Writing the Equation of a Sine Graph

Name: _____	Date: _____	Score: _____ / 48

Quick Review

This is the inverse of the previous worksheet: *given* the graph, write the equation. The strategy is always the same – read four numbers off the graph, then plug them into $y = a\sin(b(x-c)) + d$.

Step-by-step.

1. **Midline** d. Average the highest and lowest y-values: $d = \dfrac{\max + \min}{2}$.

2. **Amplitude** a. Half the vertical range: $|a| = \dfrac{\max - \min}{2}$. If the wave starts heading *down* from the midline (when a normal sine should head up), use a negative a.

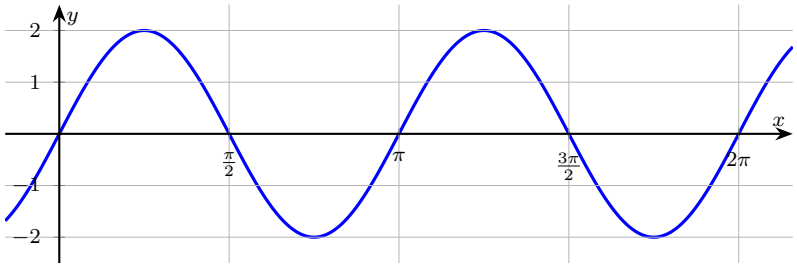
3. **Period** P, then b. Measure one full cycle on the graph (any crest to the next crest works). Then $b = \dfrac{2\pi}{P}$.

4. **Phase shift** c. Find the first x-coordinate where the wave crosses the midline heading up. For a sine, that crossing is where the parent has it at $x = 0$, so c is that x. Write it inside as $(x - c)$.

Sine vs. cosine choice. A graph is a sine if it crosses the midline heading up (or down, with a negative a) at the chosen "start." It's a cosine if it sits at a max (or min, with a negative a). You can write the same wave both ways – just pick whichever is easier to read.

Common slips. Halving the wrong thing for amplitude (it's half the range, not the range). Forgetting that the sign of a flips the wave if the picture starts *below* the midline when sine should start *at* or *above*. Reading the phase shift from a crest instead of an upward midline crossing.

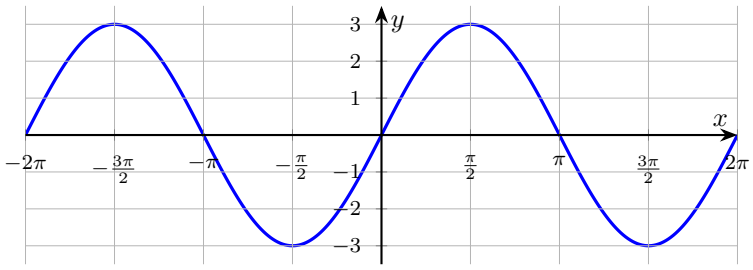
Reference shape: parent sine with amplitude 2, period π, midline $y = 0$, so the equation is $y = 2\sin(2x)$:



PRACTICE

Read amplitude, period, midline, and (where needed) phase shift; write the equation as $y = a\sin(b(x-c)) + d$ *or* $y = a\cos(\ldots)$.

1. Write the equation for this graph. _____



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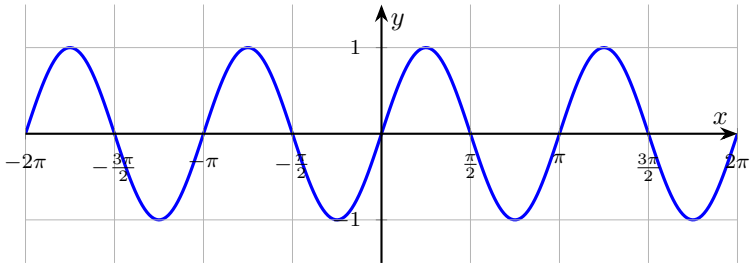
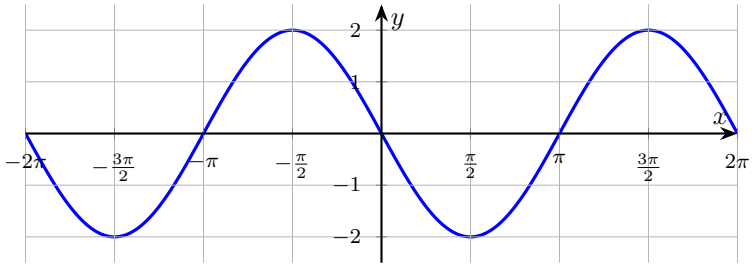
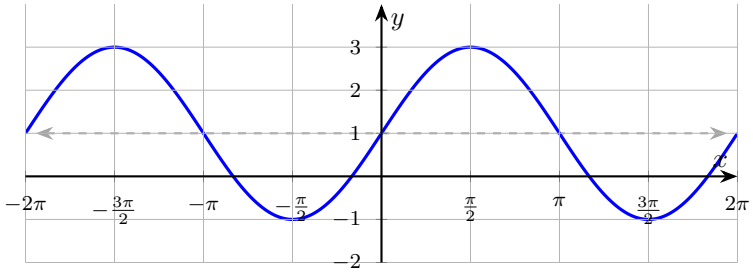
2. Write the equation for this graph. _____



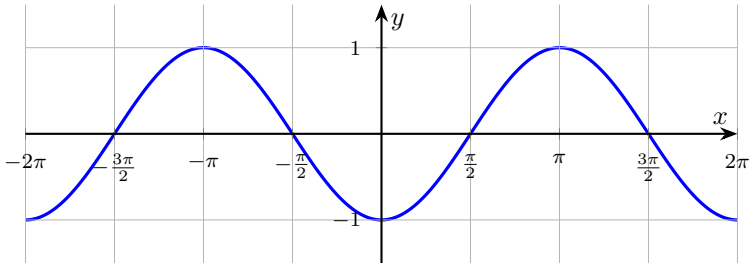
3. Write the equation for this graph. _____



4. Write the equation for this graph. _____



5. Write the equation for this graph. _____



6. A sine wave has amplitude 4, period π, midline $y = 0$, no phase shift. Equation? _____

7. A sine wave has amplitude 1, period 4π, midline $y = 2$, no phase shift. Equation? _____

8. A cosine wave has amplitude 3, period 2π, midline $y = -1$, no phase shift. Equation? _____

9. A cosine wave has amplitude 2, period π, midline $y = 0$, shifted right by $\frac{\pi}{4}$. Equation? _____

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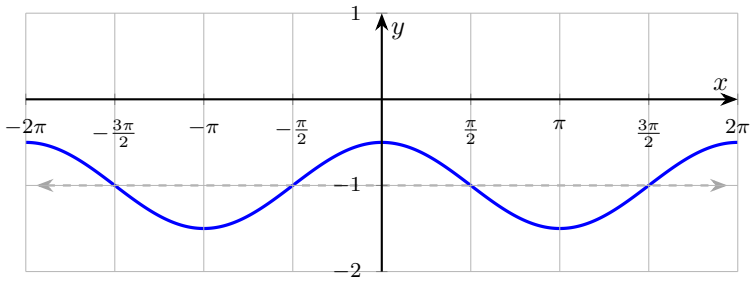
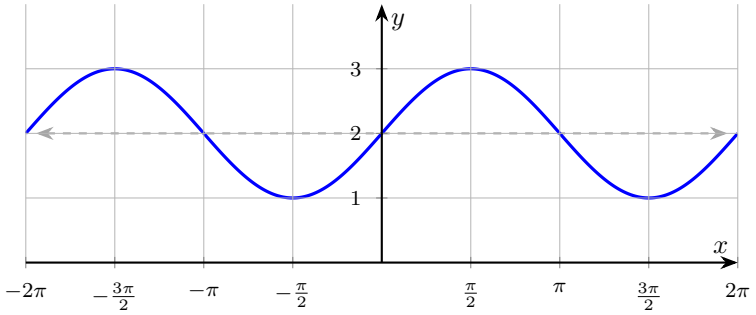
10. Write the equation for this graph. _____



11. True or false: a sine wave that starts at its maximum can be written with a positive a if a phase shift is introduced. _____

12. For the graph below, find the midline. _____



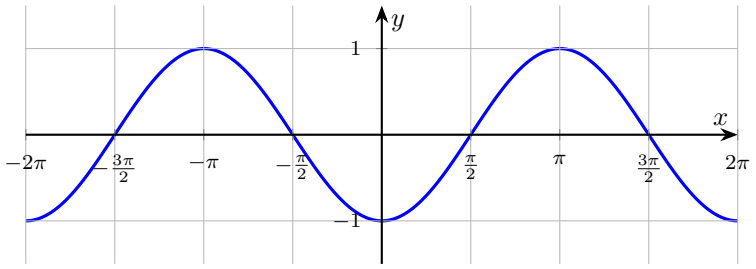
13. For a sine model, what determines whether a is positive or negative? _____

14. Write a sine equation if the graph has max 7, min 3, period π, and starts at the midline going up at $x = 0$. _____

15. Write a cosine equation if the graph has max 4, min -2, period 4π, and reaches the max at $x = 0$. _____

16. Find the equation of a sine curve with maximum at $\left(\frac{\pi}{2}, 5\right)$, minimum at $\left(\frac{3\pi}{2}, 1\right)$. _____

17. Write the equation for this graph (which is a phase-shifted cosine). _____



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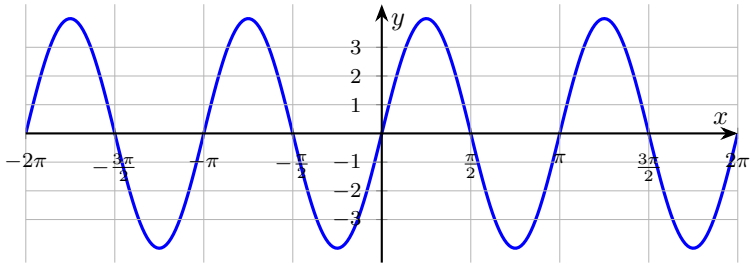
18. The wave below has period π. Read the amplitude. _____



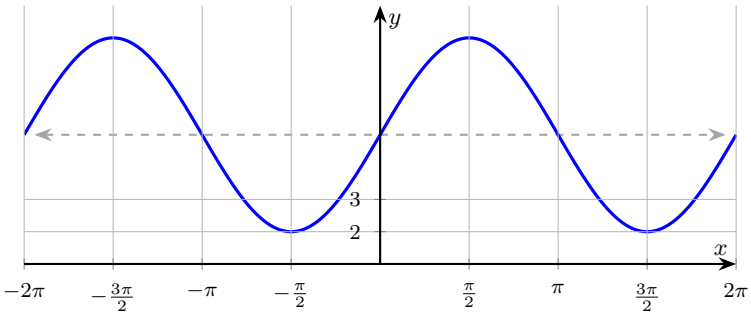
19. For a sine graph, where do you read the period most easily? _____

20. Write a sine equation with max 10, min 2, period 6, and midline-up crossing at $x = 1$. _____

♦ Word Problems

21. A Ferris wheel's height (feet) as a function of time (seconds) reaches a maximum of 50 and a minimum of 2. It completes one revolution in 40 seconds. A rider boards at the minimum at $t = 0$. Write a cosine model $h(t)$. ____

22. The graph below models a tide-height pattern. Write the equation as $y = a\sin(bx) + d$. _____



23. A swing's horizontal displacement (in feet from center) is sinusoidal. The swing reaches $+4$ ft and -4 ft, and a complete back-and-forth takes 3 seconds. The swing is at center, moving forward, at $t = 0$. Write a sine model. ____

24. Daily high temperatures over a year are modeled with a sine function whose max is $85°$F and min is $25°$F. The period is 365 days. The midline-up crossing happens on day 80. Write the equation $T(t)$. ____

Additional Practice

25. Amplitude of $y = 4\sin x$. _____

26. Period of $y = \sin(2x)$. _____

27. Amplitude of $y = -3\cos x$. _____

28. Period of $y = \tan(5x)$. _____

29. Midline of $y = 2\sin x - 7$. _____

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30. Phase shift of $y = \sin(x - \pi/3)$. _____

31. Range of $y = 5\cos x$. _____

32. Range of $y = 2\sin x + 1$. _____

33. Asymptotes of $y = \tan x$ in one period. _____

34. Domain of $y = \sec x$ excludes what? _____

35. Range of $y = \csc x$. _____

36. Range of $y = \arcsin x$. _____

37. Domain of $y = \arccos x$. _____

38. Horizontal asymptotes of $y = \arctan x$. _____

39. First maximum of $y = \sin x$ on $[0, 2\pi]$. _____

40. Zeros of $y = \sin x$ on $[0, 2\pi]$. _____

41. Amplitude of $y = -6\sin x$. _____

42. Period of $y = \cos(4x)$. _____

43. Midline of $y = -3\cos x + 2$. _____

44. Range of $y = 4\sin x - 5$. _____

45. Phase shift of $y = \cos(x + \pi)$. _____

46. Vertical asymptotes of $y = \tan(2x)$ near 0. _____

47. Domain exclusions for $y = \csc x$. _____

48. Range of $y = 3\sec x$. _____

Answer Keys

1. $y = 3\sin x$
2. $y = \sin(2x)$
3. $y = -2\sin x$
4. $y = 2\sin x + 1$
5. $y = \sin\left(x - \frac{\pi}{2}\right)$
6. $y = 4\sin(2x)$
7. $y = \sin\left(\frac{x}{2}\right) + 2$
8. $y = 3\cos x - 1$
9. $y = 2\cos\left(2\left(x - \frac{\pi}{4}\right)\right)$
10. $y = \frac{1}{2}\cos x - 1$
11. True
12. $y = 2$
13. Direction at first midline crossing
14. $y = 2\sin(2x) + 5$
15. $y = 3\cos\left(\frac{x}{2}\right) + 1$
16. $y = 2\sin x + 3$
17. $y = \cos(x - \pi)$ (or $y = -\cos x$)
18. 4
19. Crest to next crest (or any matching points)
20. $y = 4\sin\left(\frac{\pi}{3}(x - 1)\right) + 6$
21. $h(t) = -24\cos\left(\frac{\pi}{20}t\right) + 26$
22. $y = 3\sin x + 5$
23. $x(t) = 4\sin\left(\frac{2\pi}{3}t\right)$
24. $T(t) = 30\sin\left(\frac{2\pi}{365}(t - 80)\right) + 55$

Additional Practice Answers

25. 4
26. π
27. 3
28. $\frac{\pi}{5}$
29. $y = -7$
30. $\frac{\pi}{3}$ right
31. $[-5, 5]$
32. $[-1, 3]$
33. $x = \pm\frac{\pi}{2}$
34. $\cos x = 0$
35. $(-\infty, -1] \cup [1, \infty)$
36. $[-\frac{\pi}{2}, \frac{\pi}{2}]$
37. $[-1, 1]$
38. $y = \pm\frac{\pi}{2}$
39. $(\frac{\pi}{2}, 1)$
40. $0, \pi, 2\pi$
41. 6
42. $\frac{\pi}{2}$
43. $y = 2$
44. $[-9, -1]$
45. π left
46. $x = \pm\frac{\pi}{4}$
47. $x = n\pi$
48. $(-\infty, -3] \cup [3, \infty)$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Crests at 3, troughs at -3, so $|a| = 3$, $d = 0$. One cycle in 2π, so $b = 1$. The wave crosses the midline heading up at $x = 0$ – no phase shift.

2. Amplitude 1, midline 0. The wave finishes its cycle in π, so $b = \frac{2\pi}{\pi} = 2$. No phase shift.

3. Amplitude 2, but the wave heads *down* from the midline at $x = 0$. Use $a = -2$. Period 2π, midline 0.

4. Max 3, min -1. Midline $= \frac{3 + (-1)}{2} = 1$, amplitude $= \frac{3 - (-1)}{2} = 2$. Period 2π, no phase shift.

5. Amplitude 1, period 2π, midline 0. The first upward midline crossing has moved from 0 to $\frac{\pi}{2}$ – a right shift of $\frac{\pi}{2}$. (This is also $-\cos x$, but we asked for sine form.)

6. Amplitude 4 gives $a = 4$. Period π gives $b = \frac{2\pi}{\pi} = 2$. Midline $y = 0$ and no shift mean $d = 0$ and $c = 0$, so $y = 4\sin(2x)$.

7. Amplitude 1 gives $a = 1$. Period 4π gives $b = \frac{2\pi}{4\pi} = \frac{1}{2}$. Midline $y = 2$ gives $d = 2$, and there is no shift, so $y = \sin\left(\frac{x}{2}\right) + 2$.

8. Plug into $y = a\cos(bx) + d$. Amplitude 3 gives $a = 3$. Period 2π gives $b = \frac{2\pi}{2\pi} = 1$. Midline $y = -1$ gives $d = -1$, so $y = 3\cos x - 1$.

9. Amplitude 2 gives $a = 2$. Period π gives $b = \frac{2\pi}{\pi} = 2$. A right shift of $\frac{\pi}{4}$ becomes a *minus* inside the parentheses: $\left(x - \frac{\pi}{4}\right)$. Midline is 0, so no d term.

10. Max -0.5, min -1.5. Midline $= -1$, amplitude $= \frac{-0.5 - (-1.5)}{2} = 0.5$. The graph starts at the top (max), so cosine without a phase shift. $a = \frac{1}{2}$.

11. Yes – $y = A\sin(x + \frac{\pi}{2}) = A\cos x$, which starts at the top. The phase shift moves the upward midline crossing.

12. Max 3, min 1. Midline $= \frac{3 + 1}{2} = 2$. Amplitude is $\frac{3 - 1}{2} = 1$, and the equation is $y = \sin x + 2$.

13. If the wave goes up through the midline at $x = 0$ (no shift), $a > 0$. If it goes down through the midline at $x = 0$, use $a < 0$.

14. Midline $= \frac{7 + 3}{2} = 5$, amplitude $= \frac{7 - 3}{2} = 2$. Period $\pi \Rightarrow b = 2$. Goes up at $x = 0$, so $a > 0$.

15. Midline $= \frac{4 + (-2)}{2} = 1$. Amplitude $= 3$. Period $4\pi \Rightarrow b = \frac{1}{2}$. Starts at

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max with no shift – cosine territory.

16. Max 5 and min 1 give midline 3, amplitude 2. Max and min are π apart, so half-period $= \pi$ and full period $= 2\pi \Rightarrow b = 1$. The midline-up crossing is at $x = 0$, so no phase shift.

17. Parent cosine starts at max $(0, 1)$. This one starts at min $(0, -1)$ – a π horizontal shift right, or equivalently a vertical flip.

18. Keep the rule visible: Crests at 4, troughs at -4. Amplitude $= \dfrac{4-(-4)}{2} = 4$. Equation: $y = 4\sin(2x)$. That gives a quick check on the answer.

19. Period $=$ horizontal distance between any two repeats. Crest-to-crest is the cleanest measurement.

20. Midline $= \dfrac{10+2}{2} = 6$, amplitude $= 4$. Period 6 gives $b = \dfrac{2\pi}{6} = \dfrac{\pi}{3}$. Shift right by 1 to match the crossing.

21. Midline $= \dfrac{50+2}{2} = 26$, amplitude $= \dfrac{50-2}{2} = 24$. Period $40 \Rightarrow b = \dfrac{2\pi}{40} = \dfrac{\pi}{20}$. Starting at the minimum at $t = 0$ matches $-\cos$ (flipped cosine), so $a = -24$. (Check: $h(0) = -24(1) + 26 = 2$ ✓.)

22. Max 8, min 2. Midline 5, amplitude 3. Period 2π gives $b = 1$, no phase shift visible (the curve crosses midline going up at $x = 0$).

23. Amplitude 4 (max swing 4 ft each way). Period 3 gives $b = \dfrac{2\pi}{3}$. Midline 0 (centered). At $t = 0$ the swing is at center moving forward – the parent sine starts at $(0, 0)$ heading up, so no phase shift, positive a.

24. Midline $= \dfrac{85+25}{2} = 55$, amplitude $= \dfrac{85-25}{2} = 30$. Period 365 gives $b = \dfrac{2\pi}{365}$. Midline-up at day 80 means a phase shift of 80 days right – plug in as $(t - 80)$ inside.

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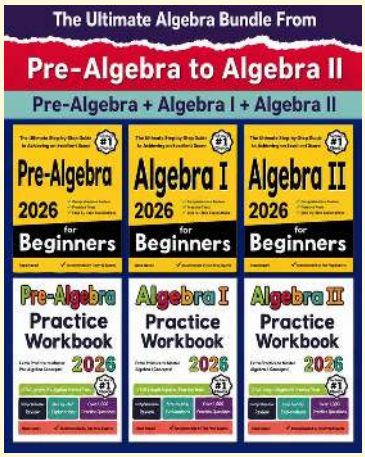

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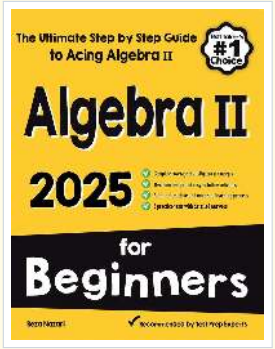

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