Systems of Equations

Name: ______________ Date: ______________ Score: ______ / 31

Quick Review

A **system** of linear equations is two (or more) line equations together. A **solution** is a point (x, y) that makes *both* equations true at once — geometrically, it's where the lines cross. Three methods. **Graphing**: draw both lines, read the intersection (best when answers are nice integers). **Substitution**: solve one equation for one variable, plug into the other ("the substitution stamp"). Works smoothly when a variable already has coefficient ± 1. **Elimination (linear combination)**: multiply equations so coefficients on one variable become opposites, then add to kill that variable. Three outcomes are possible: *one solution* (lines cross at a single point), *no solution* (parallel, never meet — you'll get a false statement like $0 = 5$), or *infinitely many* (the same line, just two disguises — you'll get a true statement like $0 = 0$). Always plug your answer back into *both* original equations to verify.**Solution check table.** After solving a system, substitute the ordered pair into both equations. One miss means the point is not the intersection.

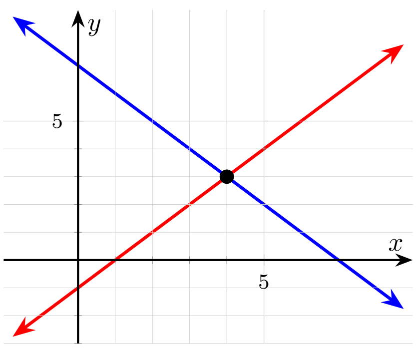
System	Candidate	Check
$y = 2x - 1,\ y = -x + 5$	$(2, 3)$	$3 = 3$ and $3 = 3$
$x + y = 6,\ 2x - y = 3$	$(3, 3)$	$6 = 6$ and $3 = 3$
$3x - y = 4,\ x + y = 8$	$(3, 5)$	$4 = 4$ and $8 = 8$

PRACTICE

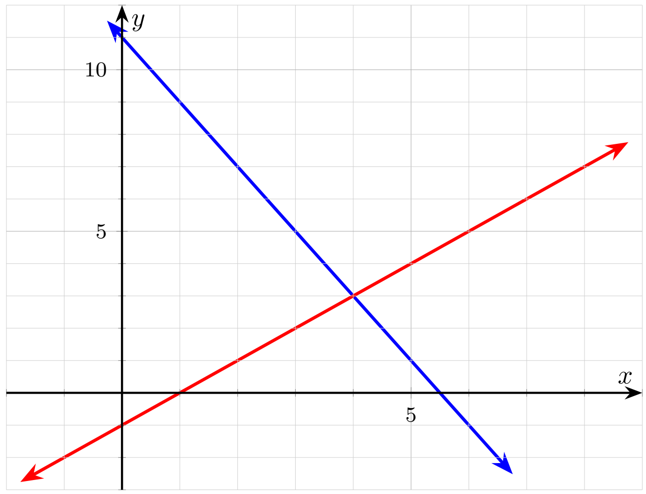
Solve each system. State whether there is one solution, none, or infinitely many.

1. Solve the system graphically: $\begin{cases} x + y = 7 \\ x - y = 1 \end{cases}$ ________

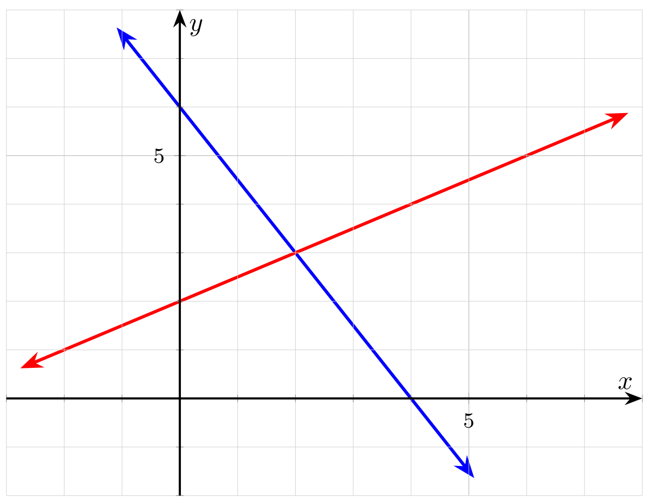
Values		
x	$y = 7 - x$	$y = x - 1$
2	5	1
3	4	2
4	3	3
5	2	4



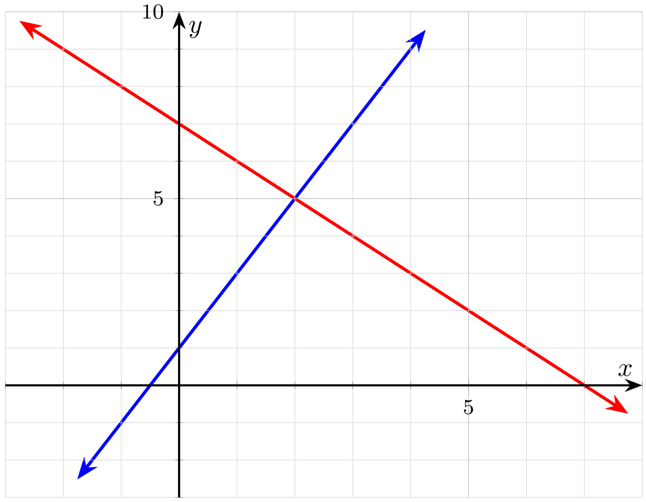
2. Solve the system graphically: $\begin{cases} 2x + y = 11 \\ y = x - 1 \end{cases}$ ______



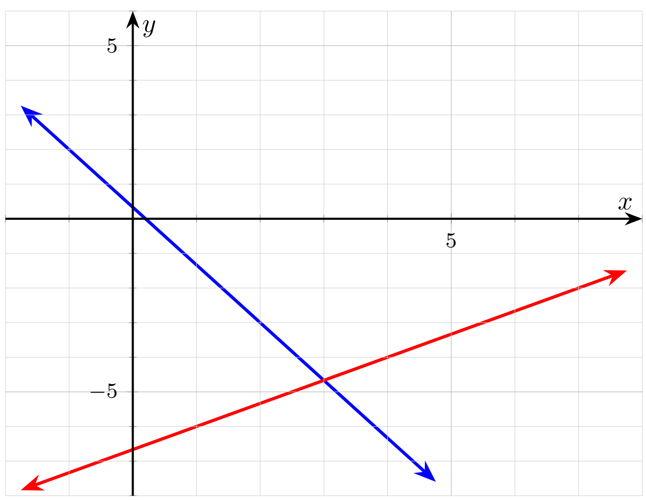
3. Solve graphically: $\begin{cases} 3x + 2y = 12 \\ x - 2y = -4 \end{cases}$ ______



4. Solve graphically: $\begin{cases} y = 2x + 1 \\ y = -x + 7 \end{cases}$ ________



5. Solve graphically: $\begin{cases} 5x + 3y = 1 \\ 2x - 3y = 20 \end{cases}$ ________



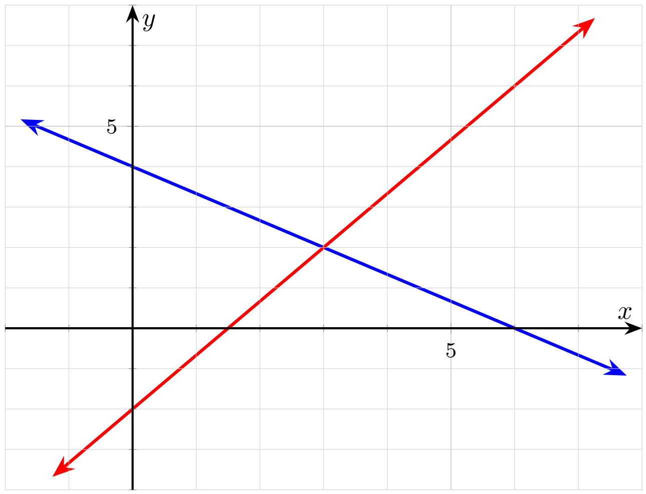
6. $\begin{cases} x + 2y = 5 \\ 2x + 4y = 10 \end{cases}$ ________

7. $\begin{cases} x + y = 4 \\ x + y = 9 \end{cases}$ ________

8. $\begin{cases} 4x - y = 5 \\ 2x + 3y = 13 \end{cases}$ ________

9. $\begin{cases} 3x + y = 7 \\ x - 2y = -5 \end{cases}$ ________

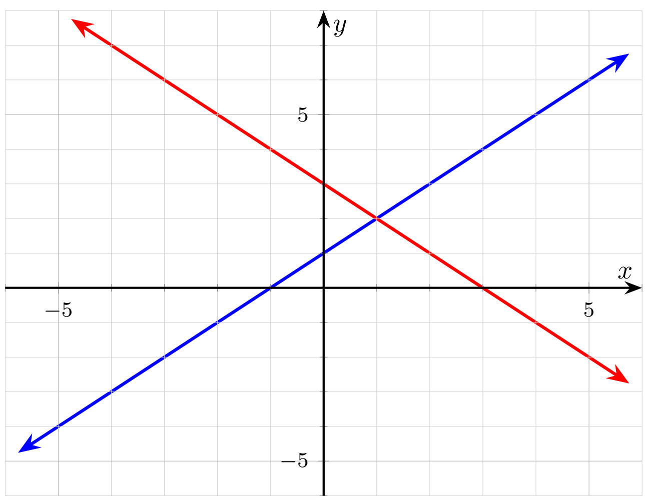
10. Solve graphically: $\begin{cases} 2x + 3y = 12 \\ 4x - 3y = 6 \end{cases}$ ______



11. $\begin{cases} y = 3x + 1 \\ 6x - 2y = -2 \end{cases}$ ______

12. $\begin{cases} y = 2x - 4 \\ y = 2x + 1 \end{cases}$ ______

13. Find the intersection of the two lines graphed below. ______



14. $\begin{cases} 3x - 4y = 7 \\ 2x + y = 9 \end{cases}$ ______

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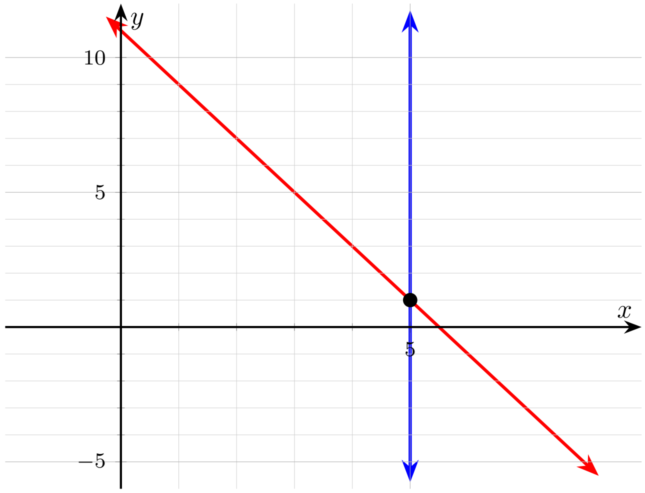
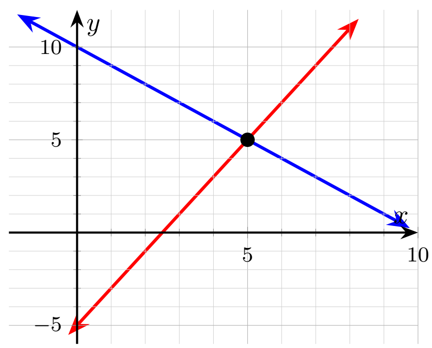
15. Solve graphically: $\begin{cases} x = 5 \\ 2x + y = 11 \end{cases}$ ________



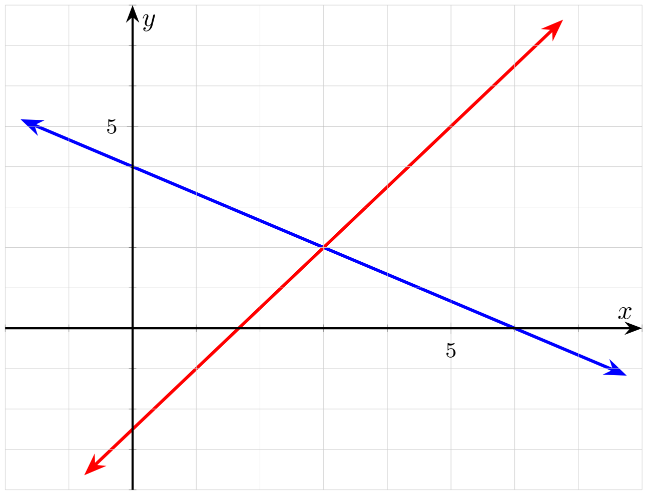
16. Solve graphically with a value table: $\begin{cases} x + y = 10 \\ 2x - y = 5 \end{cases}$ ________

Values		
x	$y = 10 - x$	$y = 2x - 5$
3	7	1
4	6	3
5	5	5
6	4	7

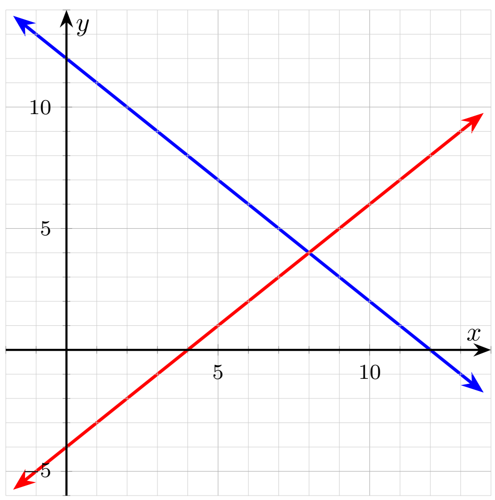


17. $\begin{cases} 4x + 5y = 23 \\ x - 2y = -1 \end{cases}$ ________

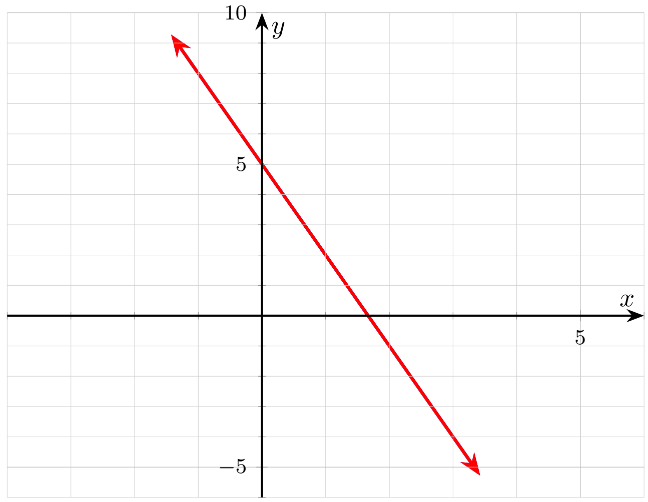
18. Solve graphically: $\begin{cases} 2x + 3y = 12 \\ 3x - 2y = 5 \end{cases}$ ________



19. Solve graphically: $\begin{cases} x + y = 12 \\ x - y = 4 \end{cases}$ ________



20. Solve graphically: $\begin{cases} 3x + y = 5 \\ 6x + 2y = 10 \end{cases}$ ______



♦ Word Problems

21. The sum of two numbers is 24 and their difference is 6. Find the two numbers. ____

22. A theater sells adult tickets for $\$8$ and child tickets for $\$5$. One evening, 50 tickets sold for $\$310$ total. How many of each type were sold? ____

23. Two cars start 300 miles apart and drive toward each other. Car A drives 60 mph and car B drives 40 mph. After how many hours do they meet? ____

24. A coffee shop sells small coffees for $\$3$ and large coffees for $\$5$. In an hour the shop made $\$84$ selling 20 coffees in total. How many of each size? ____

Additional Practice

25. Solve: $2x + y = 7$ and $x - y = 2$. ______

26. Solve: $x + y = 10$ and $x - y = 4$. ______

27. Solve: $3x - 2y = 4$ and $x + y = 6$. ______

28. Classify: $2x + 4y = 8$ and $x + 2y = 4$. ______

29. Classify: $y = 3x + 1$ and $y = 3x - 5$. ______

30. Solve: $4x + y = 1$ and $2x - y = 5$. ______

31. Eliminate y: $x + y = 8$ and $2x - y = 7$. ______

Answer Keys

1. $(4, 3)$
2. $(4, 3)$
3. $(2, 3)$
4. $(2, 5)$
5. $(3, -\frac{14}{3})$
6. infinitely many
7. no solution
8. $(2, 3)$
9. $(\frac{9}{7}, \frac{22}{7})$
10. $(3, 2)$
11. infinitely many
12. no solution
13. $(1, 2)$
14. $(\frac{43}{11}, \frac{13}{11})$
15. $(5, 1)$
16. $(5, 5)$
17. $(\frac{41}{13}, \frac{27}{13})$
18. $(3, 2)$
19. $(8, 4)$
20. infinitely many
21. 15 and 9
22. 20 adult, 30 child
23. 3 hours
24. 8 small, 12 large

Additional Practice Answers

25. $(3, 1)$
26. $(7, 3)$
27. $(\frac{16}{5}, \frac{14}{5})$
28. infinitely many
29. no solution
30. $(1, -3)$
31. $x = 5$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. From the table, both lines hit $(4, 3)$. Algebraically, add the equations: $2x = 8 \Rightarrow x = 4$. Plug into the first: $4 + y = 7 \Rightarrow y = 3$. Check: $4 - 3 = 1$. Match.

2. Both lines cross at $(4, 3)$. Substitute $y = x - 1$ into the first: $2x + (x - 1) = 11$, so $3x = 12$ and $x = 4$. Then $y = 3$.

3. Lines cross at $(2, 3)$. Add the two equations: $4x = 8 \Rightarrow x = 2$. Plug into the first: $6 + 2y = 12 \Rightarrow y = 3$.

4. Start with the key idea: The two lines meet at $(2, 5)$. Algebraically: $2x + 1 = -x + 7 \Rightarrow 3x = 6 \Rightarrow x = 2$. Then $y = 5$. That gives a quick check on the answer.

5. Both lines cross near $(3, -4.67)$. Add the equations: $7x = 21 \Rightarrow x = 3$. Plug into the first: $15 + 3y = 1 \Rightarrow 3y = -14 \Rightarrow y = -\frac{14}{3}$.

6. Multiply the first by 2: $2x + 4y = 10$ — exactly the second equation. Same line; infinitely many solutions.

7. One steady path is: Same left side, different right sides $\Rightarrow$ parallel lines. They never meet. That gives a quick check on the answer.

8. From first: $y = 4x - 5$. Plug into second: $2x + 3(4x - 5) = 13$, so $14x - 15 = 13$, $14x = 28$, $x = 2$. Then $y = 4(2) - 5 = 3$.

9. From first: $y = 7 - 3x$. Plug into second: $x - 2(7 - 3x) = -5$, so $7x - 14 = -5$, $7x = 9$, $x = \frac{9}{7}$. Then $y = 7 - 3 \cdot \frac{9}{7} = \frac{49-27}{7} = \frac{22}{7}$.

10. Keep the rule visible: Lines meet at $(3, 2)$. Add: $6x = 18 \Rightarrow x = 3$. Plug into the first: $6 + 3y = 12 \Rightarrow y = 2$. That gives a quick check on the answer.

11. One steady path is: From the second: $6x - 2y = -2 \Rightarrow y = 3x + 1$ — same as the first. Same line. That gives a quick check on the answer.

12. Start with the key idea: Same slope (2), different intercepts $\Rightarrow$ parallel. This is the part to check before moving on, because it keeps the answer tied to the original question.

13. The two lines meet where $x + 1 = -x + 3$, so $2x = 2$ and $x = 1$. Then $y = 2$. (The drawn picture overlays both lines on the same plane.)

14. From second: $y = 9 - 2x$. Plug: $3x - 4(9 - 2x) = 7$, so $11x - 36 = 7$, $11x = 43$, $x = \frac{43}{11}$. Then $y = 9 - \frac{86}{11} = \frac{13}{11}$.

15. The vertical line $x = 5$ crosses $y = -2x + 11$ at $y = -10 + 11 = 1$, so the solution is $(5, 1)$.

16. The table shows both equations give $y = 5$ when $x = 5$. Algebraically, add: $3x = 15 \Rightarrow x = 5$, so $y = 5$.

17. From second: $x = 2y - 1$. Plug: $4(2y - 1) + 5y = 23$, so $13y - 4 = 23$, $13y = 27$, $y = \frac{27}{13}$. Then $x = 2 \cdot \frac{27}{13} - 1 = \frac{41}{13}$. So $(x, y) = (\frac{41}{13}, \frac{27}{13})$.

18. Lines meet at $(3, 2)$. Multiply first by 2: $4x + 6y = 24$. Multiply second by 3: $9x - 6y = 15$. Add: $13x = 39 \Rightarrow x = 3$. Then $6 + 3y = 12 \Rightarrow y = 2$.

19. One steady path is: Lines cross at $(8, 4)$. Add: $2x = 16 \Rightarrow x = 8$. Then $y = 4$. This is the part to check before moving on, because it keeps the answer tied to the original question.

20. The two equations describe the same line: multiplying the first by 2 gives $6x + 2y = 10$, which is the second. Their graphs lie on top of one another (you only see one line).

21. System: $x + y = 24$, $x - y = 6$. Add: $2x = 30$, so $x = 15$. Then $y = 24 - 15 = 9$. Check: $15 + 9 = 24$ and $15 - 9 = 6$. Match.

22. System: $a + c = 50$, $8a + 5c = 310$. From first: $c = 50 - a$. Plug: $8a + 5(50 - a) = 310$, so $3a + 250 = 310 \Rightarrow 3a = 60 \Rightarrow a = 20$. Then $c = 30$. Both whole, positive — the numbers fit the real-world setup.

23. In t hours they cover $60t + 40t = 100t$ miles combined, and need to close 300 miles. So $100t = 300 \Rightarrow t = 3$. (You could also write a system in two positions, but a single equation does it here.)

24. System: $s + \ell = 20$, $3s + 5\ell = 84$. From first: $s = 20 - \ell$. Plug: $3(20 - \ell) + 5\ell = 84$, so $60 + 2\ell = 84 \Rightarrow 2\ell = 24 \Rightarrow \ell = 12$. Then $s = 8$. Whole positive counts — matches a real coffee shop.

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