

# Unit Circle

Name: \_\_\_\_\_

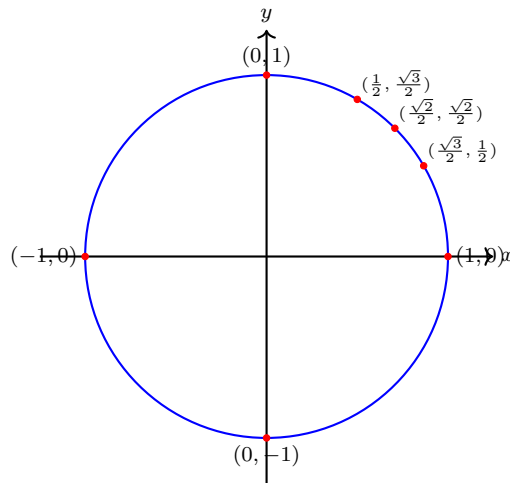
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## Q Quick Review

The **unit circle** is the circle of radius 1 centered at the origin:  $x^2 + y^2 = 1$ . It does something magical – it turns the abstract sine and cosine functions into honest geometric coordinates.

**The core fact.** For any angle  $\theta$  in standard position, the terminal side hits the unit circle at the point  $(\cos \theta, \sin \theta)$ . The  $x$ -coordinate *is* cosine; the  $y$ -coordinate *is* sine.



### Special-angle coordinates to memorize (Q1).

$$\theta = 0: (1, 0)$$

$$\theta = \frac{\pi}{6} = 30^\circ: \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$\theta = \frac{\pi}{4} = 45^\circ: \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$\theta = \frac{\pi}{3} = 60^\circ: \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$\theta = \frac{\pi}{2} = 90^\circ: (0, 1)$$

**Sign by quadrant.** Coordinates inherit the quadrant's signs. Q2 has  $x < 0, y > 0$  – so cosine negative, sine positive. Q3 has both negative. Q4 has  $x > 0, y < 0$ .

**Coterminal angles share the same point.** Adding  $360^\circ$  (or  $2\pi$ ) returns you to the same spot on the circle, so all six trig functions repeat. That's the origin of periodicity.

**Identifying angles from a point.** Given a point on the unit circle, the angle is whichever standard-position angle has that point on its terminal side. Use the quadrant and the reference angle to pick the right value.

**Common slips.** Swapping  $x$  and  $y$ :  $(\cos \theta, \sin \theta)$  – cosine first. Saying the unit circle has equation  $x + y = 1$  (it's  $x^2 + y^2 = 1$ ). Treating  $\left(\frac{3}{5}, \frac{4}{5}\right)$  as a point not on the unit circle (it is – check:  $\frac{9}{25} + \frac{16}{25} = 1$ ).

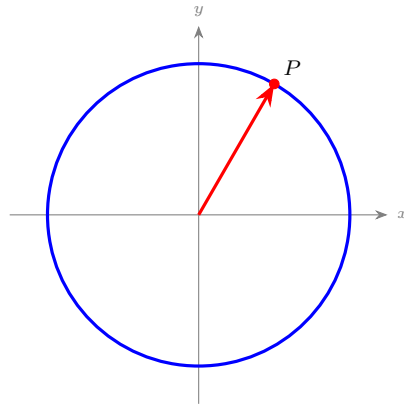


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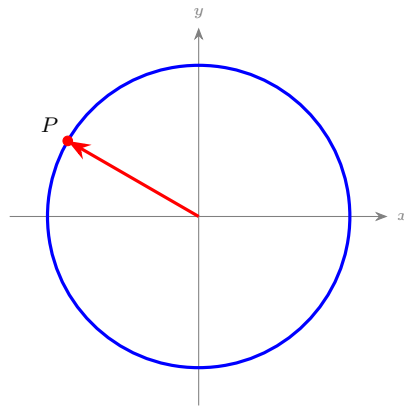
**PRACTICE**

Read coordinates and angles off the unit circle. Coordinates are  $(\cos \theta, \sin \theta)$ .

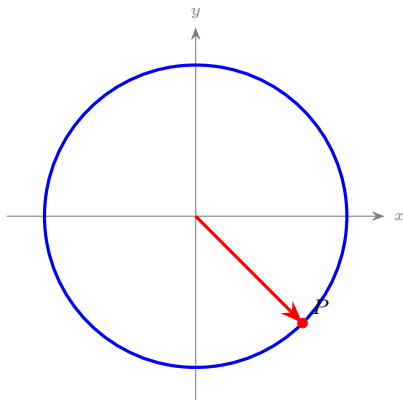
1. Point on unit circle at  $\theta = 0$ . \_\_\_\_\_
2. Point at  $\theta = \frac{\pi}{2}$ . \_\_\_\_\_
3. Point at  $\theta = \pi$ . \_\_\_\_\_
4. Point at  $\theta = \frac{3\pi}{2}$ . \_\_\_\_\_
5. Give the coordinates of the point at  $\theta = \frac{\pi}{3}$  drawn below. \_\_\_\_\_



6. Give the coordinates of the point at  $\theta = \frac{5\pi}{6}$  drawn below. \_\_\_\_\_



7. Give the coordinates of the point at  $\theta = \frac{7\pi}{4}$  drawn below. \_\_\_\_\_



8. Point at  $\theta = \frac{4\pi}{3}$ . \_\_\_\_\_

9. Point at  $\theta = \frac{11\pi}{6}$ . \_\_\_\_\_

10. Does  $(\frac{3}{5}, \frac{4}{5})$  lie on the unit circle? \_\_\_\_\_

11. Does  $(1, 1)$  lie on the unit circle? \_\_\_\_\_

12.  $\sin \theta$  at  $\theta = \frac{4\pi}{3}$ . \_\_\_\_\_

13.  $\cos \theta$  at  $\theta = \frac{5\pi}{6}$ . \_\_\_\_\_

14. Find the angle in  $[0, 2\pi)$  with unit-circle point  $(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$ . \_\_\_\_\_

15. Find the angle in  $[0, 2\pi)$  with unit-circle point  $(0, -1)$ . \_\_\_\_\_

16.  $\tan \theta$  at  $\theta = \frac{2\pi}{3}$ . \_\_\_\_\_

17. Equation of the unit circle. \_\_\_\_\_

18. True or False: coterminal angles correspond to the same point on the unit circle. \_\_\_\_\_

19. Point at  $\theta = -\frac{\pi}{4}$ . \_\_\_\_\_

20. Find  $\cos \theta$  when the unit-circle point is  $(\frac{5}{13}, -\frac{12}{13})$ . \_\_\_\_\_

◆ Word Problems

21. An ant walks counterclockwise on the unit circle from  $(1, 0)$ , sweeping out an angle of  $\frac{5\pi}{3}$ . Where does it end up? \_\_\_\_\_

22. A point  $(-\frac{4}{5}, \frac{3}{5})$  lies on the unit circle. Find  $\sin \theta$ ,  $\cos \theta$ , and  $\tan \theta$  at the corresponding angle. \_\_\_\_\_

23. For what angle  $\theta$  in  $[0, 2\pi)$  does the unit circle pass through  $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ ? \_\_\_\_\_

24. A particle on the unit circle moves so that at time  $t$  seconds, its angle from the positive  $x$ -axis is  $\theta(t) = \frac{\pi}{6}t$  radians. Find the particle's coordinates at  $t = 2$  seconds. \_\_\_\_\_



**Additional Practice**

25. Find  $\sin \theta$  if opposite = 5, hypotenuse = 13. \_\_\_\_\_
26. Find  $\cos \theta$  if adjacent = 12, hypotenuse = 13. \_\_\_\_\_
27. Find  $\tan \theta$  if opposite = 7, adjacent = 4. \_\_\_\_\_
28. Find  $\sin 30^\circ$ . \_\_\_\_\_
29. Find  $\cos 60^\circ$ . \_\_\_\_\_
30. Find  $\tan 45^\circ$ . \_\_\_\_\_
31. Convert  $180^\circ$  to radians. \_\_\_\_\_
32. Convert  $\frac{\pi}{3}$  radians to degrees. \_\_\_\_\_
33. Find a coterminal angle with  $70^\circ$ . \_\_\_\_\_
34. Reference angle for  $150^\circ$ . \_\_\_\_\_
35. Use  $\sin^2 \theta + \cos^2 \theta$ . \_\_\_\_\_
36. If  $\sin \theta = \frac{3}{5}$ ,  $\theta$  in QI, find  $\cos \theta$ . \_\_\_\_\_
37. Find  $\sec \theta$  if  $\cos \theta = \frac{2}{5}$ . \_\_\_\_\_



Answer Keys

1. $(1, 0)$	13. $-\frac{\sqrt{3}}{2}$
2. $(0, 1)$	14. $\frac{4\pi}{3}$
3. $(-1, 0)$	15. $\frac{3\pi}{2}$
4. $(0, -1)$	16. $-\sqrt{3}$
5. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	17. $x^2 + y^2 = 1$
6. $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	18. True
7. $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$	19. $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
8. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$	20. $\frac{5}{13}$
9. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	21. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
10. Yes	22. $\sin \theta = \frac{3}{5}, \cos \theta = -\frac{4}{5}, \tan \theta = -\frac{3}{4}$
11. No	23. $\theta = \frac{\pi}{4}$
12. $-\frac{\sqrt{3}}{2}$	24. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

**Additional Practice Answers**

25. $\frac{5}{13}$	31. $\pi$
26. $\frac{12}{13}$	32. $60^\circ$
27. $\frac{7}{4}$	33. $430^\circ$
28. $\frac{1}{2}$	34. $30^\circ$
29. $\frac{1}{2}$	35. 1
30. 1	36. $\frac{4}{5}$
	37. $\frac{5}{2}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

- The point is  $(\cos \theta, \sin \theta)$ . At  $\theta = 0$  the terminal side lies on the positive  $x$ -axis, where  $\cos 0 = 1$  and  $\sin 0 = 0$ , giving  $(1, 0)$ .
- At  $\frac{\pi}{2} = 90^\circ$  you're at the top of the circle:  $\cos \frac{\pi}{2} = 0$  and  $\sin \frac{\pi}{2} = 1$ , so the point is  $(0, 1)$ .
- At  $\pi = 180^\circ$  the terminal side points along the negative  $x$ -axis:  $\cos \pi = -1$ ,  $\sin \pi = 0$ , so  $(-1, 0)$ .
- At  $\frac{3\pi}{2} = 270^\circ$  you're at the bottom of the circle on the negative  $y$ -axis:  $\cos = 0$ ,  $\sin = -1$ , so  $(0, -1)$ .
- The drawn ray is the Q1 special angle  $\frac{\pi}{3} = 60^\circ$ . Coordinates are  $(\cos \theta, \sin \theta)$ , so  $\left(\cos \frac{\pi}{3}, \sin \frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ . (Cosine first – a common spot to swap  $x$  and  $y$ .)
- Keep the rule visible:  $\frac{5\pi}{6} = 150^\circ$ , Q2, reference  $\frac{\pi}{6}$ . Q2:  $x < 0, y > 0$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
- One steady path is:  $\frac{7\pi}{4} = 315^\circ$  is in Q4 with reference angle  $\frac{\pi}{4}$ . The reference-angle coordinates are both  $\frac{\sqrt{2}}{2}$ ; in Q4 the  $x$  stays positive and the  $y$

turns negative, giving  $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ . That gives a quick check on the answer.

- Start with the key idea:  $\frac{4\pi}{3} = 240^\circ$  is in Q3 with reference angle  $\frac{\pi}{3}$ , whose coordinates are  $\frac{1}{2}$  and  $\frac{\sqrt{3}}{2}$ . Q3 makes both negative:  $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ . That gives a quick check on the answer.
- A careful way to see it:  $\frac{11\pi}{6} = 330^\circ$  is in Q4 with reference angle  $\frac{\pi}{6}$  (coordinates  $\frac{\sqrt{3}}{2}$  and  $\frac{1}{2}$ ). In Q4,  $x > 0$  and  $y < 0$ , so the point is  $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ . That gives a quick check on the answer.
- Keep the rule visible: Check:  $\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{9}{25} + \frac{16}{25} = 1 \checkmark$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
- One steady path is:  $1^2 + 1^2 = 2 \neq 1$ . (Its distance from the origin is  $\sqrt{2}$ , not 1.) That gives a quick check on the answer.
- Sine is the  $y$ -coordinate of the unit-circle point. At  $\frac{4\pi}{3} = 240^\circ$  (Q3) the point



is  $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ , so  $\sin \theta = -\frac{\sqrt{3}}{2}$ .

13. Cosine is the  $x$ -coordinate. At  $\frac{5\pi}{6} = 150^\circ$  (Q2) the point is  $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ , so

$\cos \theta = -\frac{\sqrt{3}}{2}$  (negative, as cosine is throughout Q2).

14. Both coordinates negative  $\rightarrow$  Q3. Reference angle (from  $x$ -axis):  $\frac{\pi}{3}$  (since  $|x| = \frac{1}{2} = \cos \frac{\pi}{3}$ ). Q3 angle:  $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$ .

15. One steady path is:  $(0, -1)$  is the bottom of the circle, straight down the negative  $y$ -axis. The standard-position angle pointing there is  $\frac{3\pi}{2} = 270^\circ$ . That gives a quick check on the answer.

16. Start with the key idea: Point is  $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ .  $\tan = \frac{y}{x} = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$ .

This is the part to check before moving on, because it keeps the answer tied to the original question.

17. It's the circle of radius 1 centered at the origin, so by the distance formula every point  $(x, y)$  on it satisfies  $x^2 + y^2 = 1$ . (Note the squares – not  $x + y = 1$ .)

18. Coterminal angles share a terminal side, so they hit the unit circle at the same point.

19. One steady path is: Coterminal with  $\frac{7\pi}{4}$  (in Q4). Coordinates:  $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ .

That gives a quick check on the answer.

20. Cosine is the  $x$ -coordinate:  $\frac{5}{13}$ . (Quick check the point is on the circle:  $\frac{25 + 144}{169} = 1 \checkmark$ .)

21. A careful way to see it:  $\frac{5\pi}{3} = 300^\circ$ , Q4, reference  $\frac{\pi}{3}$ . Q4 coordinates:  $x > 0$ ,  $y < 0$ . At reference  $\frac{\pi}{3}$ :  $|x| = \cos \frac{\pi}{3} = \frac{1}{2}$  and  $|y| = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ . With Q4 signs, the point is  $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ . That gives a quick check on the answer.

22. On the unit circle,  $\cos \theta = x$  and  $\sin \theta = y$ . So  $\cos \theta = -\frac{4}{5}$  and  $\sin \theta = \frac{3}{5}$ .

Then  $\tan \theta = \frac{y}{x} = \frac{3/5}{-4/5} = -\frac{3}{4}$ . (Q2 – sine positive, cosine negative – consistent with ASTC.)

23. Both coordinates positive and equal point to a  $45^\circ$  angle in Q1.  $\frac{\pi}{4} = 45^\circ$  with

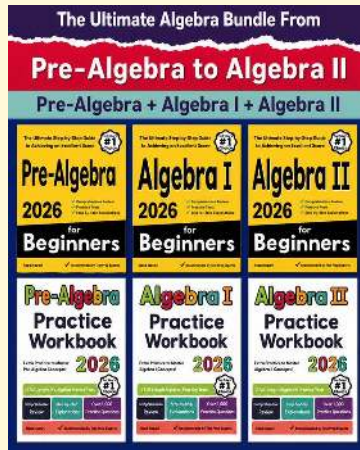
$\cos = \sin = \frac{\sqrt{2}}{2}$ .

24. At  $t = 2$ :  $\theta = \frac{\pi}{6} \cdot 2 = \frac{\pi}{3} = 60^\circ$ . Unit-circle coordinates at  $\frac{\pi}{3}$ :

$\left(\cos \frac{\pi}{3}, \sin \frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ . (After 6 seconds,  $\theta = \pi$  and the particle is at  $(-1, 0)$ .)



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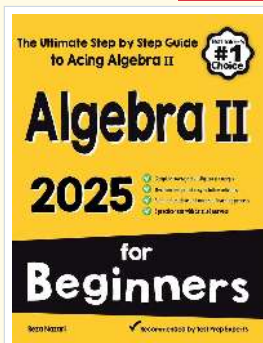
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