

# Z-Scores and the Normal Distribution

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 35

## Quick Review

A **z-score** measures *how many standard deviations* a value is from the mean:  $z = \frac{x - \mu}{\sigma}$ . A z-score of +1.5 means “1.5 standard deviations above the mean”; a z-score of -2 means “2 standard deviations below.”

**Why standardize?** Raw scores from different tests (SAT, ACT, a class quiz) live on different scales – comparing them directly is meaningless. Convert both to z-scores and you can compare apples to apples.

**The standard normal distribution** is  $N(\mu = 0, \sigma = 1)$  – a bell curve centered at 0 with standard deviation 1. Any normal distribution can be *standardized* into  $N(0, 1)$  by computing z-scores.

**The empirical rule (68-95-99.7).** For an approximately normal distribution: about 68% of values fall within  $\pm 1\sigma$  of  $\mu$ ; about 95% within  $\pm 2\sigma$ ; about 99.7% within  $\pm 3\sigma$ .

**Reversing the formula.** Given a z-score, recover the raw value with  $x = \mu + z\sigma$ . Quick check:  $\mu = 500, \sigma = 100, z = 1.5 \Rightarrow x = 500 + 1.5(100) = 650$ .

**Common slips.** Subtracting in the wrong order: it's  $x - \mu$ , not  $\mu - x$ . Forgetting to divide by  $\sigma$  (reporting a raw difference instead of a z-score). Confusing the mean (0) and standard deviation (1) of the standard normal. Using the empirical rule on a clearly non-normal distribution (it's only for bell-shaped ones).

## PRACTICE

Convert between raw scores and z-scores; apply the empirical rule.

- Formula for z-score \_\_\_\_\_
- Use row 1 of the table to compute the z-score. \_\_\_\_\_

Row	$\mu$	$\sigma$	$x$
1	70	8	86
2	500	100	400
3	1000	200	1400

- Use row 2 of the table above to compute the z-score. \_\_\_\_\_

Row	$\mu$	$\sigma$	$x$
1	70	8	86
2	500	100	400
3	1000	200	1400

- Use row 3 of the table above to compute the z-score. \_\_\_\_\_

Row	$\mu$	$\sigma$	$x$
1	70	8	86
2	500	100	400
3	1000	200	1400

- Standard normal: mean and std dev \_\_\_\_\_
- $z = -1.5$  means the value is \_\_\_\_\_
- $\mu = 500, \sigma = 100, z = 1.5$ . Find  $x$ . \_\_\_\_\_
- $\mu = 70, \sigma = 10, z = -0.5$ . Find  $x$ . \_\_\_\_\_
- Empirical rule: % within  $\pm 1\sigma$  \_\_\_\_\_
- Empirical rule: % within  $\pm 2\sigma$  \_\_\_\_\_



- 11. Empirical rule: % within  $\pm 3\sigma$  \_\_\_\_\_
- 12. Heights  $N(70, 3)$ . % between 64 and 76? \_\_\_\_\_
- 13. Student A:  $z = 1$ . Student B:  $z = 0.5$ . Who did better relative to their class? \_\_\_\_\_
- 14. Test  $\mu = 80, \sigma = 5$ . Score 90:  $z = ?$  \_\_\_\_\_
- 15. Different test  $\mu = 85, \sigma = 10$ . Score 90:  $z = ?$  \_\_\_\_\_
- 16.  $z = 0$  means \_\_\_\_\_
- 17. Can a z-score be negative? \_\_\_\_\_
- 18. SAT  $\mu = 1000, \sigma = 200$ . Score 1500:  $z = ?$  \_\_\_\_\_
- 19. IQ  $N(100, 15)$ . IQ 130 is how many SDs above mean? \_\_\_\_\_
- 20. If  $z = 2$ , what % of a normal distribution scores below? \_\_\_\_\_

◆ Word Problems

- 21. On a standardized test with mean  $\mu = 500$  and standard deviation  $\sigma = 100$ , a student scores 650. What is the student's z-score? What does that z-score say about the student's performance? \_\_\_\_\_
- 22. Heights of adult males are approximately normally distributed with mean 70 inches and standard deviation 3 inches. Approximately what percentage of adult males are between 64 and 76 inches tall? Use the empirical rule. \_\_\_\_\_
- 23. Student A scores 85 on a quiz with  $\mu = 80, \sigma = 5$ . Student B scores 90 on a different quiz with  $\mu = 85, \sigma = 10$ . Who did better relative to their class? \_\_\_\_\_
- 24. A factory produces light bulbs with a mean lifespan of  $\mu = 1200$  hours and a standard deviation of  $\sigma = 80$  hours. Lifespans are approximately normal. Approximately what percentage of bulbs last longer than 1360 hours? Use the empirical rule. \_\_\_\_\_

Additional Practice

- 25. Find the mean of 4, 6, 8, 10. \_\_\_\_\_
- 26. Find the median of 3, 9, 4, 10, 7. \_\_\_\_\_
- 27. Find the range of 12, 5, 9, 20. \_\_\_\_\_
- 28. Find the mode of 2, 3, 3, 5, 8. \_\_\_\_\_
- 29. Find  $z$  for  $x = 72$ , mean 60, standard deviation 6. \_\_\_\_\_
- 30. Interpret  $z = -1.5$ . \_\_\_\_\_
- 31. Predicted  $y$  for  $\hat{y} = 2x + 5$  at  $x = 6$ . \_\_\_\_\_
- 32. Residual if actual = 20 and predicted = 17. \_\_\_\_\_
- 33. Positive association: slope sign? \_\_\_\_\_
- 34. Margin of error = 3% around 58%. \_\_\_\_\_
- 35. Sample or census: survey every student. \_\_\_\_\_



## Answer Keys

1.  $z = \frac{x - \mu}{\sigma}$

2. 2

3. -1

4. 2

5.  $\mu = 0, \sigma = 1$

6.  $1.5\sigma$  below  $\mu$

7. 650

8. 65

9. 68%

10. 95%

11. 99.7%

12. 95%

## Additional Practice Answers

25. 7

26. 7

27. 15

28. 3

29. 2

30. 1.5 SD below mean

13. Student A

14. 2

15. 0.5

16.  $x = \mu$

17. Yes

18. 2.5

19. 2

20.  $\approx 97.5\%$

21.  $z = 1.5$ ; about 93rd percentile

22.  $\approx 95\%$

23. Student A ( $z_A = 1 > z_B = 0.5$ )

24.  $\approx 2.5\%$

31. 17

32. 3

33. positive

34. 55% to 61%

35. census

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

1. A careful way to see it: Value minus mean, divided by standard deviation. Order matters:  $x - \mu$ , not  $\mu - x$ . That gives a quick check on the answer.

2. Keep the rule visible: Row 1:  $z = \frac{86 - 70}{8} = \frac{16}{8} = 2$  – two standard deviations above the mean. That gives a quick check on the answer.

3. Row 2:  $z = \frac{400 - 500}{100} = -1$  – one std dev below the mean. The negative sign matters.

4. Start with the key idea: Row 3:  $z = \frac{1400 - 1000}{200} = \frac{400}{200} = 2$  – two std devs above. That gives a quick check on the answer.

5. A careful way to see it: The reference distribution:  $N(0, 1)$ . All z-scores live on this scale. That gives a quick check on the answer.

6. Keep the rule visible: Negative  $\Rightarrow$  below the mean. Magnitude 1.5  $\Rightarrow$  1.5 standard deviations away. That gives a quick check on the answer.

7. Reverse the z-score formula to recover the raw value:  $x = \mu + z\sigma$ . Plug in  $500 + 1.5(100) = 500 + 150 = 650$ . A positive  $z$  lands you above the mean, which is why the answer is bigger than 500.

8. Use  $x = \mu + z\sigma = 70 + (-0.5)(10) = 70 - 5 = 65$ . Keep the negative sign on  $z$  – it's what pulls the value half a standard deviation below the mean instead of above it.

9. A careful way to see it: 68% of a normal distribution sits within one std dev of the mean. That gives a quick check on the answer.

10. Keep the rule visible: 95% of a normal distribution sits within two std devs. That gives a quick check on the answer.

11. One steady path is: 99.7% within three std devs – almost all of the distribution. That gives a quick check on the answer.

12. Start with the key idea:  $64 = 70 - 2(3)$  and  $76 = 70 + 2(3)$ . Range is  $\mu \pm 2\sigma$ , so  $\approx 95\%$  by the empirical rule. That gives a quick check on the answer.

13. Higher z-score = better relative performance. Doesn't matter what the raw scores were – only the z-scores.

14. Apply  $z = \frac{x - \mu}{\sigma} = \frac{90 - 80}{5} = \frac{10}{5} = 2$ . Subtract the mean first ( $x - \mu$ , not  $\mu - x$ ), then divide by  $\sigma$ . A score of 90 sits two standard deviations above the class mean.

15. One steady path is:  $z = \frac{90 - 85}{10} = 0.5$ . Same raw score (90), but on this test it's only half a std dev above the mean. That gives a quick check on the answer.

16. Start with the key idea:  $z = 0$  tells you the value equals the mean exactly. This is the part to check before moving on, because it keeps the answer tied to the original question.

17. A careful way to see it: Negative z-scores indicate values *below* the mean. Standard, not an error. That gives a quick check on the answer.

18. Keep the rule visible:  $z = \frac{1500 - 1000}{200} = \frac{500}{200} = 2.5$ . Two and a half std devs above – impressive. That gives a quick check on the answer.

19. One steady path is:  $z = \frac{130 - 100}{30} = \frac{30}{30} = 1$ . Roughly the 97.5th percentile under a normal model. That gives a quick check on the answer.

20. By the empirical rule, 95% falls within  $\pm 2\sigma$ . By symmetry, 2.5% falls below  $-2\sigma$ , and 2.5% above  $+2\sigma$ . So 97.5% of values are below  $z = 2$ .

21. Apply the formula:  $z = \frac{x - \mu}{\sigma} = \frac{650 - 500}{100} = \frac{150}{100} = 1.5$ . The student is 1.5 standard deviations above the mean. Under a normal model, about 93% of test-takers score below this – a strong performance. (The empirical rule gives us 84% for  $z = 1$  and 97.5% for  $z = 2$ ;  $z = 1.5$  lands between, around the 93rd percentile.)

22. Convert the endpoints to z-scores:  $64 = 70 - 2(3) \Rightarrow z = -2$  and  $76 = 70 + 2(3) \Rightarrow z = 2$ . The interval is  $\mu \pm 2\sigma$ . By the empirical rule,  $\approx 95\%$  of a normal distribution falls within two standard deviations of the mean. So about 95% of adult males are between 64 and 76 inches tall.

23. Convert each to a z-score so they live on a common scale. Student A:  $z_A = \frac{85 - 80}{5} = 1$ . Student B:  $z_B = \frac{90 - 85}{10} = 0.5$ . Even though Student B's raw score (90) is higher, Student A is further above their own class's mean. Relative to their respective classes, Student A did better. (Raw-score comparisons across different tests are misleading – standardize first.)

24. Start with the key idea:  $1360 = 1200 + 2(80) \Rightarrow z = 2$ . We want the percentage of bulbs with  $z > 2$  (lifespan above 1360 hours). The empirical rule says 95% of values lie between  $z = -2$  and  $z = +2$ , leaving 5% outside. By symmetry, half of that (2.5%) sits above  $z = 2$ . So about 2.5% of bulbs outlast 1360 hours – a small “elite tail” of unusually long-lasting bulbs. That gives a quick check on the answer.



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