Using Regression Models for Prediction

Name: ______	Date: ______	Score: ____ / 30

Quick Review

Once you have a regression equation $\hat{y} = mx + b$, you can use it for two big jobs: **prediction** (plug in an x, read out an $\hat{y}$) and **interpretation** (translate the slope and intercept back into the context of the problem).

Slope interpretation template. "For each one-unit increase in x, the predicted value of y changes by m units." Always include the units. If the slope is 2.5 and y is in thousands of dollars per additional ad campaign, then each extra campaign predicts $2,500 more in revenue – not $2.50, not 2.5 raw dollars.

Intercept interpretation template. "The predicted value of y when $x = 0$." Sometimes this is meaningful (a brand-new car at $x = 0$ years), sometimes it's nonsense (a newborn baby has zero years of education – predicted income is not informative). Always sanity-check whether $x = 0$ is inside the realistic range.

Interpolation vs. extrapolation. Predicting inside the data range is *interpolation* – generally safe. Predicting far outside is *extrapolation* – risky, because the linear pattern may not hold there.

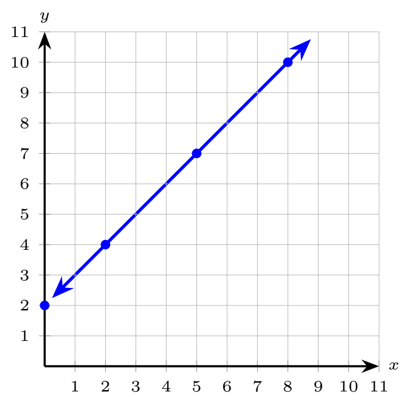
Residuals to check predictions. A predicted $\hat{y} = 82$ for a student who actually scored 87 has residual $87 - 82 = +5$ – the model underestimated. A pattern of all-positive (or all-negative) residuals in a range signals systematic bias.

Common slips. Forgetting to multiply the slope by x and just adding the intercept. Reporting the slope without units. Treating the slope as the total y-value instead of a rate of change. Mixing up the direction of the residual sign (positive residual = point is *above* the line).

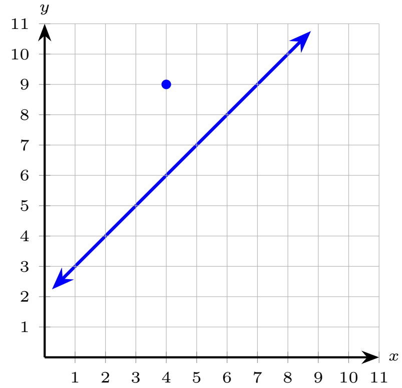
PRACTICE

Predict, interpret, and check the residual where useful.

1. $\hat{y} = 0.8x + 15,\ x = 7;\ \hat{y} = ?$ ________
2. $\hat{y} = 3x + 4,\ x = 0$ ________
3. $\hat{y} = -2x + 10,\ x = 3$ ________
4. $\hat{y} = 5x - 7,\ x = 4$ ________
5. Slope $= 2.5$ (revenue in thousands per campaign). One campaign predicts ________
6. Intercept of $\hat{y} = -1.8x + 28$ (car value in thousands of dollars, x in years). Meaning? ________
7. The regression line $\hat{y} = x + 2$ is graphed below. Use it to predict $\hat{y}$ when $x = 6$. ________



8. The line $\hat{y} = x + 2$ is graphed below with the data point $(4, 9)$ marked. Find the residual at that point. ________



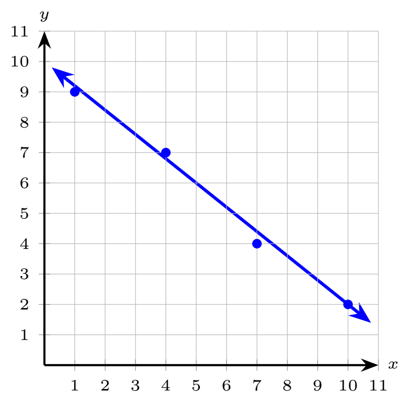
9. Actual $y = 15,\ \hat{y} = 20$. Residual? ________

10. $\hat{y} = 0.05x + 12,\ x = 200$ ________

11. Interpret slope 0.5 (predicted GPA per study hour). ________

12. $\hat{y} = 4x + 1,\ x = -2$ ________

13. From the regression line graphed below, what happens to predicted y as x rises? ________



14. $\hat{y} = 10 - 0.5x,\ x = 20$ ________

15. Predict at the data center ________

16. $\hat{y} = 2.5x + 10,\ y = 30$. Find x. ________

17. Strong vs. weak prediction power signaled by ________

18. If $r^2 = 0.04$, predictions are ________

19. $\hat{y} = -3x + 50,\ x = 12$ ________

20. When is the intercept b *not* meaningful in context? ________

Word Problems

21. A college fits $\hat{y} = 0.005x + 2.1$ where x is high-school SAT score and y is predicted college GPA. Predict the GPA for a student with an SAT score of 1200, and interpret the slope in context. ____

22. A trucking company fits $\hat{y} = 0.18x + 25$ where x is distance driven (miles) and y is fuel cost (dollars). Predict the fuel cost for a 300-mile trip, and interpret both the slope and the intercept. ____

23. A teacher fits $\hat{y} = 4x + 62$ where x is the number of homework assignments completed and y is the predicted final exam score. One student completes 7 assignments and scores 95. Compute the residual and interpret it. ____

24. A car-value model is $\hat{y} = -1.8x + 28$ in thousands of dollars, where x is age in years. The model works well for cars aged 0–10 years. A dealer asks the model to predict the value of a 20-year-old car. The model returns $\hat{y} = -8$ (i.e., $-\$8{,}000$). What went wrong, and what should the dealer do? ____

Additional Practice

25. Find the mean of $4, 6, 8, 10$. ________

26. Find the median of $3, 9, 4, 10, 7$. ________

27. Find the range of $12, 5, 9, 20$. ________

28. Find the mode of $2, 3, 3, 5, 8$. ________

29. Find z for $x = 72$, mean 60, standard deviation 6. ________

30. Interpret $z = -1.5$. ________

Answer Keys

1. 20.6	13. falls
2. 4	14. 0
3. 4	15. $(\bar{x}, \bar{y})$
4. 13	16. 8
5. $2,500 more revenue	17. r^2 (or $\lvert r \rvert$)
6. $28,000 for a brand-new car	18. barely better than the mean
7. 8	19. 14
8. 3	20. $x = 0$ outside data range
9. −5	21. $\hat{y} = 8.1$ (out-of-range); slope $= +0.005$
10. 22	22. $\hat{y} = \$79$; slope $0.18/mile; intercept $25
11. +0.5 GPA per study hour	23. residual $= +5$; outperformed by 5
12. −7	24. extrapolation; the linear model breaks at age 20

Additional Practice Answers

25. 7	28. 3
26. 7	29. 2
27. 15	30. 1.5 SD below mean

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: $0.8(7) + 15 = 5.6 + 15 = 20.6$. Multiply first, then add. This is the part to check before moving on, because it keeps the answer tied to the original question.

2. Keep the rule visible: Anything times 0 is 0, so $\hat{y} = 4$ – that's the intercept. That gives a quick check on the answer.

3. Multiply the slope by x before adding the intercept: $-2(3)+10 = -6+10 = 4$. The negative slope pulls the prediction below the intercept of 10.

4. Start with the key idea: $5(4) - 7 = 20 - 7 = 13$. This is the part to check before moving on, because it keeps the answer tied to the original question.

5. Slope 2.5 in thousands $=$ \$2,500 per unit increase in x. Watch the unit conversion.

6. Keep the rule visible: $x = 0$ years means brand-new. Predicted value is \$28,000 (intercept times the units). That gives a quick check on the answer.

7. One steady path is: Read up from $x = 6$ to the line: $\hat{y} = 6 + 2 = 8$. This is the part to check before moving on, because it keeps the answer tied to the original question.

8. Predicted value at $x = 4$: $\hat{y} = 4 + 2 = 6$. Residual $= y - \hat{y} = 9 - 6 = 3$. The point sits 3 units *above* the line.

9. A careful way to see it: $15 - 20 = -5$. Negative $\Rightarrow$ point sits below the line, model overestimated. That gives a quick check on the answer.

10. Keep the rule visible: $0.05(200) + 12 = 10 + 12 = 22$. This is the part to check before moving on, because it keeps the answer tied to the original question.

11. Slope is rate of change. Each additional study hour predicts 0.5 more GPA points (within the data range).

12. Start with the key idea: $4(-2) + 1 = -8 + 1 = -7$. Watch the sign. This is the part to check before moving on, because it keeps the answer tied to the original question.

13. The line slopes downward, so bigger x gives a smaller predicted y – a negative-slope (inverse) relationship.

14. Handle the slope term first: $0.5(20) = 10$, then subtract from the intercept: $10 - 10 = 0$. Writing the line as $10 - 0.5x$ is the same as $-0.5x + 10$ – the slope is still -0.5.

15. The regression line is guaranteed to pass through the point of means, so plugging in $\bar{x}$ predicts exactly $\bar{y}$.

16. Start with the key idea: $30 = 2.5x + 10 \Rightarrow 20 = 2.5x \Rightarrow x = 8$. Solving for x given a target y – standard inverse use of regression. That gives a quick check on the answer.

17. Higher r^2 means the line explains more of the variation, so individual predictions tend to land closer to actual values.

18. Keep the rule visible: $r^2 = 0.04$ means the model explains only 4% of the variation. The line adds very little over just predicting $\bar{y}$ every time. That gives a quick check on the answer.

19. One steady path is: $-3(12) + 50 = -36 + 50 = 14$. This is the part to check before moving on, because it keeps the answer tied to the original question.

20. If $x = 0$ is unreachable or silly (e.g., $x =$ age of an adult, $x = 0$ months wouldn't apply), the intercept is just a mathematical anchor, not a real-world value.

21. Plug in: $\hat{y} = 0.005(1200) + 2.1 = 6 + 2.1 = 8.1$. Hold up – GPA caps at 4.0. The model prediction of 8.1 is nonsense in context; this is what happens when the linear form breaks down (or the intercept and slope I specified don't realistically describe the relationship). The slope 0.005 would say each extra SAT point predicts an additional 0.005 GPA. The lesson: always sanity-check predictions against what's possible in the real world.

22. Predict: $\hat{y} = 0.18(300) + 25 = 54 + 25 = \$79$. Slope: every additional mile predicts \$0.18 more in fuel – the variable cost. Intercept: the model predicts \$25 even at 0 miles – that's a fixed cost (maybe a daily truck fee, insurance, or some other baseline expense). This dual interpretation – fixed plus variable – is common in business regressions.

23. Predicted: $\hat{y} = 4(7) + 62 = 28 + 62 = 90$. Actual: 95. Residual: $95 - 90 = +5$. Positive residual means the actual score is *above* the line – the student did better than the model predicted. If all the students who completed 7 assignments had positive residuals, that'd signal a systematic underprediction in that range – probably the line should curve up there.

24. Start with the key idea: $\hat{y} = -1.8(20) + 28 = -36 + 28 = -8$. A negative car value is impossible. The model was fit on cars aged 0–10; using it at $x = 20$ is extrapolation – the linear depreciation can't go on forever (older cars eventually level off, sometimes even rising as classics). The dealer should refit a model that includes older cars, or use a different functional form (e.g., exponential decay) that respects the floor of value \$0. That gives a quick check on the answer.

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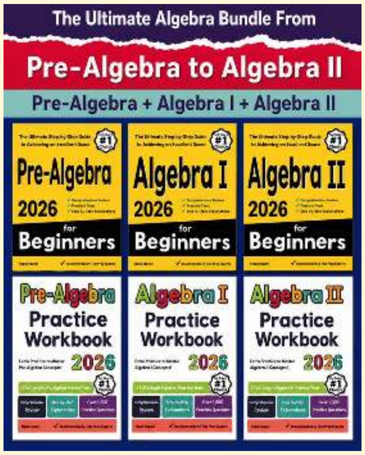

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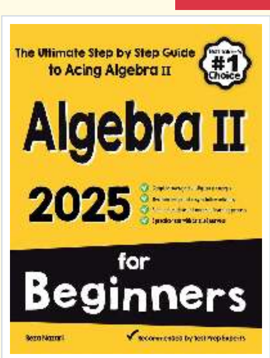

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