Solving a Quadratic Equation

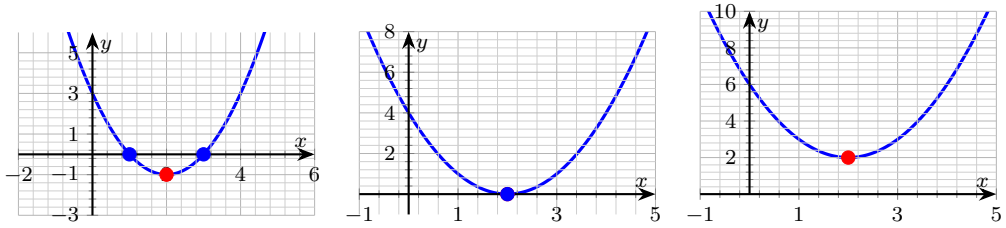
Name: _____	Date: _____	Score: _____ / 28

Quick Review

A **quadratic equation** is anything that can be rearranged into the form $ax^2 + bx + c = 0$ with $a \neq 0$. Four reliable tools solve them, and the smart move is to size up the equation before reaching for one. **Factoring** is fastest when the trinomial splits cleanly — look for two numbers that multiply to ac and add to b. **Square roots** work whenever you can isolate a perfect square: from $(x-3)^2 = 16$ jump straight to $x - 3 = \pm 4$. **Completing the square** is the all-purpose workhorse and is how the next tool was derived in the first place. The **quadratic formula** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ never fails.

The **discriminant** $\Delta = b^2 - 4ac$ tells you how many real roots before you commit: positive $\Rightarrow$ two real, zero $\Rightarrow$ one repeated, negative $\Rightarrow$ two complex conjugates (the parabola never touches the x-axis). Two traps to flag: the sign on b inside the formula — $-b$ with $b = -4$ is $+4$, not -4 — and the order of operations under the radical (square b *first*, then subtract $4ac$). Always plug your roots back into the original equation. It's the quickest way to catch a sign slip.

Three pictures, one for each discriminant sign:

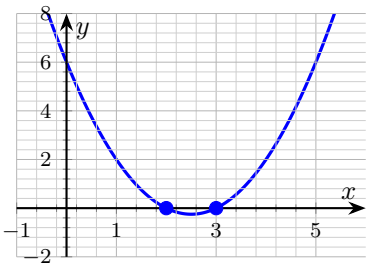


Two real roots ($\Delta > 0$), one double root ($\Delta = 0$), no real roots ($\Delta < 0$).

PRACTICE

Solve each quadratic. State both roots (real or complex).

1. Solve $x^2 - 5x + 6 = 0$. Confirm with the graph. _____



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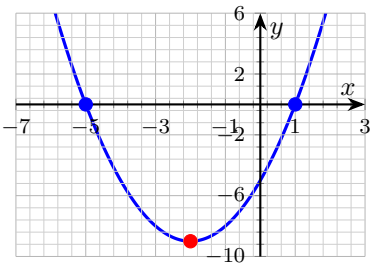
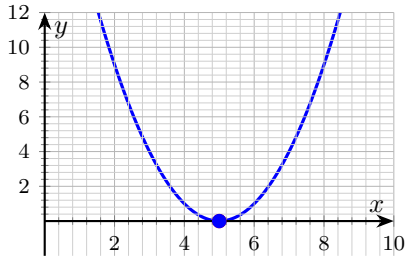
2. Solve $x^2 + 4x - 5 = 0$. Use the graph as a sanity check. _____

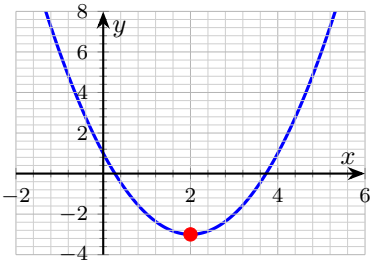


3. $2x^2 - 5x - 3 = 0$ _____

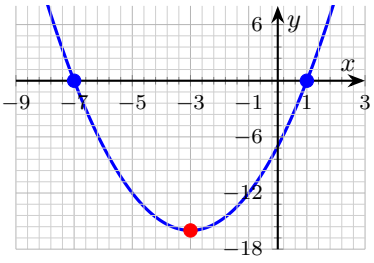
4. Solve $x^2 - 10x + 25 = 0$. (Graph is tangent.) _____



5. Solve $x^2 - 4x + 1 = 0$ (irrational roots). _____

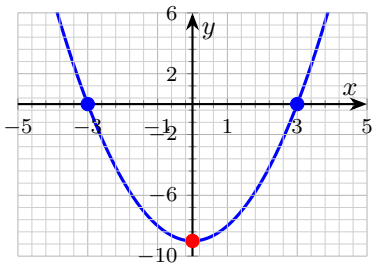


6. Solve $x^2 + 6x - 7 = 0$ and check against the graph. _____



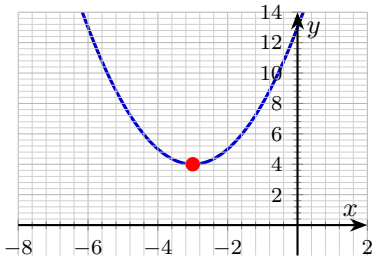
7. $3x^2 + 2x - 7 = 0$ _____

8. Solve $x^2 - 9 = 0$. Graph confirms the roots. _____

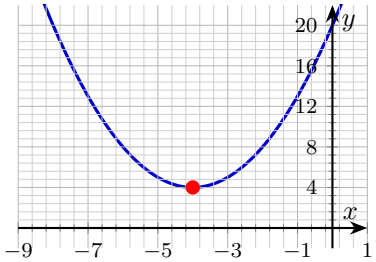


9. $3x^2 + 2x + 5 = 0$ _____

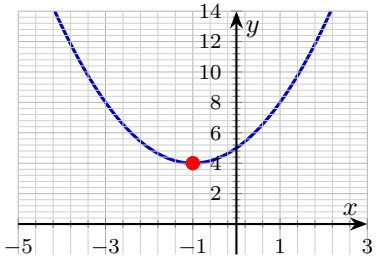
10. Solve $x^2 + 6x + 13 = 0$. (The graph confirms complex roots.) _____



11. Solve $x^2 + 8x + 20 = 0$. Graph shown. _____



12. Solve $x^2 + 2x + 5 = 0$. _____



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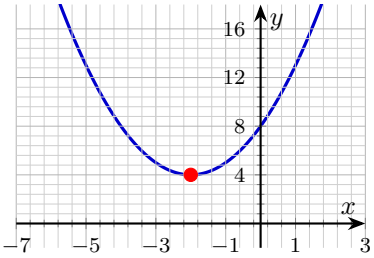
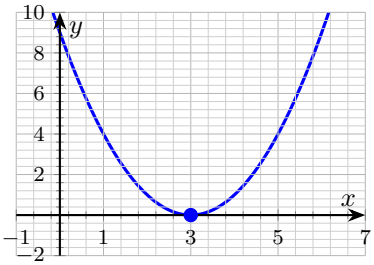
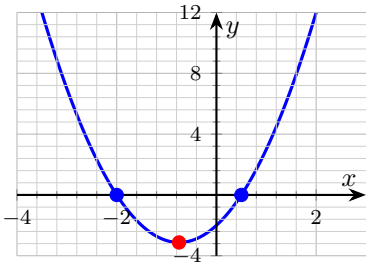
13. Solve $x^2 + 4x + 8 = 0$. (Notice the vertex's height.) _____



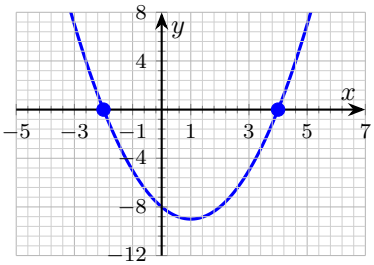
14. Solve $x^2 - 6x + 9 = 0$. (Graph is tangent to the x-axis.) _____



15. Solve $2x^2 + 3x - 2 = 0$ and use the graph as a check. _____



16. Solve $x^2 - 2x - 8 = 0$. (Graph shows the two real roots.) _____



17. How many real roots: $2x^2 + 5x + 4 = 0$? _____

18. How many real roots: $4x^2 - 12x + 9 = 0$? _____

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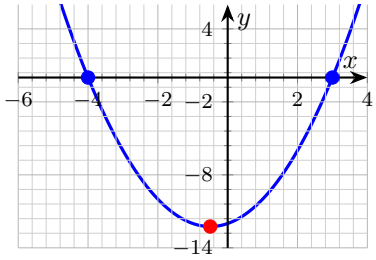
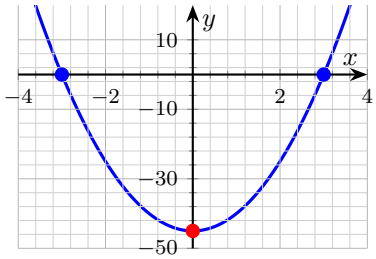
19. Solve $x^2 + x - 12 = 0$. Confirm the roots from the graph. _____



20. Solve $5x^2 = 45$. Check with the graph of $y = 5x^2 - 45$. _____



Word Problems

21. A small launched object has height $h(t) = -16t^2 + 48t + 64$ feet, where t is seconds after launch. At what positive time does it hit the ground? ____

22. A rectangular swimming pool has length 2 meters more than twice its width. The area is 60 m^2. Find the width. ____

23. A rectangular garden has length 6 feet more than twice its width. Its area is 140 ft^2. Find the width. ____

24. The product of two consecutive positive integers is 132. Find the integers. ____

Additional Practice

25. Solve $x^2 - 5x + 6 = 0$. _____

26. Solve $x^2 = 49$. _____

27. Find the vertex of $y = (x - 3)^2 - 4$. _____

28. Find the axis of symmetry of $y = x^2 + 6x + 1$. _____

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Answer Keys

1. $x = 2,\ 3$
2. $x = 1,\ -5$
3. $x = 3,\ -\frac{1}{2}$
4. $x = 5$ (double root)
5. $x = 2 \pm \sqrt{3}$
6. $x = 1,\ -7$
7. $x = \frac{-1 \pm \sqrt{22}}{3}$
8. $x = \pm 3$
9. $x = \frac{-1 \pm i\sqrt{14}}{3}$
10. $x = -3 \pm 2i$
11. $x = -4 \pm 2i$
12. $x = -1 \pm 2i$
13. $x = -2 \pm 2i$
14. $x = 3$ (double root)
15. $x = \frac{1}{2},\ -2$
16. $x = 4,\ -2$
17. 0 (two complex)
18. 1 (double root)
19. $x = 3,\ -4$
20. $x = \pm 3$
21. $t = 4$ seconds
22. 5 meters
23. 7 feet
24. 11 and 12

Additional Practice Answers

25. $x = 2,\ 3$
26. $x = -7,\ 7$
27. $(3, -4)$
28. $x = -3$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Two numbers that multiply to 6 and add to -5: -2 and -3. Factor: $(x-2)(x-3) = 0$, so $x = 2$ or $x = 3$. The graph crosses the x-axis at exactly those values.

2. Two numbers multiplying to -5 and adding to 4: 5 and -1. Factor: $(x-1)(x+5) = 0$. The signs flip from factor to root — that's the part students miss.

3. Try factoring: $(2x+1)(x-3) = 0$. So $2x+1 = 0$ gives $x = -\frac{1}{2}$, and $x-3 = 0$ gives $x = 3$. Quadratic formula confirms: $x = \frac{5 \pm \sqrt{25+24}}{4} = \frac{5 \pm 7}{4}$, matching 3 and $-\frac{1}{2}$.

4. Perfect square: $(x-5)^2 = 0$, so $x = 5$ shows up twice. Discriminant $100 - 100 = 0$ — the signature of a repeated root.

5. Doesn't factor over the integers, so reach for the formula: $x = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$. The $\pm 2\sqrt{3}$ over 2 reduces cleanly — watch that step or you'll leave a 2 behind.

6. Two numbers multiplying to -7 and adding to 6: 7 and -1. Factor: $(x+7)(x-1) = 0$, so $x = -7$ or $x = 1$.

7. Doesn't factor. Use the formula with $a = 3, b = 2, c = -7$: $x = \frac{-2 \pm \sqrt{4+84}}{6} = \frac{-2 \pm \sqrt{88}}{6} = \frac{-2 \pm 2\sqrt{22}}{6} = \frac{-1 \pm \sqrt{22}}{3}$.

8. Difference of squares: $(x-3)(x+3) = 0$, so $x = \pm 3$. (Equivalently, $x^2 = 9$ gives $x = \pm\sqrt{9} = \pm 3$.)

9. Discriminant first: $4 - 60 = -56 < 0$. Two complex roots. $x = \frac{-2 \pm \sqrt{-56}}{6} = \frac{-2 \pm 2i\sqrt{14}}{6} = \frac{-1 \pm i\sqrt{14}}{3}$.

10. Keep the rule visible: $\Delta = 36 - 52 = -16$, so complex conjugates. $x = \frac{-6 \pm \sqrt{-16}}{2} = \frac{-6 \pm 4i}{2} = -3 \pm 2i$. The parabola sits entirely above the x-axis. That gives a quick check on the answer.

11. One steady path is: $\Delta = 64 - 80 = -16$, complex roots. $x = \frac{-8 \pm 4i}{2} = -4 \pm 2i$. The graph never crosses the x-axis — exactly what complex roots look like. That gives a quick check on the answer.

12. Start with the key idea: $\Delta = 4 - 20 = -16$. $x = \frac{-2 \pm 4i}{2} = -1 \pm 2i$. The parabola hovers above the x-axis. That gives a quick check on the answer.

13. A careful way to see it: $\Delta = 16 - 32 = -16$, complex pair. $x = \frac{-4 \pm 4i}{2} = -2 \pm 2i$. The vertex $(-2, 4)$ sits above the x-axis, so no real crossings. That gives a quick check on the answer.

14. Perfect square: $(x-3)^2 = 0$. $\Delta = 36 - 36 = 0$, so the parabola is tangent to the x-axis at $x = 3$ — one repeated root.

15. Factor: $(2x-1)(x+2) = 0$. So $x = \frac{1}{2}$ or $x = -2$. Quadratic formula confirms: $x = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4}$.

16. Factor: $(x-4)(x+2) = 0$. The parabola crosses the x-axis at the two roots, exactly where the factors vanish.

17. Discriminant: $25 - 32 = -7 < 0$. Negative ⇒ no real roots. The parabola never reaches the x-axis.

18. Discriminant: $144 - 144 = 0$. A zero discriminant means one repeated real root (the vertex sits on the x-axis).

19. Two numbers multiplying to -12 and adding to 1: 4 and -3. Factor: $(x-3)(x+4) = 0$, giving $x = 3$ or $x = -4$.

20. Divide both sides by 5: $x^2 = 9$. Square roots: $x = \pm 3$. (Don't divide 45 by 5 to get 9 and then forget the $\pm$ — both roots count.)

21. Ground means $h = 0$. Set $-16t^2 + 48t + 64 = 0$, divide by -16 to clean up: $t^2 - 3t - 4 = 0$. Factor: $(t-4)(t+1) = 0$, so $t = 4$ or $t = -1$. Time can't be negative here, so the answer is $t = 4$ seconds.

22. Let w be the width; length is $2w + 2$. Area: $w(2w+2) = 60$, so $2w^2 + 2w - 60 = 0$, then $w^2 + w - 30 = 0$. Factor: $(w+6)(w-5) = 0$, giving $w = 5$ or $w = -6$. Width can't be negative, so $w = 5$ meters. (Length is $2(5) + 2 = 12$, and $5 \times 12 = 60$ checks out.)

23. Width w, length $2w + 6$. Area: $w(2w+6) = 140$, so $2w^2 + 6w - 140 = 0$, then $w^2 + 3w - 70 = 0$. Factor: $(w+10)(w-7) = 0$, so $w = 7$ or $w = -10$. Toss the negative; width is 7 feet. Check: $7 \times (2(7) + 6) = 7 \times 20 = 140$.

24. Let the integers be n and $n+1$. Then $n(n+1) = 132$, so $n^2 + n - 132 = 0$. Factor: $(n-11)(n+12) = 0$, giving $n = 11$ or $n = -12$. Positive integers only, so $n = 11$ and the pair is $11, 12$. (Quick check: $11 \times 12 = 132$.)

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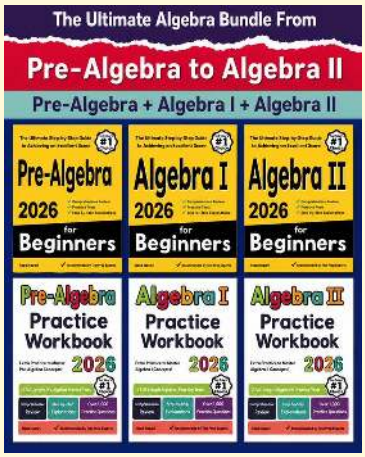

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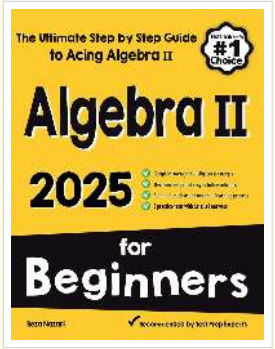

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