Sum and Difference Identities

Name: ____________________ Date: ______________ Score: ______ / 32

Quick Review

Sum and difference identities give exact values for non-standard angles by splitting them into two pieces from the unit-circle family. They also let you simplify expressions like $\sin(x + \pi/3)$ symbolically.

Sum and difference for sine.

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

Sine *keeps the same sign* as the angle operation.

Sum and difference for cosine.

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

Cosine *flips the sign* between the two products – this is the easy thing to forget.

Sum and difference for tangent.

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Choosing the split. For $75°$, write $75 = 45 + 30$ (both unit-circle standards). For $15°$, write $15 = 45 - 30$. For $105°$, try $60 + 45$. The two pieces have to be angles whose sine, cosine, and tangent values you have memorized.

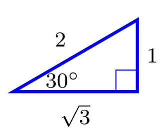
When sides are given (not exact angles). If a problem gives $\sin A$ and $\cos B$ with quadrant information, use the Pythagorean identity to fill in the missing values, watch the signs based on the quadrants, then plug into the right identity.

Common slips. Mixing up the cosine sign (using a $+$ in $\cos(A + B)$). Distributing across the function: $\sin(A + B) \neq \sin A + \sin B$ – that's the loudest trap. Forgetting to check the quadrant when filling in missing values.

PRACTICE

Use sum/difference identities. Give exact values where possible.

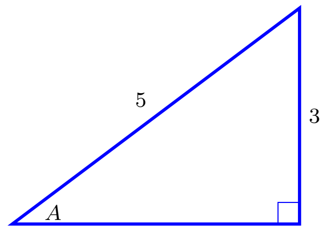
1. State the identity for $\sin(A + B)$. ________
2. State the identity for $\cos(A + B)$. ________
3. State the identity for $\tan(A + B)$. ________
4. Compute $\sin(75°)$ exactly. Split $75° = 45° + 30°$; the 30-60-90 triangle below supplies the $30°$ values you need. ________



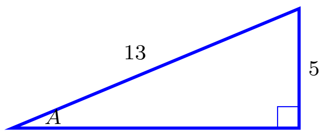
5. Compute $\cos(75°)$ exactly. ________
6. Compute $\sin(15°)$ exactly. ________
7. Compute $\cos(15°)$ exactly. ________
8. Compute $\cos(105°)$ exactly. ________
9. Compute $\tan(75°)$ exactly. ________
10. True/false: $\sin(A + B) = \sin A + \sin B$. ________

Scan Me

11. If $\sin A = \frac{3}{5}$ (Q I) and $\cos B = -\frac{5}{13}$ (Q II), find $\sin(A - B)$. The triangle below sets up angle A with $\sin A = \frac{3}{5}$; build B the same way. ________



12. If $\sin A = \frac{5}{13}$ (Q I) and $\cos B = \frac{7}{25}$ (Q IV), find $\cos(A + B)$. The triangle below sets up angle A with $\sin A = \frac{5}{13}$ (opposite over hypotenuse). ________



13. Simplify $\sin(\pi - \theta)$. ________

14. Simplify $\cos(\pi - \theta)$. ________

15. Simplify $\sin\left(\theta + \frac{\pi}{2}\right)$. ________

16. Simplify $\cos\left(\theta - \frac{\pi}{2}\right)$. ________

17. Compute $\sin(105°)$ exactly. ________

18. Compute $\tan(15°)$ exactly. ________

19. Use a sum identity to expand $\sin(x + \pi)$. ________

20. Use a sum identity to expand $\cos(x + \pi)$. ________

♦ Word Problems

21. A surveyor measures two angles: $\alpha = 30°$ and $\beta = 45°$. What is the exact value of $\sin(\alpha + \beta)$? ____

22. Two angles satisfy $\sin A = \frac{4}{5}$ with A in Q I and $\cos B = \frac{12}{13}$ with B in Q I. Find $\sin(A + B)$. ____

23. A circuit engineer needs the exact value of $\cos(15°)$ to simplify a transmission formula. Use a sum/difference identity to compute it. ____

24. A physics formula reads $f(t) = \sin(\omega t + \pi/6)$. Expand $f(t)$ using a sum identity so that the formula is in terms of $\sin(\omega t)$ and $\cos(\omega t)$ only. ____

Additional Practice

25. Find $\sin\theta$ if opposite $= 5$, hypotenuse $= 13$. ________

26. Find $\cos\theta$ if adjacent $= 12$, hypotenuse $= 13$. ________

27. Find $\tan\theta$ if opposite $= 7$, adjacent $= 4$. ________

28. Find $\sin 30°$. ________

29. Find $\cos 60°$. ________

30. Find $\tan 45°$. ________

31. Convert $180°$ to radians. ________

32. Convert $\frac{\pi}{3}$ radians to degrees. ________

Answer Keys

1. $\boxed{\sin A\cos B+\cos A\sin B}$
2. $\boxed{\cos A\cos B-\sin A\sin B}$
3. $\boxed{\dfrac{\tan A+\tan B}{1-\tan A\tan B}}$
4. $\boxed{\dfrac{\sqrt6+\sqrt2}{4}}$
5. $\boxed{\dfrac{\sqrt6-\sqrt2}{4}}$
6. $\boxed{\dfrac{\sqrt6-\sqrt2}{4}}$
7. $\boxed{\dfrac{\sqrt6+\sqrt2}{4}}$
8. $\boxed{\dfrac{\sqrt2-\sqrt6}{4}}$
9. $\boxed{2+\sqrt3}$
10. False
11. $\boxed{-\dfrac{63}{65}}$
12. $\boxed{\dfrac{204}{325}}$
13. $\boxed{\sin\theta}$
14. $\boxed{-\cos\theta}$
15. $\boxed{\cos\theta}$
16. $\boxed{\sin\theta}$
17. $\boxed{\dfrac{\sqrt6+\sqrt2}{4}}$
18. $\boxed{2-\sqrt3}$
19. $\boxed{-\sin x}$
20. $\boxed{-\cos x}$
21. $\boxed{\dfrac{\sqrt6+\sqrt2}{4}}$
22. $\boxed{\dfrac{63}{65}}$
23. $\boxed{\dfrac{\sqrt6+\sqrt2}{4}}$
24. $\boxed{\dfrac{\sqrt3}{2}\sin(\omega t)+\dfrac{1}{2}\cos(\omega t)}$

Additional Practice Answers

25. $\boxed{\frac{5}{13}}$
26. $\boxed{\frac{12}{13}}$
27. $\boxed{\frac{7}{4}}$
28. $\boxed{\frac{1}{2}}$
29. $\boxed{\frac{1}{2}}$
30. $\boxed{1}$
31. $\boxed{\pi}$
32. $\boxed{60^\circ}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Sine keeps the sign between the two products. This is the part to check before moving on, because it keeps the answer tied to the original question.

2. Keep the rule visible: Cosine flips the sign – watch this carefully. This is the part to check before moving on, because it keeps the answer tied to the original question.

3. One steady path is: Numerator: sum. Denominator: 1 minus the product. This is the part to check before moving on, because it keeps the answer tied to the original question.

4. Split $75^\circ = 45^\circ + 30^\circ$ since both are unit-circle standards, and read 30° values off the triangle. The sine sum identity keeps a plus between the products: $\sin 45\cos 30+\cos 45\sin 30 = \frac{\sqrt2}{2}\cdot\frac{\sqrt3}{2}+\frac{\sqrt2}{2}\cdot\frac{1}{2}=\frac{\sqrt6}{4}+\frac{\sqrt2}{4}=\frac{\sqrt6+\sqrt2}{4}$.

5. A careful way to see it: $\cos(45+30) = \cos 45\cos 30 - \sin 45\sin 30 = \frac{\sqrt6}{4}-\frac{\sqrt2}{4}=\frac{\sqrt6-\sqrt2}{4}$. (Notice the minus sign from cosine.) That gives a quick check on the answer.

6. Keep the rule visible: Split: $15 = 45 - 30$. $\sin 45\cos 30 - \cos 45\sin 30 = \frac{\sqrt6}{4}-\frac{\sqrt2}{4}=\frac{\sqrt6-\sqrt2}{4}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

7. One steady path is: $\cos(45-30) = \cos 45\cos 30 + \sin 45\sin 30$ (cosine *flips* signs, so a difference becomes plus) $= \frac{\sqrt6+\sqrt2}{4}$. That gives a quick check on the answer.

8. Start with the key idea: $105 = 60 + 45$. $\cos 60\cos 45 - \sin 60\sin 45 = \frac{1}{2}\cdot\frac{\sqrt2}{2}-\frac{\sqrt3}{2}\cdot\frac{\sqrt2}{2}=\frac{\sqrt2-\sqrt6}{4}$. (Negative – 105° is in Q2.) That gives a quick check on the answer.

9. Use $\tan(45+30)$: $\frac{1+1/\sqrt3}{1-1\cdot 1/\sqrt3}=\frac{\sqrt3+1}{\sqrt3-1}$. Rationalize by multiplying by $\frac{\sqrt3+1}{\sqrt3+1}$: $\frac{(\sqrt3+1)^2}{3-1}=\frac{4+2\sqrt3}{2}=2+\sqrt3$.

10. Classic trap. Sine doesn't distribute over addition. Use the sum identity instead.

11. Fill in the missing values with quadrant care. In Q1, cosine is positive, so from the 3-4-5 triangle $\cos A = \frac{4}{5}$. In Q2, sine is positive, so with $\cos B = -\frac{5}{13}$ the 5-12-13 triangle gives $\sin B = +\frac{12}{13}$. Now the sine difference identity: $\sin(A-B) = \sin A\cos B - \cos A\sin B = \frac{3}{5}\left(-\frac{5}{13}\right)-\frac{4}{5}\cdot\frac{12}{13} = -\frac{15}{65}-\frac{48}{65}=-\frac{63}{65}$.

12. Fill in missing values by quadrant. In Q1 cosine is positive, so the 5-12-13 triangle gives $\cos A = \frac{12}{13}$. In Q4 sine is negative, so with $\cos B = \frac{7}{25}$ the 7-24-25 triangle gives $\sin B = -\frac{24}{25}$. The cosine sum identity flips the sign between products: $\cos(A+B) = \cos A\cos B - \sin A\sin B = \frac{12}{13}\cdot\frac{7}{25}-\frac{5}{13}\left(-\frac{24}{25}\right)=\frac{84}{325}+\frac{120}{325}=\frac{204}{325}$.

13. A careful way to see it: $\sin(\pi-\theta) = \sin\pi\cos\theta - \cos\pi\sin\theta = 0\cdot\cos\theta - (-1)\sin\theta = \sin\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

14. Keep the rule visible: $\cos(\pi-\theta) = \cos\pi\cos\theta + \sin\pi\sin\theta = -\cos\theta + 0 = -\cos\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

15. One steady path is: $\sin\theta\cos(\pi/2)+\cos\theta\sin(\pi/2) = 0+\cos\theta = \cos\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

16. Start with the key idea: $\cos\theta\cos(\pi/2)+\sin\theta\sin(\pi/2) = 0+\sin\theta = \sin\theta$. This is the part to check before moving on, because it keeps the answer tied to the

Scan Me

original question.

17. A careful way to see it: $105 = 60 + 45$. $\sin 60 \cos 45 + \cos 60 \sin 45 = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$. (Same value as $\sin 75^\circ$ – supplementary angles share sine.) That gives a quick check on the answer.

18. Keep the rule visible: $\tan(45 - 30) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$. Rationalize: $\frac{(\sqrt{3}-1)^2}{2} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$. That gives a quick check on the answer.

19. One steady path is: $\sin x \cos \pi + \cos x \sin \pi = -\sin x + 0 = -\sin x$. (Makes sense – shifting sine by π flips its sign.) That gives a quick check on the answer.

20. Start with the key idea: $\cos x \cos \pi - \sin x \sin \pi = -\cos x - 0 = -\cos x$. This is the part to check before moving on, because it keeps the answer tied to the original question.

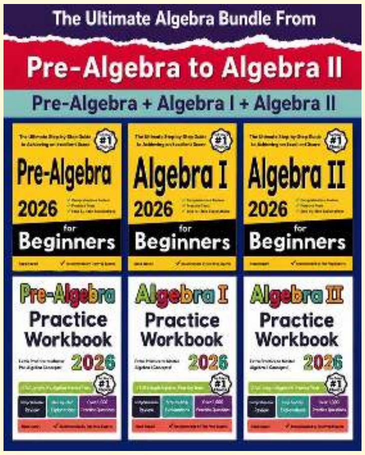
21. A careful way to see it: $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$. (Same value as $\sin 75^\circ$.) That gives a quick check on the answer.

22. Both in Q1, so cosines and sines are positive. $\cos A = \frac{3}{5}, \sin B = \frac{5}{13}$. Then $\sin(A+B) = \sin A \cos B + \cos A \sin B = \frac{4}{5} \cdot \frac{12}{13} + \frac{3}{5} \cdot \frac{5}{13} = \frac{48}{65} + \frac{15}{65} = \frac{63}{65}$.

23. Split $15 = 45 - 30$. By the cosine difference identity (which uses $+$ between the two products): $\cos 15^\circ = \cos 45 \cos 30 + \sin 45 \sin 30 = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$.

24. By the sine sum identity, $\sin(\omega t + \pi/6) = \sin(\omega t)\cos(\pi/6) + \cos(\omega t)\sin(\pi/6) = \frac{\sqrt{3}}{2}\sin(\omega t) + \frac{1}{2}\cos(\omega t)$. This phase-split form is what engineers use to combine multiple sine waves into one amplitude and phase.

Build Algebra Confidence From Pre-Algebra Through Algebra II



The Complete Algebra Success Bundle

Pre-Algebra, Algebra I, and Algebra II in one clear path

Friendly lessons, focused practice, and full-review support for every stage.

Scan Me

Scan for the Bundle

6 Books
3 Courses
1 Path

Bundle Value: Six coordinated books help students review missing skills, learn new algebra topics, and practice until the steps feel natural.

Complete Course Path

- ✓ Starts with Pre-Algebra foundations
- ✓ Moves smoothly into Algebra I skills
- ✓ Extends learning through Algebra II topics
- ✓ Great for review, tutoring, and summer study

One bundle, one steady path.

Step-by-Step Lessons

- ✓ Plain-English explanations students can follow
- ✓ Worked examples that show every important step
- ✓ Common mistakes called out before they stick
- ✓ Skill-building practice after each lesson
- ✓ Helpful for independent study or class support

Less guessing. More understanding.

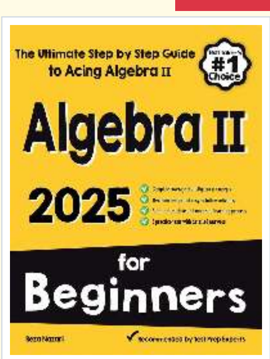
Practice That Sticks

- ✓ Matching practice workbooks for extra repetition
- ✓ Review sets to keep older skills fresh
- ✓ Answer explanations for checking thinking
- ✓ Strong support before tests and final exams
- ✓ Designed to build fluency and confidence

Practice today. Remember tomorrow.

STUDENT FAVORITE • Master Algebra II From the Ground Up

Algebra II for Beginners



✓ 100% Guaranteed
✓ Lifetime Support
✓ Trusted by Teachers

Start Your Algebra Journey Today! →

Written by a top math teacher & aligned with national and state Algebra II courses. From polynomial functions to logarithms, trigonometry, and rational expressions — explained the easy way.

- ✓ **Complete coverage** of every Algebra II concept — perfect companion to these worksheets
- ✓ **Step-by-step explanations** with worked examples on every topic
- ✓ **QR codes in every chapter** for free video lessons & bonus practice
- ✓ **2 full-length practice tests** with detailed answer keys

⋆ STUDENT'S #1 CHOICE ⋆

Teacher-recommended • 12,000+ Happy Students

↓ PDF EDITION

Scan Me

Instant download • any device

PAPERBACK

Scan Me

Paperback on Amazon

Hold it in your hands

Pair these free worksheets with *Algebra II for Beginners* and you have a complete self-paced course — concept lessons, daily practice, and full exam-style reviews, all in one path. → EffortlessMath.com/product/algebra-ii-for-beginners