Systems of Three Linear Equations

Name: ________________	Date: ____________	Score: _____ / 32

Quick Review

A **3×3 linear system** has three equations in three unknowns (often x, y, z). Geometrically, each equation is a plane in 3D space, and a solution is a point where all three planes meet. The standard plan: **eliminate one variable** using a pair of equations, then **eliminate the same variable** using a different pair — you now have a 2×2 system in the remaining two variables. Solve it, then back-substitute to find the third. The cleanest way is elimination (linear combination), though substitution works when a variable already has coefficient ± 1. Outcomes: *one solution* (three planes meet at a point), *no solution* (planes are parallel or arrange so they don't all meet), or *infinitely many* (planes share a line or a plane). Watch out for sign errors when you combine equations, and *always plug your answer back into all three originals* — one wrong sign can produce a near-right answer that fails one check.

PRACTICE

Solve each system for (x, y, z), or determine that no unique solution exists.

1. $\begin{cases} x + y + z = 6 \\ x - y + z = 2 \\ x + y - z = 0 \end{cases}$ ________

2. Solve the system summarized by this augmented table (each row is one equation; columns are the coefficients of x, y, z and the constant). ________

x	y	z	$=$
1	1	0	5
0	1	1	8
1	0	1	7

3. Solve the system shown in this augmented table (rows are equations; columns are the coefficients of x, y, z and the constant). ________

x	y	z	$=$
1	1	1	4
2	1	1	5
1	2	1	6

4. $\begin{cases} x + y + z = 1 \\ x + y + z = 2 \\ x + y + z = 3 \end{cases}$ ________

5. $\begin{cases} x + y + z = 3 \\ 2x + 2y + 2z = 6 \\ x + 2y + z = 4 \end{cases}$ ________

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6. Solve the system encoded in this augmented table (rows are equations; columns give the coefficients of x, y, z and the constant). ________

x	y	z	$=$
3	1	1	8
−2	−1	−1	−6
1	3	1	6

7. $\begin{cases} -x-y-2z=-6 \\ 2x-2y+3z=6 \\ 2x-2y-2z=-4 \end{cases}$ ________

8. $\begin{cases} x+2y-2z=1 \\ -2x-2y-z=-10 \\ -x-2y+z=-3 \end{cases}$ ________

9. $\begin{cases} 2x+3y+2z=12 \\ x+2y-2z=-2 \\ x+y+3z=11 \end{cases}$ ________

10. $\begin{cases} 2x-y+3z=4 \\ 3x+3y+z=16 \\ -2x+2y+3z=5 \end{cases}$ ________

11. $\begin{cases} 2x+3y+3z=8 \\ 2x+3y+2z=6 \\ x+3y-2z=-3 \end{cases}$ ________

12. $\begin{cases} -x-2y+2z=-5 \\ x-y+z=2 \\ x+3y+2z=11 \end{cases}$ ________

13. $\begin{cases} -2x+2y+2z=8 \\ 3x+y-2z=2 \\ -x+3y-2z=4 \end{cases}$ ________

14. $\begin{cases} x-2y+z=3 \\ -x+2y+3z=1 \\ x+y+z=3 \end{cases}$ ________

15. $\begin{cases} 2x-y+3z=13 \\ 2x-y-z=5 \\ 3x+2y+2z=18 \end{cases}$ ________

16. $\begin{cases} 2x-2y-2z=-2 \\ -x+2y-z=3 \\ -2x+y+z=0 \end{cases}$ ________

17. $\begin{cases} x+2y+2z=12 \\ -2x-2y+3z=1 \\ -2x+3y+z=5 \end{cases}$ ________

18. $\begin{cases} 2x+y+3z=10 \\ 3x+3y-2z=10 \\ -x+3y-2z=-2 \end{cases}$ ________

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19. $\begin{cases} 3x + y + z = 9 \\ x - 2y - z = -7 \\ 2x + 2y + z = 10 \end{cases}$ ________

20. $\begin{cases} x - y + z = 4 \\ 2x + y - z = 2 \\ 3x + y + 3z = 16 \end{cases}$ ________

Word Problems

21. A snack shop sells three items. A bag of chips (c), a soda (s), and a cookie (k). Two chips, one soda, and one cookie cost $\$8$. One chip, two sodas, and one cookie cost $\$9$. One chip, one soda, and two cookies cost $\$7$. Find each item's price. ____

22. The sum of three numbers is 30. The first plus twice the second equals 25. The third is twice the first. Find all three numbers. ____

23. A school store sells notebooks (n), pencils (p), and erasers (e). One notebook, two pencils, and three erasers cost $\$7$. Two notebooks, one pencil, and one eraser cost $\$6$. Three notebooks, two pencils, and one eraser cost $\$9$. Find each item's price. ____

24. At a fruit stand, apples (a), bananas (b), and cherries (c) cost: $2a + b + c = \$4,\ a + 2b + c = \$5,\ a + b + 2c = \$6$. Find the price of each fruit. ____

Additional Practice

25. Solve: $2x + y = 7$ and $x - y = 2$. ________

26. Solve: $x + y = 10$ and $x - y = 4$. ________

27. Solve: $3x - 2y = 4$ and $x + y = 6$. ________

28. Classify: $2x + 4y = 8$ and $x + 2y = 4$. ________

29. Classify: $y = 3x + 1$ and $y = 3x - 5$. ________

30. Solve: $4x + y = 1$ and $2x - y = 5$. ________

31. Eliminate y: $x + y = 8$ and $2x - y = 7$. ________

32. Eliminate x: $3x + 2y = 12$ and $3x - y = 3$. ________

Answer Keys

1.	$(1, 2, 3)$	**13.**	$(1, 3, 2)$
2.	$(2, 3, 5)$	**14.**	$(2, 0, 1)$
3.	$(1, 2, 1)$	**15.**	$(4, 1, 2)$
4.	no solution	**16.**	$(1, 2, 0)$
5.	infinitely many	**17.**	$(2, 2, 3)$
6.	$(2, 1, 1)$	**18.**	$(3, 1, 1)$
7.	$(1, 1, 2)$	**19.**	$(1, 2, 4)$
8.	$(3, 1, 2)$	**20.**	$(2, 1, 3)$
9.	$(0, 2, 3)$	**21.**	$c = \$2,\ s = \$3,\ k = \$1$
10.	$(2, 3, 1)$	**22.**	$7, 9, 14$
11.	$(1, 0, 2)$	**23.**	$n = \$2,\ p = \$1,\ e = \$1$
12.	$(3, 2, 1)$	**24.**	$a = \$0.25,\ b = \$1.25,\ c = \$2.25$

Additional Practice Answers

25.	$(3, 1)$	**29.**	no solution
26.	$(7, 3)$	**30.**	$(1, -3)$
27.	$(\frac{16}{5}, \frac{14}{5})$	**31.**	$x = 5$
28.	infinitely many	**32.**	$y = 3$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Add eq 1 and eq 3: $2x + 2y = 6 \Rightarrow x + y = 3$. Subtract eq 2 from eq 1: $2y = 4 \Rightarrow y = 2$. So $x = 1$. From eq 1: $z = 6 - 1 - 2 = 3$. Verify all three.

2. The rows read $x + y = 5,\ y + z = 8,\ x + z = 7$. Add all three: $2(x + y + z) = 20 \Rightarrow x + y + z = 10$. Subtract each pair: $z = 10 - 5 = 5$, $x = 10 - 8 = 2$, $y = 10 - 7 = 3$. Verify: each pair sums correctly.

3. The rows read $x + y + z = 4,\ 2x + y + z = 5,\ x + 2y + z = 6$. Subtract eq 1 from eq 2: $x = 1$. Subtract eq 1 from eq 3: $y = 2$. Plug into eq 1: $1 + 2 + z = 4 \Rightarrow z = 1$. Verify: $4 = 4, 5 = 5, 6 = 6$.

4. Same left side, three different right sides — the three planes are pairwise parallel, no common point.

5. Eq 2 is $2\cdot$ eq 1, so only two independent equations. Subtract eq 1 from eq 3: $y = 1$. Then $x + z = 2$ is the only remaining constraint — a line of solutions.

6. The rows read $3x + y + z = 8,\ -2x - y - z = -6,\ x + 3y + z = 6$. Add eq 1 and eq 2: $x = 2$. Plug into eq 1: $6 + y + z = 8 \Rightarrow y + z = 2$. Plug $x = 2$ into eq 3: $2 + 3y + z = 6 \Rightarrow 3y + z = 4$. Subtract: $2y = 2 \Rightarrow y = 1,\ z = 1$. Verify all three.

7. Multiply eq 1 by -1: $x + y + 2z = 6$. Subtract eq 3 from eq 2: $5z = 10 \Rightarrow z = 2$. Plug into eq 1: $x + y = 2$. Plug $z = 2$ into eq 2: $2x - 2y + 6 = 6 \Rightarrow x - y = 0 \Rightarrow x = y$. So $x = y = 1$. Verify all.

8. Add eq 1 and eq 3: the x and y terms cancel and you get $-z = -2$, so $z = 2$. Plug into eq 1: $x + 2y - 4 = 1 \Rightarrow x + 2y = 5$. Plug into eq 2: $-2x - 2y - 2 = -10 \Rightarrow x + y = 4$. Subtract: $y = 1,\ x = 3$. Verify: $3 + 2 - 4 = 1, -6 - 2 - 2 = -10, -3 - 2 + 2 = -3$. All check.

9. Add eq 2 and eq 3: $2x + 3y + z = 9$. Subtract from eq 1: $z = 3$. Plug into eq 2: $x + 2y = 4$. Plug into eq 3: $x + y = 2$. Subtract: $y = 2,\ x = 0$. Verify all three.

10. Multiply eq 1 by 3 and add eq 2: $9x + 10z = 28$. Multiply eq 1 by 2 and add eq 3: $2x + 9z = 13$. Solve that 2×2 system: $z = 1$ and $x = 2$. From eq 1, $y = 2x + 3z - 4 = 4 + 3 - 4 = 3$. Verify: $4 - 3 + 3 = 4, 6 + 9 + 1 = 16, -4 + 6 + 3 = 5$.

11. Subtract eq 2 from eq 1: $z = 2$. Plug into eq 2: $2x + 3y + 4 = 6 \Rightarrow 2x + 3y = 2$. Plug into eq 3: $x + 3y - 4 = -3 \Rightarrow x + 3y = 1$. Subtract: $x = 1,\ y = 0$. Verify all.

12. Add eq 1 and eq 2: $-3y + 3z = -3 \Rightarrow y - z = 1$. Add eq 1 and eq 3: $y + 4z = 6$. Subtract the first from the second: $5z = 5 \Rightarrow z = 1$, so $y = 2$. From eq 2, $x = 2 + y - z = 2 + 2 - 1 = 3$. Verify all three.

13. Subtract eq 3 from eq 2: $4x - 2y = -2$, i.e. $y = 2x + 1$. Divide eq 1 by 2: $-x + y + z = 4$, so $z = 4 + x - y = 4 + x - (2x + 1) = 3 - x$. Plug both into eq 3: $-x + 3(2x + 1) - 2(3 - x) = 4$, which simplifies to $7x - 3 = 4$, giving $x = 1$. Then $y = 3$ and $z = 2$. Verify all three.

14. Add eq 1 and eq 2: $4z = 4 \Rightarrow z = 1$. Subtract eq 1 from eq 3: $3y = 0 \Rightarrow y = 0$. From eq 3: $x = 2$. Verify all.

15. Subtract eq 2 from eq 1: $4z = 8 \Rightarrow z = 2$. Plug into eq 2: $2x - y - 2 = 5 \Rightarrow 2x - y = 7$. Plug into eq 3: $3x + 2y + 4 = 18 \Rightarrow 3x + 2y = 14$. Multiply first by 2: $4x - 2y = 14$, add to $3x + 2y = 14$: $7x = 28 \Rightarrow x = 4$. Then $y = 1$. Verify all.

16. Add eq 1 and eq 3: $-y - z = -2 \Rightarrow y + z = 2$. Add eq 2 and eq 3: $-3x + 3y = 3 \Rightarrow y - x = 1$, i.e. $y = x + 1$. From eq 1 $\div 2$: $x - y - z = -1 \Rightarrow z = x - y + 1 = x - (x + 1) + 1 = 0$. Then $y + z = 2 \Rightarrow y = 2,\ x = 1$. Verify all.

17. Add eq 1 and eq 2: $-x + 5z = 13$. Multiply eq 1 by 2 and add eq 3: $7y + 5z = 29$. From eq 1: $x = 12 - 2y - 2z$. Plug into $-x + 5z = 13$: $-12 + 2y + 2z + 5z = 13 \Rightarrow 2y + 7z = 25$. Combined with $7y + 5z = 29$: multiply first by 7, second by -2: $14y + 49z = 175,\ -14y - 10z = -58$. Add: $39z = 117 \Rightarrow z = 3$. Then $2y + 21 = 25 \Rightarrow y = 2$. And $x = 12 - 4 - 6 = 2$. Verify all.

18. Subtract eq 3 from eq 2: $4x = 12 \Rightarrow x = 3$. Plug into eq 1: $6 + y + 3z = 10 \Rightarrow y + 3z = 4$. Plug into eq 3: $-3 + 3y - 2z = -2 \Rightarrow 3y - 2z = 1$. Multiply the first by 3: $3y + 9z = 12$, then subtract the second: $11z = 11 \Rightarrow z = 1$, so $y = 1$. Verify all.

19. Add eq 1 and eq 2: $4x - y = 2$. Add eq 2 and eq 3: $3x = 3 \Rightarrow x = 1$. Then $4 - y = 2 \Rightarrow y = 2$. From eq 1: $3 + 2 + z = 9 \Rightarrow z = 4$. Verify all.

20. Add eq 1 and eq 2: $3x = 6 \Rightarrow x = 2$. Plug into eq 1: $2 - y + z = 4 \Rightarrow z - y = 2$. Plug into eq 3: $6 + y + 3z = 16 \Rightarrow y + 3z = 10$. From first $y = z - 2$. Plug: $(z - 2) + 3z = 10 \Rightarrow 4z = 12 \Rightarrow z = 3,\ y = 1$. Verify all.

21. System: $2c + s + k = 8,\ c + 2s + k = 9,\ c + s + 2k = 7$. Subtract eq 1 from eq 2: $-c + s = 1$. Subtract eq 1 from eq 3: $-c + k = -1$. So $s = c + 1,\ k = c - 1$. Plug into eq 1: $2c + (c + 1) + (c - 1) = 8 \Rightarrow 4c = 8 \Rightarrow c = 2$. Then $s = 3,\ k = 1$. All positive; reasonable.

22. Let a, b, c. $a + b + c = 30,\ a + 2b = 25,\ c = 2a$. Substitute $c = 2a$ into eq 1: $3a + b = 30$, so $b = 30 - 3a$. Plug into $a + 2b = 25$: $a + 2(30 - 3a) = 25 \Rightarrow -5a = -35 \Rightarrow a = 7$. Then $b = 9,\ c = 14$. Verify: $7 + 9 + 14 = 30$ and $7 + 18 = 25$.

23. System: $n + 2p + 3e = 7,\ 2n + p + e = 6,\ 3n + 2p + e = 9$. Subtract eq 2 from eq 3: $n + p = 3$. From eq 2: $e = 6 - 2n - p$. Plug into eq 1: $n + 2p + 3(6 - 2n - p) = 7 \Rightarrow -5n - p + 18 = 7 \Rightarrow 5n + p = 11$. Combine with $n + p = 3$ by subtracting: $4n = 8 \Rightarrow n = 2$. Then $p = 3 - n = 1$, and $e = 6 - 4 - 1 = 1$. Verify: $2 + 2 + 3 = 7$ ✓, $4 + 1 + 1 = 6$ ✓, $6 + 2 + 1 = 9$ ✓. All prices are whole dollars.

24. Subtract eq 1 from eq 2: $-a + b = 1 \Rightarrow b = a + 1$. Subtract eq 2 from eq 3: $-b + c = 1 \Rightarrow c = b + 1 = a + 2$. Plug both into eq 1:

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$2a + (a + 1) + (a + 2) = 4 \Rightarrow 4a + 3 = 4 \Rightarrow a = \frac{1}{4} = \$0.25$. Then $b = a + 1 = \$1.25$ and $c = a + 2 = \$2.25$. Verify by plug-back: $2(0.25)+1.25+2.25 = 0.50+1.25+2.25 = 4.00\,\checkmark$; $0.25+2(1.25)+2.25 = 0.25+2.50+2.25 = 5.00\,\checkmark$; $0.25+1.25+2(2.25) = 0.25+1.25+4.50 = 6.00$ $\checkmark$. Each fruit costs \$1 more than the previous, starting at $a = \$0.25$.

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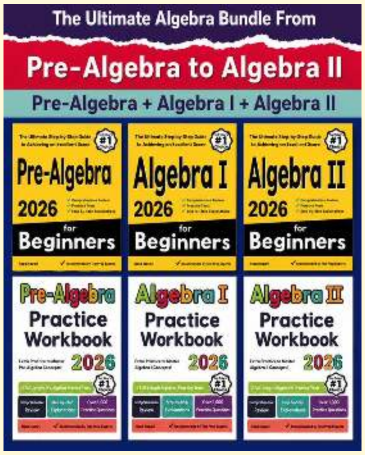

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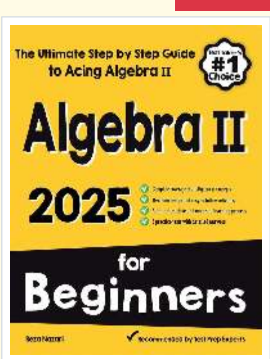

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