{b}{2a}$ . The vertex sits right on that line — it's the lowest point if the U opens up, or the highest if it opens down. That makes the vertex the key to any "maximum height" or "minimum cost" problem.

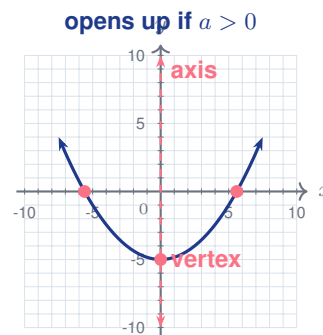
#### Visual: key parabola features

##### Parabola checklist

Find the axis first:  $x = -\frac{b}{2a}$ .

The vertex sits on the axis. The  $x$ -intercepts are where the curve crosses the  $x$ -axis.

Since  $a > 0$ , the parabola opens upward.



### Solving Quadratic Equations

#### The Quadratic Formula — it always works

$$ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (a \neq 0)$$

#### Tutor's Note

You have a toolbox of methods — reach for the easiest one that fits:

- **Factoring** (fastest *when* it factors): if  $pq = 0$ , then  $p = 0$  or  $q = 0$ .
- **Square roots**: if  $x^2 = k$ , then  $x = \pm\sqrt{k}$  for  $k \geq 0$ .
- **Completing the square**: rewrites it in vertex form.

#### Completing the square

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$



**Tutor's Note**

The “zero-product” idea is sneaky-powerful: if two things multiply to zero, at least one of them must be zero. That’s why factoring solves equations — set each factor to zero. When factoring fails, the quadratic formula never does; it’s the universal key that works on every quadratic, no matter how ugly.

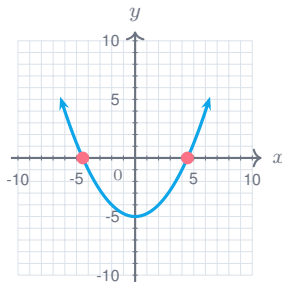
**◆ The Discriminant — a Sneak Peek at the Answers**

$D = b^2 - 4ac$  counts the real solutions

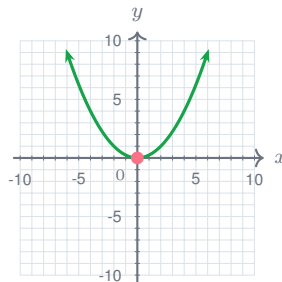
- $D > 0$ : two different real solutions
- $D = 0$ : one repeated real solution
- $D < 0$ : no real solutions (the parabola misses the  $x$ -axis)

**Tutor's Note**

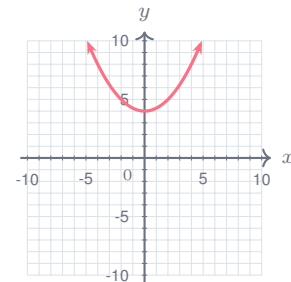
The discriminant is the part under the square root. Before solving, it tells you what to expect: a positive value means the parabola crosses the  $x$ -axis twice, zero means it just touches once, and a negative means it floats entirely above or below — no real crossing, because you can’t take the square root of a negative in real numbers.

**Visual: discriminant and  $x$ -intercepts**

$D > 0$ : two  $x$ -intercepts



$D = 0$ : one touch



$D < 0$ : no real intercepts

**Example**  
 $x^2 - 5x + 6 = 0$ . It factors:  $(x - 2)(x - 3) = 0$ , so  $x = 2$  or  $x = 3$ . Check:  $D = (-5)^2 - 4(1)(6) = 25 - 24 = 1 > 0$ , confirming two real answers.

**7 Radicals & Rational Expressions**

A radical (square root) undoes a square, and a rational expression is just a fraction with variables. Both follow rules you already know from numbers — we’re simply letting letters join in.



## ◆ Radical Rules

### Product

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad (a, b \geq 0)$$

### Radical = Exponent

$$\sqrt[n]{a} = a^{1/n} \quad (n \in \mathbb{Z}^+)$$

### Quotient

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad (a \geq 0, b > 0)$$

### Power

$$a^{m/n} = (\sqrt[n]{a})^m$$

**Real-number note:** even roots need a nonnegative radicand. Also,  $\sqrt{x^2} = |x|$  because the principal square root is never negative. Odd roots can have negative radicands.

### Tutor's Note

The most freeing idea here: a root is just a *fractional exponent*. A square root is the  $\frac{1}{2}$  power, a cube root is the  $\frac{1}{3}$  power. Once you see that, all the exponent laws from Section 4 apply to radicals too — you don't need a separate rule book. The product rule also lets you “break apart” a root to simplify it.

## ◆ Simplifying & Rationalizing

### Simplify a radical

Pull out perfect-square factors:  $\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$ .

### Rationalize the denominator

Multiply by a clever form of 1 to clear the root from the bottom:

$$\frac{1}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a} \quad (a > 0) \quad \frac{1}{a + \sqrt{b}} \cdot \frac{a - \sqrt{b}}{a - \sqrt{b}} = \frac{a - \sqrt{b}}{a^2 - b} \quad (b \geq 0, a^2 \neq b)$$

The flipped-sign partner  $a - \sqrt{b}$  is called the **conjugate**.

### Tutor's Note

To simplify, hunt for the biggest perfect square hiding inside (4, 9, 16, 25, ...) and take its root out front. “Rationalizing” just means cleaning roots out of the denominator, which is considered tidy form. Multiplying by the conjugate works because of difference of squares — it turns  $\sqrt{b}$  into plain  $b$  and the messy root vanishes.



## ◆ Solving Radical Equations

### Isolate, then undo the root

Get the radical alone, raise both sides to the matching power, then **always check** your answers in the original equation.

$$\sqrt{x+3} = 5 \Rightarrow x+3 = 25 \Rightarrow x = 22$$

### Tutor's Note

Squaring both sides is the move that frees  $x$  from under the root. But squaring can secretly introduce a fake answer (an “extraneous” solution), so checking isn’t optional here — it’s part of the method. Plug your answer back in; if it doesn’t actually work, toss it.



### CHECK

After squaring both sides, check in the *original* equation, not the squared version. The squared equation can accidentally accept fake answers.

## ◆ Rational Expressions

**Simplify** by canceling common factors:

$$\frac{x^2 - 9}{x + 3} = \frac{(x - 3)(x + 3)}{x + 3} = x - 3, \quad x \neq -3$$

**Multiply / Divide** like any fractions:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, \quad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \quad (b, d, c \neq 0)$$

**Restriction:** a denominator can never equal 0. Exclude any  $x$  that would make the bottom zero — those values are off-limits.

### Tutor's Note

To simplify a rational expression, factor the top and bottom first, then cancel matching factors — exactly like reducing  $\frac{6}{9}$  to  $\frac{2}{3}$ . Just remember the one rule the calculator can’t bend: dividing by zero is undefined, so always note which  $x$ -values are forbidden before you simplify.

## 8 Systems of Equations & Inequalities

A system is two (or more) equations that must be true at the same time. The solution is the point where they agree — where their graphs cross.



### ◆ Three Ways to Solve

#### Graphing

Plot both lines; the solution is the point where they intersect.

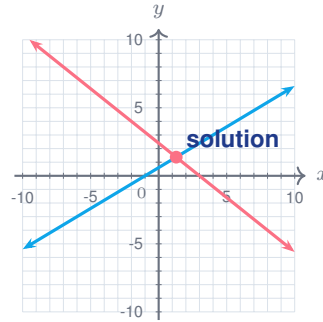
#### Visual: solution is the intersection

##### Graphing a system

Draw both equations on the same plane.

The solution is the point that lies on both lines at the same time.

If the lines do not meet, there is no solution.  
If they are the same line, there are infinitely many.



#### Substitution

Solve one equation for a variable, then plug it into the other.

$$\begin{cases} y = 2x \\ x + y = 6 \end{cases} \Rightarrow x + 2x = 6 \Rightarrow x = 2, y = 4$$

#### Elimination

Add or subtract the equations to cancel a variable.

$$\begin{cases} x + y = 7 \\ x - y = 1 \end{cases} \Rightarrow 2x = 8 \Rightarrow x = 4, y = 3$$

#### Tutor's Note

Which method should I pick? **Graphing** is great for seeing what's going on, but it's only as exact as your drawing. **Substitution** shines when one variable is already alone (like  $y = 2x$ ). **Elimination** is fastest when the equations line up so a variable cancels when you add them. All three reach the same answer — choose whichever fits the problem in front of you.

### ◆ How Many Solutions?

#### One solution

Lines cross once.  
Different slopes.

#### Infinitely many

Same line.  
Same slope and same intercept.

#### No solution

Parallel lines.  
Same slope, different intercept.



**Tutor's Note**

Think geometrically. Two lines that lean differently must cross at exactly one point — one solution. Two parallel lines never meet — no solution. And if both equations secretly describe the *same* line, every point works — infinitely many. The slopes and intercepts tell you which case you're in before you even solve.

**◆ Systems of Inequalities****Graph, then shade**

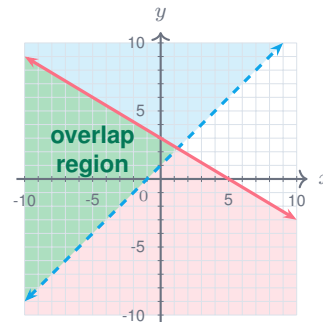
- Solid line for  $\leq$ ,  $\geq$ ; dashed line for  $<$ ,  $>$ .
- Shade the side of each line that makes its inequality true.
- The **solution** is the region where the shadings overlap.

**Visual: overlapping solution region****Shade and overlap**

Use a dashed boundary for  $<$  or  $>$  and a solid boundary for  $\leq$  or  $\geq$ .

The answer is the region where both shaded half-planes overlap.

Test a point, often  $(0, 0)$ , to decide which side to shade.

**Tutor's Note**

With inequalities, the answer is a whole *region*, not one point. A solid boundary counts; a dashed boundary does not. The overlapping shaded zone is the solution. **Tip:** To choose a side, test  $(0, 0)$  if it is not on the boundary.

**9 Sequences & Statistics**

We finish with two practical topics: patterns of numbers (sequences) and ways to summarize data (statistics). Both show up constantly in real life and on tests.



## ◆ Arithmetic Sequences

Add the same amount each time ( $d$ )

$$a_n = a_1 + (n - 1)d$$

$a_1$  is the first term and  $d$  is the common difference.

### Tutor's Note

An arithmetic sequence grows by *adding* a fixed step every time — like stairs of equal height. The formula just says: start at  $a_1$ , then take  $(n - 1)$  steps of size  $d$  to reach the  $n$ th term. The  $(n - 1)$  catches a lot of students — the first term takes *zero* steps, so we subtract one.

3, 7, 11, 15, ... here  $a_1 = 3$  and  $d = 4$ , so  $a_n = 3 + (n - 1)4 = 4n - 1$ .

**Example**

## ◆ Geometric Sequences

Multiply by the same amount each time ( $r$ )

$$a_n = a_1 \cdot r^{n-1}$$

$r$  is the common ratio.

### Tutor's Note

A geometric sequence grows by *multiplying* by a fixed ratio — this is how money compounds and how populations grow. Same structure as before, but repeated multiplication replaces repeated addition, so  $r$  is raised to the  $(n - 1)$  power.

2, 6, 18, 54, ... here  $a_1 = 2$  and  $r = 3$ , so  $a_n = 2 \cdot 3^{n-1}$ .

**Example**

## ◆ Measures of Center & Spread

**Mean** (average)

$$\bar{x} = \frac{\sum x}{n}$$

**Median**

the middle of ordered data; if there are two middles, average them

**Mode**

the most frequent value

**Range**

max - min



# 1 Functions, Transformations & Inverses

## Core function formulas

### Function notation

$y = f(x)$  means the output when the input is  $x$ .

### Average rate of change

$$\frac{f(b) - f(a)}{b - a}, \text{ where } b \neq a.$$

### Composition

$(f \circ g)(x) = f(g(x))$ . Use the domain of  $g$  and any added restrictions from  $f$ .

### Inverse functions

$f^{-1}(x)$  reverses  $f(x)$ , and  $f(f^{-1}(x)) = x$  on the allowed domain.

### Transformation model

$g(x) = a f(b(x - h)) + k$ . Vertical shift  $k$ , horizontal shift  $h$ , vertical scale  $|a|$ , horizontal scale  $\frac{1}{|b|}$ .

## Tutor's Note

Algebra 2 functions are machines you can move, stretch, combine, and reverse. Always track domain: denominators cannot be zero, even roots need nonnegative radicands, and logs need positive arguments.

If  $f(x) = x^2$  and  $g(x) = 2f(x - 3) - 5$ , then  $g(x) = 2(x - 3)^2 - 5$ . The graph shifts right 3, stretches vertically by 2, and shifts down 5. **Example**



INVERSE

To find an inverse, write  $y = f(x)$ , swap  $x$  and  $y$ , solve for  $y$ , and then write  $f^{-1}(x)$ . If the original function is not one-to-one, restrict the domain before taking an inverse.

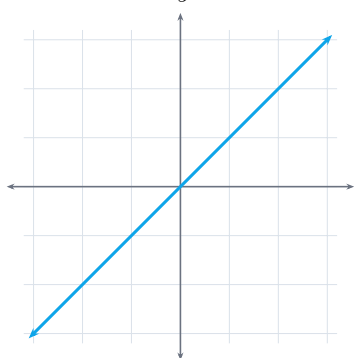
**Before you move on** A function rule, a line, and a system all answer the same question in different ways: **what output matches this input?** Check the input restrictions first, then use slope, substitution, or graph intersections to compare outputs.



## 2 Parent Functions & Transformations

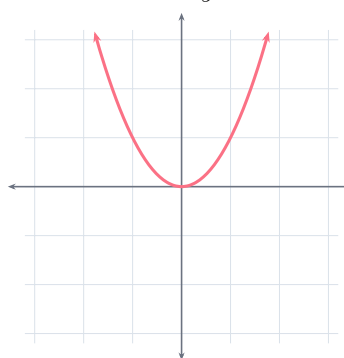
Every Algebra 2 graph is a transformed version of one of these parent shapes. Learn the parents — their graphs, domains, and ranges — then apply shifts, reflections, and stretches. Each curve continues past the arrows.

**Linear**  $y = x$



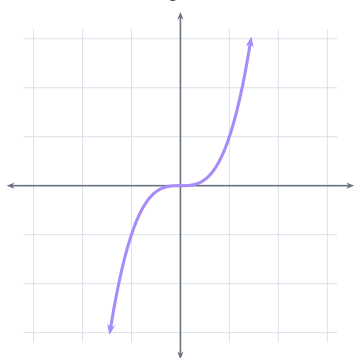
Domain  $\mathbb{R}$  • Range  $\mathbb{R}$

**Quadratic**  $y = x^2$



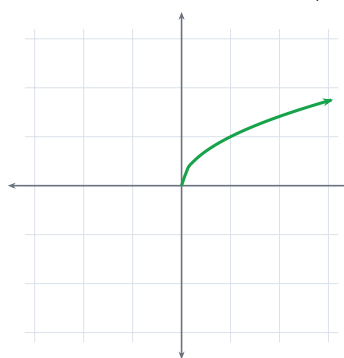
Domain  $\mathbb{R}$  • Range  $y \geq 0$

**Cubic**  $y = x^3$



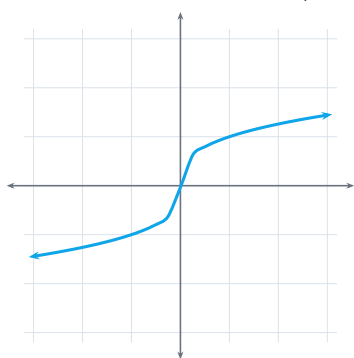
Domain  $\mathbb{R}$  • Range  $\mathbb{R}$

**Square root**  $y = \sqrt{x}$



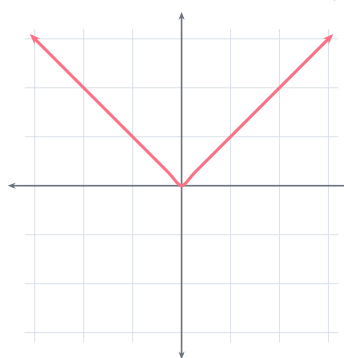
Domain  $x \geq 0$  • Range  $y \geq 0$

**Cube root**  $y = \sqrt[3]{x}$



Domain  $\mathbb{R}$  • Range  $\mathbb{R}$

**Absolute value**  $y = |x|$

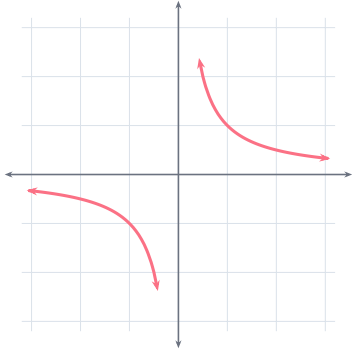


Domain  $\mathbb{R}$  • Range  $y \geq 0$



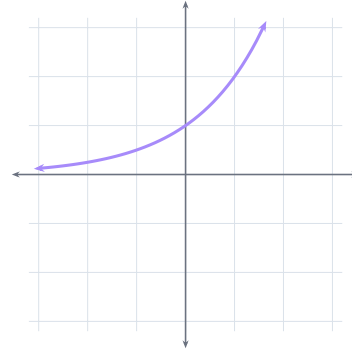
## More parent functions

**Reciprocal**  $y = \frac{1}{x}$



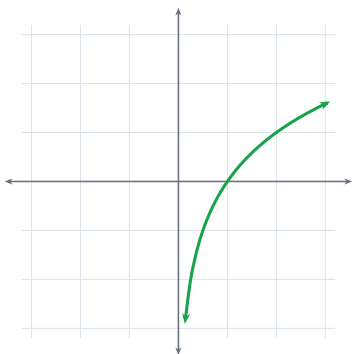
Domain  $x \neq 0$  • Range  $y \neq 0$

**Exponential**  $y = b^x$



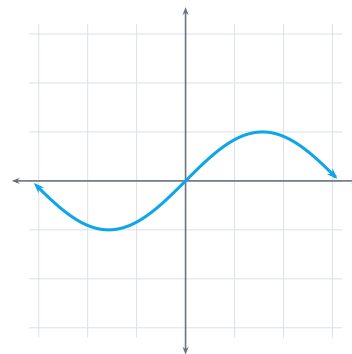
Domain  $\mathbb{R}$  • Range  $y > 0$

**Logarithm**  $y = \log_b x$



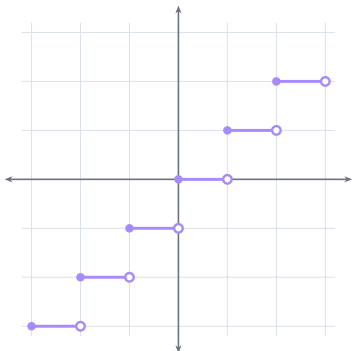
Domain  $x > 0$  • Range  $\mathbb{R}$

**Sine**  $y = \sin x$



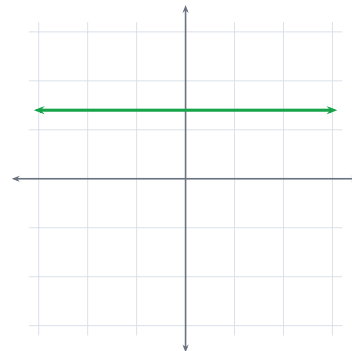
Domain  $\mathbb{R}$  • Range  $[-1, 1]$

**Greatest integer**  $y = \lfloor x \rfloor$



Domain  $\mathbb{R}$  • Range integers

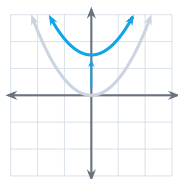
**Constant**  $y = b$



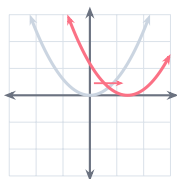
Domain  $\mathbb{R}$  • Range  $\{b\}$



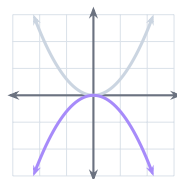
**Transformations of  $y = x^2$**  The gray curve is the parent  $y = x^2$ . Each colored curve shows one change from  $g(x) = a f(x - h) + k$ .



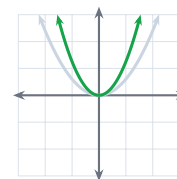
$+k$ : up 1.5



$x - h$ : right 1.4



$a < 0$ : reflect



$a > 1$ : stretch

**Transformation rules for  $g(x) = a f(b(x - h)) + k$**

**Vertical shift**

$+k$  moves the graph up  $k$ ;  $-k$  moves it down.

**Horizontal shift**

$(x - h)$  moves the graph right  $h$ ;  $(x + h)$  moves it left.

**Vertical stretch/shrink**

$|a| > 1$  stretches;  $0 < |a| < 1$  shrinks toward the  $x$ -axis.

**Horizontal stretch/shrink**

factor  $\frac{1}{|b|}$ ;  $|b| > 1$  shrinks,  $0 < |b| < 1$  stretches.

**Reflections**

$a < 0$  reflects over the  $x$ -axis;  $b < 0$  reflects over the  $y$ -axis.

**Order of operations**

stretch/reflect first, then shift; horizontal changes work in reverse.

**Tutor's Note**

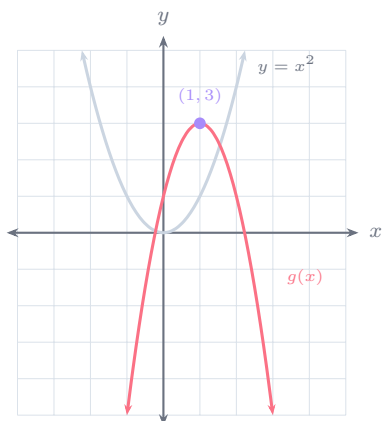
Read a transformed rule from the inside out. Inside the function, changes affect  $x$  and behave "backwards" (right/left, horizontal scaling). Outside the function, changes affect  $y$  and behave as written (up/down, vertical scaling, reflection).



**DOMAIN & RANGE**

Transformations move domain and range too. A vertical shift changes the range; a horizontal shift changes the domain; reflections can swap which way an inequality points.

**Worked example: build  $g(x) = -2(x - 1)^2 + 3$  from the parent  $y = x^2$**



Read  $g(x) = -2(x - 1)^2 + 3$  as four moves on the parent  $y = x^2$ :

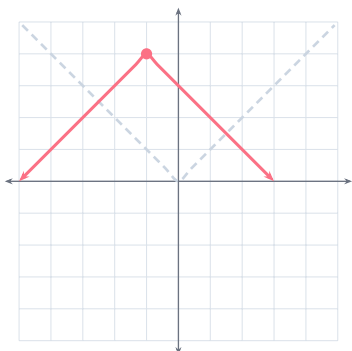
1.  $x - 1$  inside: shift **right** 1.
2.  $\times 2$ : **stretch** vertically (narrower).
3. leading  $-$ : **reflect** over the  $x$ -axis (opens down).
4.  $+3$  outside: shift **up** 3, so the vertex lands at  $(1, 3)$ .



## Transforming every parent function

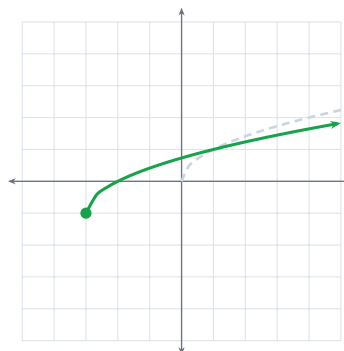
The gray dashed curve is always the parent; the **colored** curve is the transformed function with its key point marked. The same moves — shift, reflect, stretch — work on every family, so once you can read one, you can read them all.

**Absolute value**  $y = |x| \rightarrow -|x+1| + 4$



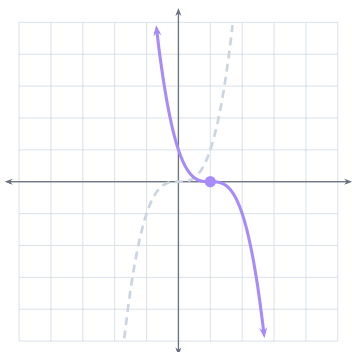
Reflect down, left 1, up 4 — vertex  $(-1, 4)$ .

**Square root**  $y = \sqrt{x} \rightarrow \sqrt{x+3} - 1$



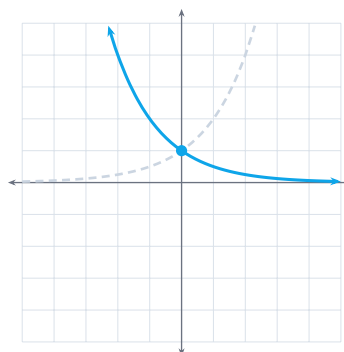
Left 3, down 1 — starts at  $(-3, -1)$ .

**Cubic**  $y = x^3 \rightarrow -(x-1)^3$



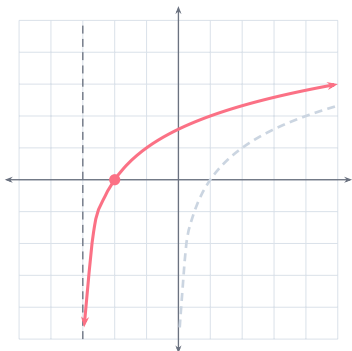
Reflect, right 1 — inflection at  $(1, 0)$ .

**Exponential**  $y = 2^x \rightarrow 2^{-x}$



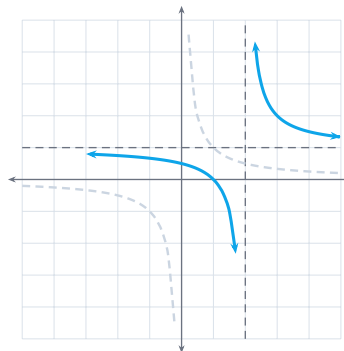
Reflect over the  $y$ -axis — same asymptote  $y = 0$ .

**Logarithm**  $y = \log_2 x \rightarrow \log_2(x+3)$



Left 3 — asymptote moves to  $x = -3$ .

**Reciprocal**  $y = \frac{1}{x} \rightarrow \frac{1}{x-2} + 1$



Right 2, up 1 — asymptotes  $x = 2, y = 1$ .

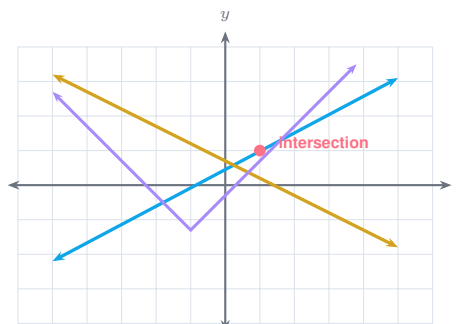


### 3 Linear, Absolute Value & Systems

#### Quick visual: line, V-shape, or intersection?

Lines track constant rate, absolute value graphs measure distance from a center, and systems ask where two rules agree. Before solving, sketch the story in one sentence.

**Slope**  $m = \frac{\text{change in } y}{\text{change in } x}$     **System** shared point



#### Lines and systems

**Slope**

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ with } x_2 \neq x_1.$$

**Slope-intercept form**

$$y = mx + b.$$

**Point-slope form**

$$y - y_1 = m(x - x_1).$$

**Standard form**

$Ax + By = C$ , usually with integer coefficients.

**Parallel lines**

Same slope.

**Perpendicular lines**

Slopes multiply to  $-1$ , unless one line is vertical and the other is horizontal.

**Absolute value equation**

$|x - a| = b$  gives  $x = a \pm b$  when  $b \geq 0$ . If  $b < 0$ , no solution.

**System types**

One intersection: one solution. Parallel distinct lines: no solution. Same line: infinitely many solutions.

#### Tutor's Note

Systems ask for values that make every equation true at the same time. Substitution is great when one variable is already isolated. Elimination is great when coefficients can be made opposites.

Solve  $\begin{cases} 2x + y = 7 \\ x - y = 2 \end{cases}$

. Add the equations to get  $3x = 9$ , so  $x = 3$ . Then  $3 - y = 2$ , so  $y = 1$ . The solution

is  $(3, 1)$ .

**Example**



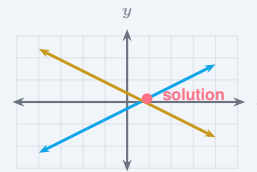
**INEQUALITIES**

When multiplying or dividing an inequality by a negative number, reverse the inequality symbol. For  $|x - a| < b$ , write  $a - b < x < a + b$ . For  $|x - a| > b$ , write  $x < a - b$  or  $x > a + b$ .



### System story check

Before solving, predict the answer count: crossing lines mean **one** solution, parallel lines mean **none**, and the same line means **infinitely many**. Choose substitution when a variable is isolated; choose elimination when coefficients can cancel.



Intersection = shared answer

**System solving choice** Is one variable already alone? Use substitution. Do matching coefficients appear? Use elimination. Are the equations in graph form? Compare slopes and intercepts first.

## 4 Quadratics & Complex Numbers

### Quadratic toolbox

**Standard form**

$$f(x) = ax^2 + bx + c, \text{ where } a \neq 0.$$

**Vertex & axis**

$$h = -\frac{b}{2a}, k = f(h); \text{ axis of symmetry } x = h.$$

**Vertex form**

$$f(x) = a(x - h)^2 + k. \text{ Opens up if } a > 0 \text{ (min), down if } a < 0 \text{ (max).}$$

**Factored form**

$$f(x) = a(x - r_1)(x - r_2) \text{ shows zeros } r_1, r_2.$$

**Quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

**Discriminant**

$D = b^2 - 4ac$ :  $D > 0$  two real roots,  $D = 0$  one real double root,  $D < 0$  two complex roots.

**Complex unit**

$$i^2 = -1, \text{ so } \sqrt{-a} = i\sqrt{a} \text{ for } a > 0.$$

**Powers of  $i$**

cycle every 4:  $i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1$ , then repeat.

**Complex conjugate**

The conjugate of  $a + bi$  is  $a - bi$ ;  $(a + bi)(a - bi) = a^2 + b^2$ .

### Tutor's Note

The same quadratic can be useful in different forms. Standard form is good for the quadratic formula and the  $y$ -intercept. Vertex form is best for maximum/minimum. Factored form is best for zeros.

For  $f(x) = 2x^2 - 8x + 3$ , the vertex has  $h = -\frac{-8}{2(2)} = 2$  and  $k = f(2) = 8 - 16 + 3 = -5$ . **Example** Vertex:  $(2, -5)$ .

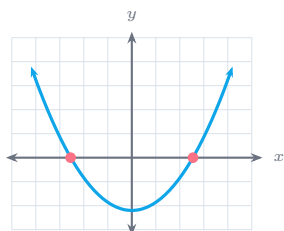


COMPLEX

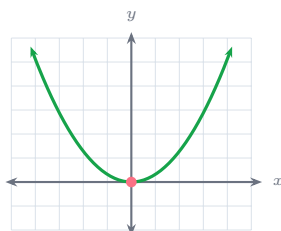
When  $D < 0$ , the graph has no real  $x$ -intercepts, but the equation still has complex solutions. Keep  $i^2 = -1$  handy when simplifying.



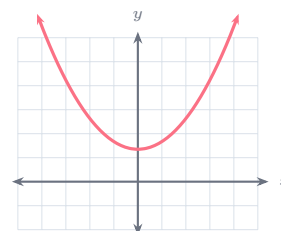
**Discriminant snapshot** Let  $D = b^2 - 4ac$ . It tells how many real  $x$ -intercepts the parabola has before you solve.



$D > 0$ : two



$D = 0$ : one



$D < 0$ : none real

## 5 Polynomial Functions & Factoring

### Polynomial formulas

**Degree**

Highest exponent after the polynomial is simplified.

**Square of a binomial**

$$(a \pm b)^2 = a^2 \pm 2ab + b^2.$$

**Cube of a binomial**

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3; (a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3.$$

**Difference of squares**

$$a^2 - b^2 = (a + b)(a - b).$$

**Sum/difference of cubes**

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2); a^3 - b^3 = (a - b)(a^2 + ab + b^2).$$

**Division algorithm**

$$P(x) = D(x)Q(x) + R(x), \text{ with } \deg R < \deg D \text{ and } D(x) \neq 0.$$

**Remainder theorem**

When  $P(x)$  is divided by  $x - c$ , the remainder is  $P(c)$ . For  $ax - b$ , it is  $P(\frac{b}{a})$ .

**Factor theorem**

$x - c$  is a factor of  $P(x)$  exactly when  $P(c) = 0$ .

**Rational root candidates**

For integer coefficients, candidates are  $\pm \frac{p}{q}$  where  $p$  divides the constant and  $q$  divides the leading coefficient.

**Fundamental theorem**

A degree- $n$  polynomial has exactly  $n$  complex roots, counting multiplicity.

**Multiplicity**

Even multiplicity: graph is tangent to (bounces off) the  $x$ -axis. Odd multiplicity: graph crosses, flattening if  $> 1$ .

**End behavior**

The leading term  $a_n x^n$  controls the far left and far right ends.

### Tutor's Note

Factoring is reverse multiplication. First take out a GCF, then look for special patterns, grouping, or trinomial factoring. For higher-degree polynomials, test possible rational zeros and use division to reduce the degree.

Factor  $x^3 - 8$ . This is a difference of cubes:  $x^3 - 2^3 = (x - 2)(x^2 + 2x + 4)$ .

**Example**

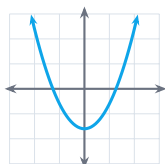




**END BEHAVIOR**

Even degree polynomials point the same way on both ends. Odd degree polynomials point opposite ways. A positive leading coefficient makes the right end go up; a negative leading coefficient makes the right end go down.

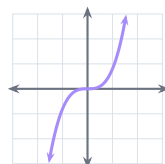
**End behavior: degree & leading coefficient** Only the leading term  $a_n x^n$  controls the far ends. Even degree: both ends match. Odd degree: ends go opposite ways. The sign of  $a_n$  sets the right end.



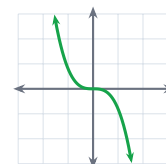
Even,  $a_n > 0$



Even,  $a_n < 0$



Odd,  $a_n > 0$



Odd,  $a_n < 0$

## 6 Rational Expressions & Variation

### Rational expression rules

**Restriction**

Any value that makes a denominator 0 is not allowed.

**Multiply**

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, \text{ with } b, d \neq 0.$$

**Divide**

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}, \text{ with } b, c, d \neq 0.$$

**Add/subtract**

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}, \text{ with } b, d \neq 0.$$

**Direct variation**

$$y = kx.$$

**Inverse variation**

$$y = \frac{k}{x} \text{ or } xy = k, \text{ with } x \neq 0.$$

**Joint variation**

$$y = kxz.$$

**Combined variation**

$$y = \frac{kx}{z}, \text{ with } z \neq 0.$$

**Tutor's Note**

Simplify rational expressions by factoring first, then cancel common factors. Never cancel pieces across addition or subtraction. For rational equations, multiply by the LCD, solve, then check every answer in the original equation.

**Example**

$$\frac{x^2 - 9}{x^2 - 3x} = \frac{(x - 3)(x + 3)}{x(x - 3)} = \frac{x + 3}{x}, \text{ but the original restrictions are } x \neq 0 \text{ and } x \neq 3.$$

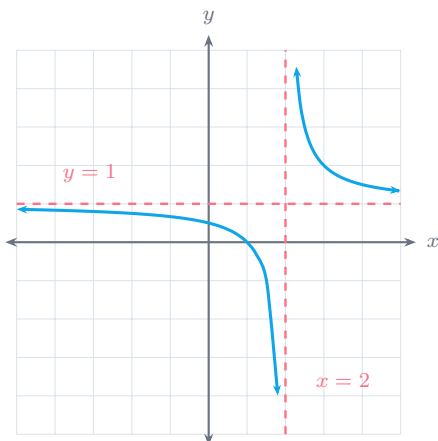


**ASYMPTOTES**

Vertical asymptotes often come from denominator zeros that remain after simplifying. Horizontal asymptotes compare degrees: lower numerator degree gives  $y = 0$ ; equal degrees give the ratio of leading coefficients.



**Reading a rational graph** Denominator zeros that survive give *vertical asymptotes*; comparing degrees gives the *horizontal asymptote*; a canceled factor leaves a *hole*.



**Vertical asymptote:** set the surviving denominator to 0 ( $x = 2$  here).

**Horizontal asymptote:** compare degrees — bottom-heavy gives  $y = 0$ , equal degrees give the ratio of leading coefficients, top-heavy gives none (slant instead).

**Hole:** a factor that cancels from top and bottom removes a single point, not a whole line.

## 7 Radicals & Rational Exponents

### Laws of exponents

**Product of powers**

$$a^m \cdot a^n = a^{m+n}.$$

**Quotient of powers**

$$\frac{a^m}{a^n} = a^{m-n}, \text{ with } a \neq 0.$$

**Power of a power**

$$(a^m)^n = a^{mn}.$$

**Power of a product/quotient**

$$(ab)^n = a^n b^n \text{ and } \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \text{ with } b \neq 0.$$

**Zero exponent**

$$a^0 = 1, \text{ with } a \neq 0.$$

**Negative exponent**

$$a^{-n} = \frac{1}{a^n} \text{ and } \frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}, \text{ with nonzero bases.}$$

### Radical and exponent forms

**Rational exponent**

$$a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m \text{ when the real expression is defined.}$$

**Product rule**

$$\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b} \text{ when both sides are real.}$$

**Quotient rule**

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, \text{ with } b \neq 0.$$

**$n$ th root of  $a^n$**

$$\sqrt[n]{a^n} = a \text{ if } n \text{ is odd; } \sqrt[n]{a^n} = |a| \text{ if } n \text{ is even.}$$

**Even roots**

$\sqrt{x}$  means the principal, nonnegative square root. For real even roots, the radicand must be nonnegative.

**Conjugates**

$$(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b.$$

**Radical equations**

Isolate the radical, raise both sides to the index power, solve, then check.

