Standard Form of a Circle

Name: _____ Date: _____ Score: _____ / 33

Quick Review

A **circle** is the set of points in the plane that all sit at the same distance (the **radius** r) from a fixed point (the **center** (h, k)). That definition is what forces the equation.

Standard form. $(x-h)^2+(y-k)^2=r^2$. The distance from any point (x, y) on the circle to the center (h, k) is r, so by the distance formula $\sqrt{(x-h)^2+(y-k)^2}=r$ – squaring both sides gives the standard form.

Reading center and radius. From $(x-h)^2+(y-k)^2=r^2$, the center is (h, k) (sign-flipped from what you see inside the parentheses) and the radius is $\sqrt{r^2}$, not r^2 itself. If you see $(x-5)^2+(y+3)^2=16$, the center is $(5,-3)$ and the radius is $\sqrt{16}=4$.

Build from a center and radius. Plug into the formula directly. Center $(2,-4)$, radius 5: $(x-2)^2+(y+4)^2=25$. (Square the radius – a common slip is to write r on the right instead of r^2.)

Build from a center and a point on the circle. The radius equals the distance from the center to that point. Center $(0, 0)$ and point $(3, 4)$: $r=\sqrt{9+16}=5$, so $x^2+y^2=25$.

Build from a diameter's endpoints. The center is the *midpoint* of the diameter; the radius is half the *length* of the diameter (or, equivalently, the distance from the midpoint to either endpoint).

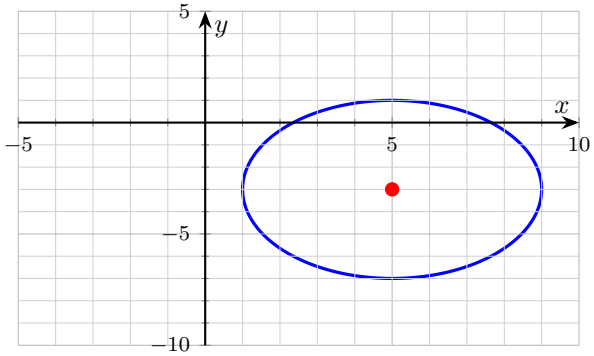
Converting from general form $x^2+y^2+Dx+Ey+F=0$**.** Complete the square on each variable separately. Quick check: $x^2+y^2-4x+6y-12=0 \Rightarrow (x^2-4x+4)+(y^2+6y+9)=12+4+9 \Rightarrow (x-2)^2+(y+3)^2=25$.

Common slips. Forgetting to square the radius. Misreading the sign on the center ($(y+3)$ means $k=-3$, not $k=3$). Not balancing the constant added on the left when completing the square on the right. Confusing circles with ellipses (circles always have equal coefficients on x^2 and y^2).

PRACTICE

For each circle, find the center and radius, or build the equation from the given features. Always square the radius for standard form.

1. What is the center of $(x-5)^2+(y+3)^2=16$? _____



2. What is the radius of $(x+1)^2+(y-2)^2=49$? _____

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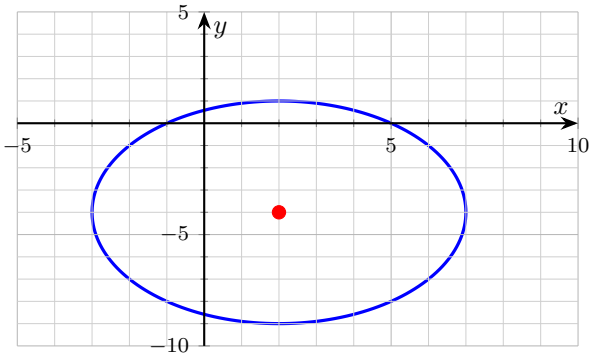
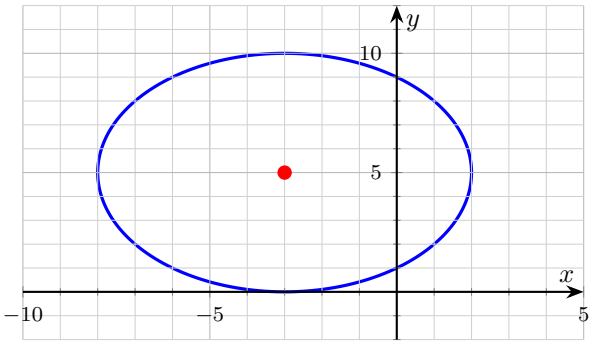
3. Write the equation of a circle with center $(2, -4)$ and radius 5. _____



4. A circle has center $(0, 0)$ and passes through $(3, 4)$. Find its equation. _____

5. Convert $x^2 + y^2 - 4x + 6y - 12 = 0$ to standard form. _____

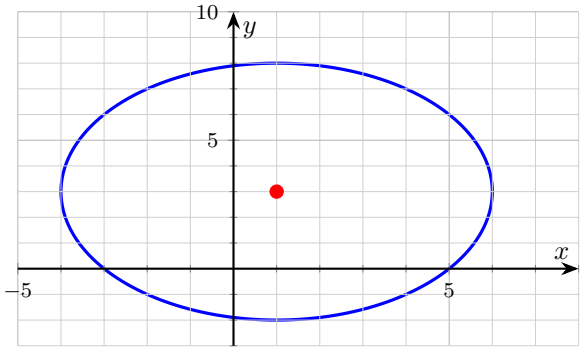
6. A circle has center $(-3, 5)$ and passes through $(0, 1)$. Find its equation. _____



7. Mark TRUE or FALSE: A circle in standard form can have a negative number on the right side. _____

8. Find the center and radius of $x^2 + y^2 + 8x - 10y + 32 = 0$. _____

9. A circle has a diameter with endpoints $(-2, 7)$ and $(4, -1)$. Find its equation. _____



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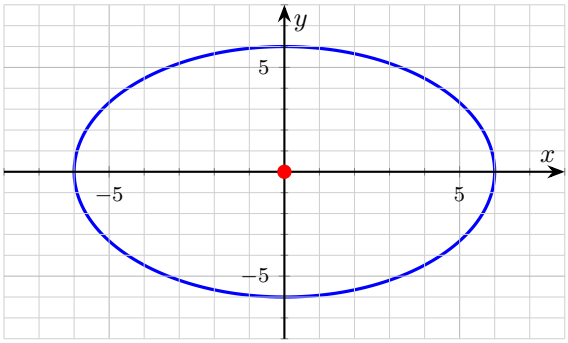
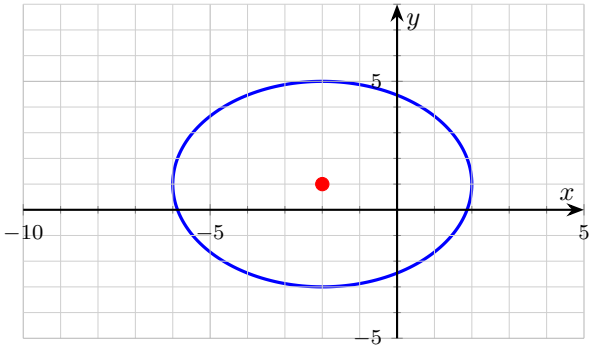
10. What is the radius of $x^2 + y^2 = 36$? _____



11. Find the equation of a circle with center $(-2, 1)$ and radius 4. _____



12. What is the radius of the circle $x^2 + y^2 - 6x - 8y = 0$? _____

13. Mark TRUE or FALSE: For a circle in standard form, the coefficients of x^2 and y^2 must be equal. _____

14. A circle has center $(4, -1)$ and passes through the origin. Find its equation. _____

15. Convert $x^2 + y^2 + 2x - 4y - 4 = 0$ to standard form. _____

16. Find the equation of a circle with center $(1, 2)$ and radius 4. _____

17. A circle is tangent to the x-axis with center $(3, 5)$. Find its equation. _____

18. What's the center of the circle whose diameter has endpoints $(-4, 2)$ and $(6, 8)$? _____

19. Find the equation of a circle centered at the origin with radius $\sqrt{10}$. _____

20. Find the equation of a circle with center $(0, 0)$ that passes through $(-5, 12)$. _____

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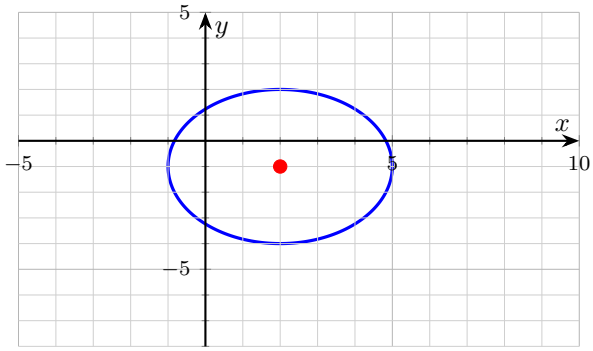
Word Problems

21. A circular cell-tower coverage region is centered at $(2, -1)$ on a map (in miles) and has a coverage radius of 3 miles. Write the equation of the coverage boundary, and determine whether a town at $(4, 1)$ falls inside the coverage area. ____



22. An archaeologist marks two stones at $(-3, 4)$ and $(5, -2)$ on a grid (in meters). The straight line between them is a diameter of a circular site boundary. Find the equation of the boundary in standard form. ____

23. A circular running track is described by the equation $x^2 + y^2 - 12x + 4y - 9 = 0$ (in meters). Convert this to standard form, identify the center and radius, and determine the track's circumference. ____

24. A surveyor needs a circle that passes through the three points $(0, 0)$, $(6, 0)$, and $(0, 8)$. Find the equation of the circle in standard form. (Hint: these three points form a right angle at the origin, so the hypotenuse is a diameter.) ____

Additional Practice

25. Center and radius of $(x - 3)^2 + (y + 2)^2 = 25$. _____

26. Write a circle with center $(0, 0)$ and radius 7. _____

27. Find the radius of $x^2 + y^2 = 64$. _____

28. Find the center of $(x + 5)^2 + (y - 1)^2 = 9$. _____

29. Vertex of $y = (x - 4)^2 + 6$. _____

30. Axis of symmetry of $y = (x + 2)^2 - 3$. _____

31. Classify $x^2 + y^2 = 36$. _____

32. Classify $\frac{x^2}{9} + \frac{y^2}{4} = 1$. _____

33. Classify $\frac{x^2}{16} - \frac{y^2}{9} = 1$. _____

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Answer Keys

1. $(5, -3)$
2. 7
3. $(x-2)^2+(y+4)^2=25$
4. $x^2+y^2=25$
5. $(x-2)^2+(y+3)^2=25$
6. $(x+3)^2+(y-5)^2=25$
7. FALSE
8. center $(-4,5)$; $r=3$
9. $(x-1)^2+(y-3)^2=25$
10. 6
11. $(x+2)^2+(y-1)^2=16$
12. 5
13. TRUE
14. $(x-4)^2+(y+1)^2=17$
15. $(x+1)^2+(y-2)^2=9$
16. $(x-1)^2+(y-2)^2=16$
17. $(x-3)^2+(y-5)^2=25$
18. $(1, 5)$
19. $x^2+y^2=10$
20. $x^2+y^2=169$
21. $(x-2)^2+(y+1)^2=9$; yes, inside
22. $(x-1)^2+(y-1)^2=25$
23. $(x-6)^2+(y+2)^2=49$; center $(6,-2)$; $r=7$
24. $(x-3)^2+(y-4)^2=25$

Additional Practice Answers

25. $(3,-2)$, $r=5$
26. $x^2+y^2=49$
27. 8
28. $(-5,1)$
29. $(4,6)$
30. $x=-2$
31. circle
32. ellipse
33. hyperbola

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Compare to $(x-h)^2+(y-k)^2=r^2$. $h=5$ (from $(x-5)$), $k=-3$ (from $(y+3)=y-(-3)$). The sign-flip is the classic trap.

2. In standard form $(x-h)^2+(y-k)^2=r^2$, the right side is r^2, not r. Here $r^2=49$, so take the square root: $r=\sqrt{49}=7$. Calling the radius 49 is the classic slip.

3. Plug into $(x-h)^2+(y-k)^2=r^2$ with $h=2$, $k=-4$, $r^2=25$. The $(y-(-4))=(y+4)$ catches some students.

4. Start with the key idea: $r=\sqrt{3^2+4^2}=\sqrt{25}=5$. So $x^2+y^2=25$. (The 3-4-5 right triangle shows up a lot.) That gives a quick check on the answer.

5. Group: $(x^2-4x)+(y^2+6y)=12$. Complete: half of -4 is -2, square is 4. Half of 6 is 3, square is 9. $(x^2-4x+4)+(y^2+6y+9)=12+4+9=25 \Rightarrow (x-2)^2+(y+3)^2=25$. Center $(2,-3)$, radius 5.

6. Keep the rule visible: $r=\sqrt{3^2+(-4)^2}=5$ (another 3-4-5). Use $h=-3$, $k=5$, $r^2=25$. That gives a quick check on the answer.

7. The right side equals $r^2 \geq 0$. A negative right side describes *no real circle* at all – there's no real point that satisfies the equation.

8. Move the constant over and group by variable: $(x^2+8x)+(y^2-10y)=-32$. Half of 8 is 4 ($4^2=16$); half of -10 is -5 ($(-5)^2=25$). Add both to each side: $(x+4)^2+(y-5)^2=-32+16+25=9$. Center $(-4,5)$ (signs flip), $r=\sqrt{9}=3$.

9. Center is the midpoint of the diameter: $\left(\frac{-2+4}{2}, \frac{7+(-1)}{2}\right)=(1,3)$. Radius is the distance from the center to either endpoint: $\sqrt{3^2+(-4)^2}=5$. So $(x-1)^2+(y-3)^2=25$.

10. Keep the rule visible: $r^2=36$, so $r=6$. The center is at the origin. This is the part to check before moving on, because it keeps the answer tied to the original question.

11. Plug in: $h=-2$, $k=1$, $r^2=16$. $(x-(-2))^2+(y-1)^2=16$ simplifies to $(x+2)^2+(y-1)^2=16$.

12. Group: $(x^2-6x)+(y^2-8y)=0$. Half of -6 is -3 (9); half of -8 is -4 (16). Add both to each side: $(x-3)^2+(y-4)^2=0+9+16=25$. Then $r=\sqrt{25}=5$.

13. Standard form $(x-h)^2+(y-k)^2=r^2$ has implicit coefficient 1 on each squared term. Unequal coefficients describe an ellipse, not a circle. (If you see, say, $4x^2+4y^2+\ldots=0$, divide through by 4 to recover standard form.)

14. The radius is the distance from center $(4,-1)$ to the origin $(0,0)$: $r=\sqrt{(4-0)^2+(-1-0)^2}=\sqrt{16+1}=\sqrt{17}$, so $r^2=17$. Plug into standard form: $(x-4)^2+(y+1)^2=17$ (the right side is r^2, no need to simplify the radical).

15. Move the -4 over: $(x^2+2x)+(y^2-4y)=4$. Half of 2 is 1 (1); half of -4 is -2 (4). Add both to each side: $(x+1)^2+(y-2)^2=4+1+4=9$. Center $(-1,2)$, radius $\sqrt{9}=3$.

16. Plug straight into $(x-h)^2+(y-k)^2=r^2$ with $h=1$, $k=2$, and $r^2=4^2=16$ (square the radius). That gives $(x-1)^2+(y-2)^2=16$.

17. A circle tangent to the x-axis just touches it, so the radius equals the center's distance from that axis – namely the $|y|$-coordinate, $|5|=5$. Then $r^2=25$ and the equation is $(x-3)^2+(y-5)^2=25$.

18. The center of a circle is the midpoint of any diameter. Average the endpoints' coordinates: $\left(\frac{-4+6}{2}, \frac{2+8}{2}\right)=\left(\frac{2}{2}, \frac{10}{2}\right)=(1,5)$.

19. Centered at the origin means $h=k=0$, so the form is $x^2+y^2=r^2$. Squaring the radius, $r^2=(\sqrt{10})^2=10$. Equation: $x^2+y^2=10$.

20. Start with the key idea: $r=\sqrt{(-5)^2+12^2}=\sqrt{169}=13$ (a 5-12-13 right triangle). $r^2=169$. That gives a quick check on the answer.

21. Center $(2,-1)$, radius 3: equation $(x-2)^2+(y+1)^2=9$. To check the town at $(4,1)$: distance from center is $\sqrt{(4-2)^2+(1-(-1))^2}=\sqrt{4+4}=\sqrt{8}\approx 2.83$ miles. Since $2.83<3$, the town is inside the coverage area. **Algebra check:** plug $(4,1)$ into the left side: $(2)^2+(2)^2=8<9$, confirming inside (strict inequality $<r^2$). Points *on* the circle satisfy equality; points *inside* give a smaller value.

22. Center is the midpoint of the diameter: $\left(\frac{-3+5}{2}, \frac{4+(-2)}{2}\right)=(1,1)$. Radius is half the diameter length: diameter $=\sqrt{(5-(-3))^2+(-2-4)^2}=\sqrt{64+36}=\sqrt{100}=10$, so $r=5$. Equation: $(x-1)^2+(y-1)^2=25$. **Verify:** plug $(5,-2)$ in: $(4)^2+(-3)^2=16+9=25$ ✓. (Another 3-4-5 in disguise – the legs from center $(1,1)$ to endpoint $(5,-2)$ are 4 and 3.)

23. Group and complete the square: $(x^2-12x)+(y^2+4y)=9$. Half of -12 is -6; square is 36. Half of 4 is 2; square is 4. $(x^2-12x+36)+(y^2+4y+4)=9+36+4=49 \Rightarrow (x-6)^2+(y+2)^2=49$. Center $(6,-2)$, $r=\sqrt{49}=7$

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m. Circumference $= 2\pi r = 14\pi$ m ≈ 43.98 m. (A typical recreational track is about 400 m, so this would be a smaller training loop.)

24. The triangle formed by $(0,0)$, $(6,0)$, $(0,8)$ has a right angle at the origin (the legs are along the axes). By Thales's theorem, the hypotenuse from $(6,0)$ to $(0,8)$ is a diameter of the circle through all three. Midpoint: $\left(\frac{6+0}{2}, \frac{0+8}{2}\right) = (3,4)$ (the center). Radius: distance from $(3,4)$ to $(0,0)$ is $\sqrt{9+16} = 5$. Equation: $(x-3)^2+(y-4)^2 = 25$. **Verify all three points:** $(6-3)^2+(0-4)^2 = 9+16 = 25$ ✓; $(0-3)^2+(8-4)^2 = 9+16 = 25$ ✓; $(0-3)^2+(0-4)^2 = 9+16 = 25$ ✓.

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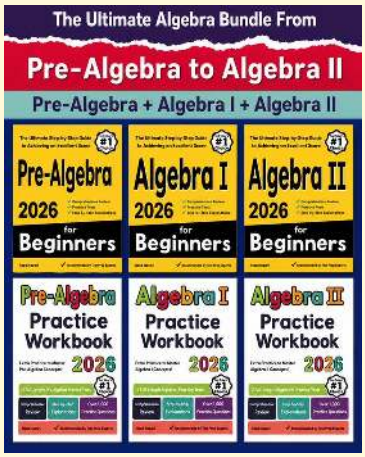

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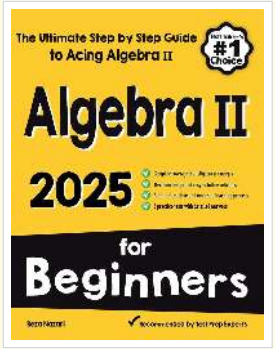

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