

The Inverse of a Matrix

Name: _____ Date: _____ Score: _____ / 35

Q Quick Review

The **inverse** of a square matrix A , written A^{-1} , is the matrix that undoes A : $AA^{-1} = A^{-1}A = I$, where I is the identity. Only square matrices can have an inverse, and even then only if $\det(A) \neq 0$. A matrix with $\det(A) = 0$ is called **singular** and has no inverse.

2 × 2 inverse formula: for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with $\det(A) = ad - bc \neq 0$, $A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. Swap the diagonal entries, negate the off-diagonals, then divide by the determinant. The most common slip is forgetting to divide by $\det(A)$ — write that scalar factor first, then build the adjugate.

Three identities to remember. (1) $(A^{-1})^{-1} = A$ — inverting twice returns the original. (2) $(AB)^{-1} = B^{-1}A^{-1}$ — inverses of a product reverse the order (think *socks before shoes, take off shoes before socks*). (3) $\det(A^{-1}) = \frac{1}{\det(A)}$.

Always verify $AA^{-1} = I$ when you're learning. It's a quick way to catch a sign error: compute AA^{-1} , and if you don't see 1s on the diagonal and 0s everywhere else, something's off.

PRACTICE

Compute each inverse, or state that the matrix is singular (no inverse).

1. State the condition that determines whether a square matrix A has an inverse. _____

2. A^{-1} where $A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ _____

3. Find A^{-1} for the matrix A laid out below: $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$. _____

| | Col 1 | Col 2 |
|-------|-------|-------|
| Row 1 | 2 | 5 |
| Row 2 | 1 | 3 |

4. Which is *not* invertible: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$, $\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$? _____

5. Compute AB for $A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$, and confirm $B = A^{-1}$. _____

6. A^{-1} where $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ _____

7. Find $\det(A)$ for the matrix A laid out below: $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$. _____

| | Col 1 | Col 2 |
|-------|-------|-------|
| Row 1 | 5 | 2 |
| Row 2 | 3 | 1 |

8. A^{-1} where $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$ _____

9. Find k such that $\begin{bmatrix} k & 6 \\ 2 & 3 \end{bmatrix}$ has no inverse. _____

10. A^{-1} where $A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$ _____

11. $(A^{-1})^{-1}$ for any invertible A _____

12. $(AB)^{-1}$ when both inverses exist _____



13. A^{-1} where $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ _____

14. $\det(A^{-1})$ if $\det(A) = 4$ _____

15. Find A^{-1} for the matrix A laid out below: $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$. _____

| | Col 1 | Col 2 |
|-------|-------|-------|
| Row 1 | 3 | 5 |
| Row 2 | 2 | 4 |

16. True or false: the identity matrix I_n is its own inverse. _____

17. A^{-1} where $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ _____

18. $\det(A)$ if $A^{-1} = \frac{1}{5} \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}$ _____

19. A^{-1} where $A = \begin{bmatrix} 6 & 4 \\ 3 & 2 \end{bmatrix}$ _____

20. A^{-1} where $A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ _____

◆ Word Problems

21. Solve the matrix equation $A\vec{x} = \vec{b}$ where $A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$. Use $\vec{x} = A^{-1}\vec{b}$. _____

22. A linear transformation matrix is $M = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$. Find M^{-1} , which is the transformation that undoes M . _____

23. A code-breaking matrix encrypts plaintext via $\vec{c} = E\vec{p}$ where $E = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$. Find the decryption matrix E^{-1} , which recovers \vec{p} from \vec{c} via $\vec{p} = E^{-1}\vec{c}$. _____

24. For what value(s) of k does the matrix $M = \begin{bmatrix} k+1 & 2 \\ 3 & k \end{bmatrix}$ fail to be invertible? _____

Additional Practice

25. State the dimensions of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$. _____

26. Add $\begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 5 & 6 \end{bmatrix}$. _____

27. Subtract $\begin{bmatrix} 7 & 2 \\ 1 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ -2 & 1 \end{bmatrix}$. _____

28. Find $\det \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$. _____

29. Find entry a_{21} in $\begin{bmatrix} 8 & 9 \\ -3 & 4 \end{bmatrix}$. _____

30. Can a 2×3 matrix multiply a 3×4 matrix? _____

31. Product size: $(2 \times 3)(3 \times 4)$. _____



32. Multiply $\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$.

33. Find the identity matrix of order 2.

34. Find $\det \begin{bmatrix} 1 & 6 \\ 2 & 12 \end{bmatrix}$.

35. Does a matrix with determinant 0 have an inverse?



Answer Keys

| | |
|--|--|
| 1. $\det(A) \neq 0$ | 13. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ |
| 2. $\frac{1}{-2} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$ | 14. $\frac{1}{4}$ |
| 3. $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ | 15. $\frac{1}{2} \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}$ |
| 4. $\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$ | 16. true |
| 5. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ | 17. $\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ |
| 6. $\frac{1}{2} \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$ | 18. $\bar{5}$ |
| 7. -1 | 19. singular (no inverse) |
| 8. $\begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}$ | 20. $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ |
| 9. $k = 4$ | 21. $\vec{x} = \begin{bmatrix} 13 \\ -4 \end{bmatrix}$ |
| 10. $\begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$ | 22. $M^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$ |
| 11. A | 23. $E^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ |
| 12. $B^{-1}A^{-1}$ | 24. $k = 2$ or $k = -3$ |

Additional Practice Answers

| | |
|---|--|
| 25. 2×3 | 30. yes |
| 26. $\begin{bmatrix} 4 & 3 \\ 7 & 6 \end{bmatrix}$ | 31. 2×4 |
| 27. $\begin{bmatrix} 4 & -2 \\ 3 & 4 \end{bmatrix}$ | 32. 13 |
| 28. 2 | 33. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ |
| 29. -3 | 34. 0 |
| | 35. no |

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

- An $n \times n$ matrix is invertible if and only if its determinant is nonzero. A matrix with $\det(A) = 0$ is singular — no inverse, no unique solution to $A\vec{x} = \vec{b}$.
- Keep the rule visible: $\det(A) = (4)(1) - (3)(2) = -2$. Swap diagonals, negate off-diagonals: adjugate is $[1, -3; -2, 4]$. Inverse is $\frac{1}{-2}$ times that. (Equivalently, $[-0.5, 1.5; 1, -2]$ — but the unsimplified factored form is fine to leave.) That gives a quick check on the answer.
- One steady path is: $\det(A) = (2)(3) - (5)(1) = 1$. Adjugate: swap diagonal, negate off-diagonal $\rightarrow [3, -5; -1, 2]$. Divide by $\det = 1$ (no change). Verify: $A \cdot A^{-1}$ top-left is $(2)(3) + (5)(-1) = 1 \checkmark$. That gives a quick check on the answer.
- Check each determinant: $1 \cdot 1 - 0 = 1$; $3 \cdot 4 - 1 \cdot 2 = 10$; $2 \cdot 2 - 4 \cdot 1 = 0$ (singular); $0 \cdot 0 - 1 \cdot 1 = -1$. The third matrix has $\det = 0$, so it has no inverse.
- Compute AB : $(1, 1): (1)(7) + (2)(-3) = 1$. $(1, 2): (1)(-2) + (2)(1) = 0$. $(2, 1): (3)(7) + (7)(-3) = 0$. $(2, 2): (3)(-2) + (7)(1) = 1$. The product is I , confirming $B = A^{-1}$.
- Keep the rule visible: $\det(A) = (2)(3) - (4)(1) = 2$. Adjugate $[3, -4; -1, 2]$, scale by $\frac{1}{2}$: $A^{-1} = \frac{1}{2} [3, -4; -1, 2]$. (Or, equivalently, $[1.5, -2; -0.5, 1]$). That gives a quick check on the answer.
- One steady path is: $(5)(1) - (2)(3) = 5 - 6 = -1$. (Setting up the determinant is always step one toward the inverse.) That gives a quick check on the answer.
- From the previous, $\det(A) = -1$. Adjugate $[1, -2; -3, 5]$. Multiply by $\frac{-1}{-1} = -1$: $A^{-1} = [-1, 2; 3, -5]$. Verify AA^{-1} top-left: $(5)(-1) + (2)(3) = -5 + 6 = 1 \checkmark$.
- Set $\det = 3k - 12 = 0$. Solve: $k = 4$. At $k = 4$, the matrix is singular — rows $(4, 6)$ and $(2, 3)$ are scalar multiples.
- Keep the rule visible: $\det(A) = (1)(7) - (2)(3) = 1$. Adjugate $[7, -2; -3, 1]$. Divide by 1 — no change. This is the inverse used in the

- matrix-equation worksheet next section. That gives a quick check on the answer.
- The inverse of the inverse is the original matrix. (Applying undo twice is the same as doing nothing.) Formally, $(A^{-1})^{-1} \cdot A^{-1} = I = A \cdot A^{-1}$, and inverses are unique, so $(A^{-1})^{-1} = A$.
 - The product inverse reverses the order: $(AB)^{-1} = B^{-1}A^{-1}$. Quick mnemonic: socks before shoes; shoes off before socks. Writing $A^{-1}B^{-1}$ is a common error.
 - A careful way to see it: $\det(A) = (0)(0) - (1)(1) = -1$. Adjugate $[0, -1; -1, 0]$. Divide by -1 : $A^{-1} = [0, 1; 1, 0] = A$. So this swap matrix is its own inverse — applying it twice returns the original. That gives a quick check on the answer.
 - For invertible A , $\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{4}$. (Geometrically: if A scales area by 4, A^{-1} shrinks area by $\frac{1}{4}$ — the undo factor.)
 - One steady path is: $\det = (3)(4) - (5)(2) = 2$. Adjugate $[4, -5; -2, 3]$, scale by $\frac{1}{2}$. Final form: $\frac{1}{2} [4, -5; -2, 3]$, or equivalently $[2, -2.5; -1, 1.5]$. That gives a quick check on the answer.
 - Start with the key idea: $I_n \cdot I_n = I_n$, so $I_n^{-1} = I_n$. (Multiplying by the identity is doing nothing; doing nothing is also its own undo.) That gives a quick check on the answer.
 - Diagonal matrices invert entrywise: replace each diagonal entry by its reciprocal. $\det = 2$; the formula gives the same answer. Geometrically, the y -stretch by 2 undoes via a y -stretch by $\frac{1}{2}$.
 - The scalar $\frac{1}{5}$ out front is exactly $\frac{1}{\det(A)}$. So $\det(A) = 5$. (Reading off the determinant from the inverse formula is a quick trick.)



19. One steady path is: $\det(A) = (6)(2) - (4)(3) = 0$. Singular — no inverse. (Row 1 is $2 \times$ row 2, so the rows are linearly dependent.) That gives a quick check on the answer.

20. Start with the key idea: $\det(A) = (-1)(-1) - (2)(1) = 1 - 2 = -1$. Adjugate (swap diagonals, negate off-diagonals): $[-1, -2; -1, -1]$. Divide by $\det = -1$ — every entry flips sign — giving $[1, 2; 1, 1]$. Verify: AA^{-1} top-left $(-1)(1) + (2)(1) = 1 \checkmark$, top-right $(-1)(2) + (2)(1) = 0 \checkmark$, bottom-left $(1)(1) + (-1)(1) = 0 \checkmark$, bottom-right $(1)(2) + (-1)(1) = 1 \checkmark$. That gives a quick check on the answer.

21. A careful way to see it: $\det(A) = 7 - 6 = 1$, so $A^{-1} = [7, -2; -3, 1]$. Multiply: $A^{-1}\vec{b} = [(7)(5) + (-2)(11); (-3)(5) + (1)(11)] = [35 - 22; -15 + 11] = [13; -4]$. So $x_1 = 13$, $x_2 = -4$. (Always verify by plugging back: $A\vec{x} = [(1)(13) + (2)(-4); (3)(13) + (7)(-4)] = [5; 11] \checkmark$.)

That gives a quick check on the answer.

22. Keep the rule visible: $\det(M) = (3)(2) - (1)(5) = 1$. Adjugate $[2, -1; -5, 3]$ (diagonals swapped, off-diagonals negated). Divide by $\det = 1$: no change. So $M^{-1} = [2, -1; -5, 3]$. Verify: MM^{-1} top-left $(3)(2) + (1)(-5) = 1 \checkmark$, off-diagonal $(3)(-1) + (1)(3) = 0 \checkmark$. That gives a quick check on the answer.

23. One steady path is: $\det(E) = (2)(2) - (3)(1) = 1$. Adjugate $[2, -3; -1, 2]$. Scale by $\frac{1}{1}$ — no change. So $E^{-1} = [2, -3; -1, 2]$. (Hill ciphers in cryptography literally do this — encrypt with a matrix, decrypt with its inverse.) That gives a quick check on the answer.

24. Set $\det(M) = 0$: $(k + 1)(k) - (2)(3) = k^2 + k - 6 = 0$. Factor: $(k + 3)(k - 2) = 0$, so $k = -3$ or $k = 2$. At either value, the determinant is zero and M is singular. (Two roots, two ways to break invertibility.)



Build Algebra Confidence From Pre-Algebra Through Algebra II



The Complete Algebra Success Bundle

Pre-Algebra, Algebra I, and Algebra II in one clear path

Friendly lessons, focused practice, and full-review support for every stage.



Scan for the Bundle

6 Books
3 Courses
1 Path

Bundle Value: Six coordinated books help students review missing skills, learn new algebra topics, and practice until the steps feel natural.

Complete Course Path

- ✓ Starts with Pre-Algebra foundations
- ✓ Moves smoothly into Algebra I skills
- ✓ Extends learning through Algebra II topics
- ✓ Great for review, tutoring, and summer study

One bundle, one steady path.

Step-by-Step Lessons

- ✓ Plain-English explanations students can follow
- ✓ Worked examples that show every important step
- ✓ Common mistakes called out before they stick
- ✓ Skill-building practice after each lesson
- ✓ Helpful for independent study or class support

Less guessing. More understanding.

Practice That Sticks

- ✓ Matching practice workbooks for extra repetition
- ✓ Review sets to keep older skills fresh
- ✓ Answer explanations for checking thinking
- ✓ Strong support before tests and final exams
- ✓ Designed to build fluency and confidence

Practice today. Remember tomorrow.

STUDENT FAVORITE • Master Algebra II From the Ground Up



Algebra II for Beginners

Written by a top math teacher & aligned with national and state Algebra II courses. From polynomial functions to logarithms, trigonometry, and rational expressions — explained the easy way.

- ✓ **Complete coverage** of every Algebra II concept — perfect companion to these worksheets
- ✓ **Step-by-step explanations** with worked examples on every topic
- ✓ **QR codes in every chapter** for free video lessons & bonus practice
- ✓ **2 full-length practice tests** with detailed answer keys

- ✓ 100% Guaranteed
- ✓ Lifetime Support
- ✓ Trusted by Teachers

Start Your Algebra Journey Today! →

★ STUDENT'S #1 CHOICE ★

Teacher-recommended • 12,000+ Happy Students

PDF EDITION



Instant download • any device

PAPERBACK



Paperback on Amazon

Hold it in your hands

Pair these free worksheets with *Algebra II for Beginners* and you have a complete self-paced course — concept lessons, daily practice, and full exam-style reviews, all in one path. → EffortlessMath.com/product/algebra-ii-for-beginners