

Theoretical Probability

Name: _____ Date: _____ Score: _____ / 35

Q Quick Review

Theoretical probability is what the math *predicts* when every outcome is equally likely. The formula is the cleanest in the whole subject:

$$P(E) = \frac{\text{favorable outcomes}}{\text{total outcomes}}$$

Range. Every probability lives in $[0, 1]$. A 0 means the event is impossible; a 1 means certain; values in between give the relative frequency you'd expect over many repeats.

Counting matters. For a deck, sample space has 52 cards. For a die, 6 outcomes. For two dice, 36 ordered pairs. Pin down the size of the sample space first – that's your denominator. Then count favorable outcomes (your numerator).

Complement trick. "At least one" problems are usually easier through the complement: $P(\text{at least one}) = 1 - P(\text{none})$. The "none" case is often a clean multiplication.

Geometric probability. When outcomes are points in a region, compare *areas* (or lengths, volumes): $P = \frac{\text{favorable area}}{\text{total area}}$.

Combination/permutation problems. Use $\binom{n}{r}$ when order doesn't matter, $P(n, r)$ when it does. The same rule applies for numerator and denominator – match them.

Common slips. Forgetting that "with replacement" and "without replacement" give different totals. Counting an outcome twice in an OR question (forgot inclusion-exclusion). Reporting a probability outside $[0, 1]$ – always a sign of double-counting.

PRACTICE

Find each theoretical probability by counting equally-likely outcomes.

- 1. Die: $P(\text{rolling a 4})$ _____
- 2. Die: $P(\text{even})$ _____
- 3. Card: $P(\text{heart})$ _____
- 4. Using the deck breakdown below, find $P(\text{red king})$. _____

	Red	Black	Total
Kings	2	2	4
All cards	26	26	52

- 5. Using the bag below, find $P(\text{blue})$. _____

Red	Blue	Green	Total
4	5	6	15

- 6. Two coins. $P(\text{both heads})$ _____
- 7. The table gives the number of ordered outcomes (out of 36) for each two-dice sum. Find $P(\text{sum}= 7)$. _____

sum	2	3	4	5	6	7	8	9	10	11	12
#	1	2	3	4	5	6	5	4	3	2	1

- 8. Two dice. $P(\text{doubles})$ _____
- 9. Spinner $\{1, \dots, 8\}$. $P(\text{prime})$ _____
- 10. Coin flipped 4 times. $P(\text{exactly 3 heads})$ _____
- 11. Coin flipped 5 times. $P(\text{exactly 3 heads})$ _____
- 12. Die rolled 4 times. $P(\text{exactly two 6s})$ _____
- 13. Card: $P(\text{ace or king})$ _____



- 14. Two cards drawn no replacement. $P(\text{both hearts})$ _____
- 15. PIN 4 digits, can repeat. $P(\text{at least one repeat})$ _____
- 16. $\binom{6}{2} \binom{5}{2} / \binom{11}{4}$ _____
- 17. $\binom{6}{4} / \binom{11}{4}$ _____
- 18. Two dice. $P(\text{sum is at least } 10)$ _____
- 19. Geometric: dart on 8×8 square, hit unit circle at center _____
- 20. Card: $P(\text{not a heart})$ _____

◆ Word Problems

- 21. A jar contains 4 red, 5 blue, and 6 green marbles. One marble is drawn at random. What's the theoretical probability that the marble is blue? _____
- 22. A committee of 4 students is chosen at random from 6 juniors and 5 seniors. What's the probability the committee has exactly 2 juniors? _____
- 23. A 4-digit PIN is chosen at random from 0000 through 9999. What's the probability that at least one digit repeats? _____
- 24. A school has 80 students in three clubs: 42 in robotics, 35 in coding, 28 on math team. The pairwise overlaps are 18 (robotics and coding), 12 (robotics and math), 10 (coding and math). 6 students are in all three. If a student is picked at random, what's the probability they're in at least one of the three clubs? _____

Additional Practice

- 25. Probability of rolling an even number on a fair die. _____
- 26. Probability of drawing a heart from a standard deck. _____
- 27. Complement of $P(A) = 0.37$. _____
- 28. If events are independent, $P(A) = 0.4$, $P(B) = 0.5$, find $P(A \cap B)$. _____
- 29. Find $P(A \cup B)$ if $P(A) = 0.6$, $P(B) = 0.3$, $P(A \cap B) = 0.1$. _____
- 30. Choose 3 from 8. _____
- 31. Arrange 4 distinct books. _____
- 32. Find 7P_2 . _____
- 33. Find 7C_2 . _____
- 34. Probability of two heads in two coin flips. _____
- 35. Expected wins in 80 trials with $p = 0.25$. _____



Answer Keys

1. $\frac{1}{6}$

2. $\frac{1}{2}$

3. $\frac{1}{4}$

4. $\frac{1}{26}$

5. $\frac{1}{3}$

6. $\frac{1}{4}$

7. $\frac{1}{6}$

8. $\frac{1}{6}$

9. $\frac{1}{2}$

10. $\frac{1}{4}$

11. $\frac{5}{16}$

12. $\frac{25}{216}$

Additional Practice Answers

25. $\frac{1}{2}$

26. $\frac{1}{4}$

27. 0.63

28. 0.20

29. 0.8

13. $\frac{2}{13}$

14. $\frac{1}{17}$

15. $\frac{62}{125}$

16. $\frac{5}{11}$

17. $\frac{1}{22}$

18. $\frac{1}{6}$

19. $\frac{\pi}{64}$

20. $\frac{3}{4}$

21. $\frac{1}{3}$

22. $\frac{5}{11}$

23. $\frac{62}{125}$

24. $\frac{71}{80}$

30. 56

31. 24

32. 42

33. 21

34. $\frac{1}{4}$

35. 20

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Theoretical probability is favorable over total. A fair die has 6 equally-likely faces and only one is a 4, so $\frac{1}{6}$.

2. Keep the rule visible: Even faces: {2, 4, 6}, so 3 out of 6, which reduces to $\frac{1}{2}$. That gives a quick check on the answer.

3. A deck has 52 cards split into 4 suits, so 13 are hearts. Favorable over total: $\frac{13}{52} = \frac{1}{4}$.

4. Start with the key idea: Two red kings (hearts and diamonds) out of 52: $\frac{2}{52} = \frac{1}{26}$. That gives a quick check on the answer.

5. A careful way to see it: 5 blue out of 15 total: $\frac{5}{15} = \frac{1}{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

6. Keep the rule visible: Four equally-likely outcomes HH/HT/TH/TT, one favorable: $\frac{1}{4}$. That gives a quick check on the answer.

7. Pairs summing to 7: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) – six pairs out of 36. So $\frac{6}{36} = \frac{1}{6}$.

8. Doubles means both dice match: (1, 1), (2, 2), ..., (6, 6) – six favorable pairs out of the 36 equally-likely ordered outcomes. So $\frac{6}{36} = \frac{1}{6}$.

9. List the primes among 1 through 8: {2, 3, 5, 7} – that's four favorable (remember

1 is not prime). Out of 8 equal sections: $\frac{4}{8} = \frac{1}{2}$.

10. Count favorable arrangements over all sequences. There are $\binom{4}{3} = 4$ ways to choose which 3 of the 4 flips are heads, out of $2^4 = 16$ equally-likely H/T sequences. So $\frac{4}{16} = \frac{1}{4}$.

11. Choose which 3 of the 5 flips are heads: $\binom{5}{3} = 10$ favorable sequences. Total sequences: $2^5 = 32$. So $\frac{10}{32} = \frac{5}{16}$. Use the combination because the three heads are interchangeable.

12. Start with the key idea: Binomial with $n = 4$, $k = 2$, $p = \frac{1}{6}$: $\binom{4}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 = 6 \cdot \frac{25}{1296} = \frac{25}{216}$. That gives a quick check on the answer.

13. A single card can't be both an ace and a king, so the events are mutually exclusive and we just add. There are 4 aces and 4 kings: $\frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$.

14. Keep the rule visible: $\frac{13}{52} \cdot \frac{12}{51} = \frac{156}{2652} = \frac{1}{17}$. Dependent: second draw uses an updated denominator. That gives a quick check on the answer.

15. Complement: $P(\text{all distinct}) = \frac{10 \cdot 9 \cdot 8 \cdot 7}{10^4} = \frac{5040}{10000} = \frac{63}{125}$. So $P(\text{repeat}) = 1 - \frac{63}{125} = \frac{62}{125}$.

16. This counts $P(2 \text{ juniors}, 2 \text{ seniors})$ from 6 juniors and 5 seniors. Use combinations (a committee ignores order): $\binom{6}{2} = 15$ ways for juniors, $\binom{5}{2} = 10$



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for seniors, over $\binom{11}{4} = 330$ total. $\frac{15 \cdot 10}{330} = \frac{150}{330} = \frac{5}{11}$.

17. This is P (all four juniors) from 6 juniors out of 11 people. Choosing 4 juniors:

$\binom{6}{4} = 15$ ways, out of $\binom{11}{4} = 330$ total committees. $\frac{15}{330} = \frac{1}{22}$.

18. Count ordered pairs for each sum: 10 has $(4, 6), (5, 5), (6, 4) = 3$; 11 has $(5, 6), (6, 5) = 2$; 12 has $(6, 6) = 1$. That's $3 + 2 + 1 = 6$ favorable out of 36

total pairs: $\frac{6}{36} = \frac{1}{6}$.

19. Geometric probability compares areas. The square has area $8 \times 8 = 64$ and the unit circle has area $\pi \cdot 1^2 = \pi$. Assuming the dart lands uniformly, $P = \frac{\pi}{64} \approx 0.049$.

20. Start with the key idea: Complement: $1 - \frac{13}{52} = \frac{39}{52} = \frac{3}{4}$. Three suits' worth. That gives a quick check on the answer.

21. Total marbles: $4 + 5 + 6 = 15$. Blue marbles: 5. Theoretical probability: $\frac{5}{15} =$

$\frac{1}{3} \approx 0.333$. Theoretical because we're assuming the marbles are physically interchangeable and the draw is uniformly random – no rigged hand-picking.

22. Use combinations because order doesn't matter in a committee. Total committees: $\binom{11}{4} = 330$. Committees with exactly 2 juniors and 2 seniors:

$\binom{6}{2} \binom{5}{2} = 15 \cdot 10 = 150$. Probability: $\frac{150}{330} = \frac{5}{11} \approx 0.455$. (Match: combinations on top, combinations on bottom – consistency is the trick.)

23. "At least one repeat" is hard to count directly, so use the complement. Total PINs: $10^4 = 10000$. PINs with all distinct digits: $10 \cdot 9 \cdot 8 \cdot 7 = 5040$. So PINs with at least one repeat: $10000 - 5040 = 4960$. $P = \frac{4960}{10000} = \frac{62}{125} = 0.496$. Pretty close to half – repeats are more common than people guess.

24. Three-set inclusion-exclusion: add the three individual totals, subtract the three pairwise overlaps, add back the triple overlap. $42 + 35 + 28 - 18 - 12 - 10 + 6 = 71$.

So 71 of the 80 students are in at least one club, giving $P = \frac{71}{80} = 0.8875$.

(Sanity check: the 9 left over are the "no club" students. That feels reasonable for a club-heavy school.)



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