

# Solving Trigonometric Equations

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 29

## Q Quick Review

Solving a trig equation is like solving any algebraic equation, with one extra layer: trig functions are *periodic*, so a single equation usually has infinitely many solutions. The problem will tell you what interval to report.

**Step 1: Isolate the trig function.** Get to  $\sin x = c$  (or cos, tan, etc.) before doing anything else. Move constants, divide, factor – normal algebra.

**Step 2: Find the reference angle.** Pretend the equation is positive; ask what acute angle gives that value. Memorized specials:  $\sin x = \frac{1}{2} \Rightarrow \text{ref } \frac{\pi}{6}$ ;  $\sin x = \frac{\sqrt{2}}{2} \Rightarrow \text{ref } \frac{\pi}{4}$ ;  $\sin x = \frac{\sqrt{3}}{2} \Rightarrow \text{ref } \frac{\pi}{3}$ .

**Step 3: Place the solutions in the right quadrants.** Sine positive: Q1, Q2. Sine negative: Q3, Q4. Cosine positive: Q1, Q4. Cosine negative: Q2, Q3. Tangent positive: Q1, Q3. Tangent negative: Q2, Q4. (Memory aid: **ASTC** – All, Sine, Tangent, Cosine – starting in Q1 and going counterclockwise tells you which is positive in each quadrant.)

**Step 4: Use the interval.** If the problem asks for  $[0, 2\pi)$ , list every solution in that range. For a general solution, add  $2\pi k$  (sine, cosine) or  $\pi k$  (tangent) for integer  $k$ .

**Trig equations that look quadratic.** If you see  $2\sin^2 x - \sin x = 0$ , factor:  $\sin x(2\sin x - 1) = 0$ . Then  $\sin x = 0$  or  $\sin x = \frac{1}{2}$ .

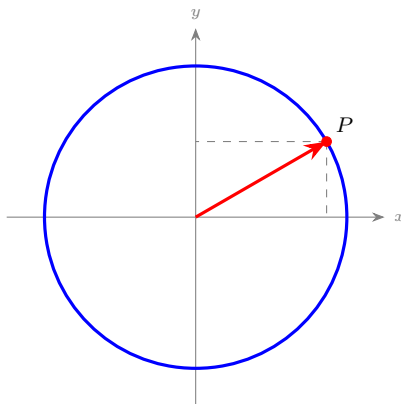
**Scaled inputs.** For  $\sin(2x) = \frac{1}{2}$  on  $[0, 2\pi)$ , let  $u = 2x$  so  $u \in [0, 4\pi)$  – twice the usual range. Find all  $u$  first, then divide by 2 to get  $x$ . Don't lose half the solutions.

**Watch out for.** Dividing by a trig expression that could be zero – you'll lose solutions. Factor instead. After squaring, check for extraneous roots.

## PRACTICE

Find all solutions in  $[0, 2\pi)$  unless told otherwise. Give exact answers using unit-circle angles.

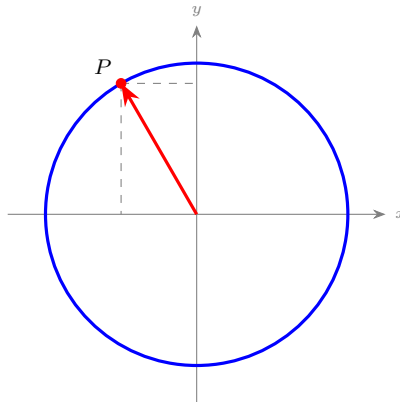
1. Solve  $\sin x = \frac{1}{2}$  on  $[0, 2\pi)$ . One solution is marked on the unit circle below; sine is positive, so find its partner in the other quadrant too. \_\_\_\_\_



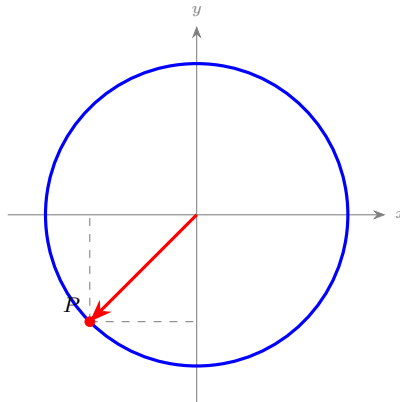
2. Solve  $\cos x = 0$  on  $[0, 2\pi)$ . \_\_\_\_\_
3. Solve  $2\sin x - 1 = 0$  on  $[0, 2\pi)$ . \_\_\_\_\_



4. Solve  $2 \cos x + 1 = 0$  on  $[0, 2\pi)$ . First isolate the cosine; one of the two solution angles is shown on the circle below – find the second. \_\_\_\_\_



- 5. Solve  $\tan x = 1$  on  $[0, 2\pi)$ . \_\_\_\_\_
- 6. Solve  $\tan x = -1$  on  $[0, 2\pi)$ . \_\_\_\_\_
- 7. Solve  $\sin x = 0$  on  $[0, 2\pi)$ . \_\_\_\_\_
- 8. Solve  $\cos x = 1$  on  $[0, 2\pi)$ . \_\_\_\_\_
- 9. Solve  $\sin x = -\frac{\sqrt{2}}{2}$  on  $[0, 2\pi)$ . The Q3 solution is marked below; sine is negative, so there is also a Q4 partner – find both. \_\_\_\_\_



- 10. Solve  $2 \sin^2 x - 1 = 0$  on  $[0, 2\pi)$ . \_\_\_\_\_
- 11. Solve  $\cos^2 x = \sin^2 x$  on  $[0, 2\pi)$ . \_\_\_\_\_
- 12. Solve  $\sin^2 x - \sin x = 0$  on  $[0, 2\pi)$ . \_\_\_\_\_
- 13. Solve  $2 \cos^2 x - 1 = 0$  on  $[0, 2\pi)$ . \_\_\_\_\_
- 14. Solve  $\tan x = \sqrt{3}$  on  $[0, 2\pi)$ . \_\_\_\_\_
- 15. Solve  $\sin x = 1$  on  $[0, 2\pi)$ . \_\_\_\_\_
- 16. Solve  $\cos x = -1$  on  $[0, 2\pi)$ . \_\_\_\_\_
- 17. Solve  $\sqrt{3} \tan x - 1 = 0$  on  $[0, 2\pi)$ . \_\_\_\_\_
- 18. Solve  $\sin(2x) = \frac{1}{2}$  on  $[0, 2\pi)$ . \_\_\_\_\_
- 19. Solve  $2 \sin x \cos x = 0$  on  $[0, 2\pi)$ . \_\_\_\_\_
- 20. Solve  $2 \sin^2 x + \sin x - 1 = 0$  on  $[0, 2\pi)$ . \_\_\_\_\_



## ◆ Word Problems

21. A pendulum's angle from vertical is modeled by  $\theta(t) = 0.5 \sin(t)$  radians, with  $t$  in seconds. For what times in  $[0, 2\pi)$  is the angle exactly  $0.25$  rad? \_\_\_\_\_
22. A daily-temperature model is  $T(t) = 70 + 10 \cos t$ , where  $T$  is degrees Fahrenheit and  $t$  is in radians ( $[0, 2\pi)$  represents one day). For what times in  $[0, 2\pi)$  does  $T(t) = 65^\circ$ ? \_\_\_\_\_
23. A buoy's vertical position is  $y(t) = 2 \sin t$  meters. How many times does the buoy reach its maximum height of  $2$  meters during the interval  $[0, 4\pi)$ ? \_\_\_\_\_
24. An AC voltage signal is  $V(t) = 120 \sin(t)$  volts. Find all times in  $[0, 2\pi)$  when  $V(t) = -60\sqrt{3}$  volts. \_\_\_\_\_

## Additional Practice

25. Find  $\sin \theta$  if opposite = 5, hypotenuse = 13. \_\_\_\_\_
26. Find  $\cos \theta$  if adjacent = 12, hypotenuse = 13. \_\_\_\_\_
27. Find  $\tan \theta$  if opposite = 7, adjacent = 4. \_\_\_\_\_
28. Find  $\sin 30^\circ$ . \_\_\_\_\_
29. Find  $\cos 60^\circ$ . \_\_\_\_\_



Answer Keys

<p>1. <math>x = \frac{\pi}{6}, \frac{5\pi}{6}</math></p> <p>2. <math>x = \frac{\pi}{2}, \frac{3\pi}{2}</math></p> <p>3. <math>x = \frac{\pi}{6}, \frac{5\pi}{6}</math></p> <p>4. <math>x = \frac{2\pi}{3}, \frac{4\pi}{3}</math></p> <p>5. <math>x = \frac{\pi}{4}, \frac{5\pi}{4}</math></p> <p>6. <math>x = \frac{3\pi}{4}, \frac{7\pi}{4}</math></p> <p>7. <math>x = 0, \pi</math></p> <p>8. <math>x = 0</math></p> <p>9. <math>x = \frac{5\pi}{4}, \frac{7\pi}{4}</math></p> <p>10. <math>x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}</math></p> <p>11. <math>x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}</math></p> <p>12. <math>x = 0, \frac{\pi}{2}, \pi</math></p> <p><b>Additional Practice Answers</b></p> <p>25. <math>\frac{5}{13}</math></p> <p>26. <math>\frac{12}{13}</math></p> <p>27. <math>\frac{7}{4}</math></p>	<p>13. <math>x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}</math></p> <p>14. <math>x = \frac{\pi}{3}, \frac{4\pi}{3}</math></p> <p>15. <math>x = \frac{\pi}{2}</math></p> <p>16. <math>x = \pi</math></p> <p>17. <math>x = \frac{\pi}{6}, \frac{7\pi}{6}</math></p> <p>18. <math>x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}</math></p> <p>19. <math>x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}</math></p> <p>20. <math>x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}</math></p> <p>21. <math>t = \frac{\pi}{6}, \frac{5\pi}{6}</math></p> <p>22. <math>t = \frac{2\pi}{3}, \frac{4\pi}{3}</math></p> <p>23. 2 times</p> <p>24. <math>t = \frac{4\pi}{3}, \frac{5\pi}{3}</math></p> <p>28. <math>\frac{1}{2}</math></p> <p>29. <math>\frac{1}{2}</math></p>
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Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

- The reference angle whose sine is  $\frac{1}{2}$  is  $\frac{\pi}{6}$ . Since  $\sin x$  is positive,  $x$  lives where the  $y$ -coordinate is positive – Q1 and Q2 (the marked point and its mirror across the  $y$ -axis). That gives  $x = \frac{\pi}{6}$  in Q1 and  $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$  in Q2. Don't stop at the first solution – the interval  $[0, 2\pi)$  holds both.
- Cosine is the  $x$ -coordinate on the unit circle; it equals zero at the top and bottom:  $\frac{\pi}{2}$  and  $\frac{3\pi}{2}$ .
- One steady path is: Isolate first:  $\sin x = \frac{1}{2}$ . Same as the first problem –  $\frac{\pi}{6}$  and  $\frac{5\pi}{6}$ . That gives a quick check on the answer.
- Isolate first:  $2 \cos x + 1 = 0$  gives  $\cos x = -\frac{1}{2}$ . The reference angle is  $\frac{\pi}{3}$ . Cosine (the  $x$ -coordinate) is negative on the left side of the circle – Q2 and Q3 – so  $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$  (the marked Q2 point) and  $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$  in Q3.
- A careful way to see it: Tangent is positive in Q1 and Q3; reference  $\frac{\pi}{4}$ . So  $x = \frac{\pi}{4}$  and  $x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$ . That gives a quick check on the answer.
- Tangent is negative in Q2 and Q4; reference  $\frac{\pi}{4}$ .  $x = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$  and  $x = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$ .
- Sine is zero on the  $x$ -axis: at  $x = 0$  (start) and  $x = \pi$  (halfway). Don't include  $2\pi$  – the interval is half-open.
- Cosine maxes out at 1 only at  $x = 0$  (in the interval  $[0, 2\pi)$ ).  $2\pi$  would also work, but it's excluded.
- The reference angle for  $\sin = \frac{\sqrt{2}}{2}$  is  $\frac{\pi}{4}$ . Sine is negative where the  $y$ -coordinate dips below the axis – the bottom half, Q3 and Q4. So  $x = \pi + \frac{\pi}{4} =$

- $\frac{5\pi}{4}$  (the marked Q3 point) and  $x = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$  in Q4. A common slip is reporting the Q3 angle alone and forgetting its Q4 partner.
- Keep the rule visible:  $\sin^2 x = \frac{1}{2} \Rightarrow \sin x = \pm \frac{\sqrt{2}}{2}$ . All four reference- $\frac{\pi}{4}$  angles work because the sign on both branches is covered. That gives a quick check on the answer.
- One steady path is:  $|\cos x| = |\sin x|$ , so the reference angle is  $\frac{\pi}{4}$  and every quadrant supplies a solution. (Or: divide both sides by  $\cos^2 x$  to get  $\tan^2 x = 1$ .) That gives a quick check on the answer.
- Factor:  $\sin x(\sin x - 1) = 0$ . So  $\sin x = 0 \Rightarrow x = 0, \pi$  or  $\sin x = 1 \Rightarrow x = \frac{\pi}{2}$ . Three solutions. (Don't divide both sides by  $\sin x$  – you'd lose  $x = 0, \pi$ .)
- A careful way to see it:  $\cos^2 x = \frac{1}{2} \Rightarrow \cos x = \pm \frac{\sqrt{2}}{2}$ . All four  $\frac{\pi}{4}$ -reference angles. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Keep the rule visible: Tangent positive: Q1 and Q3. Reference  $\frac{\pi}{3}$ . So  $x = \frac{\pi}{3}$  and  $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ . That gives a quick check on the answer.
- One steady path is: Sine reaches its max of 1 only at  $x = \frac{\pi}{2}$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
- Start with the key idea: Cosine bottoms out at  $-1$  only at  $x = \pi$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
- Isolate:  $\tan x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ . Reference  $\frac{\pi}{6}$ ; tangent positive in Q1, Q3. So  $x = \frac{\pi}{6}$  and  $x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ .
- Let  $u = 2x$ , so  $u \in [0, 4\pi)$  – two full periods.  $\sin u = \frac{1}{2}$  at  $u =$



$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ . Divide each by 2 to get  $x$ .

**19.** Zero product:  $\sin x = 0$  gives  $x = 0, \pi$ ;  $\cos x = 0$  gives  $x = \frac{\pi}{2}, \frac{3\pi}{2}$ . Four solutions total. (Bonus:  $2 \sin x \cos x = \sin 2x$ , so this is just  $\sin 2x = 0$ .)

**20.** Factor like a quadratic in  $\sin x$ :  $(2 \sin x - 1)(\sin x + 1) = 0$ . Then  $\sin x = \frac{1}{2}$  gives  $x = \frac{\pi}{6}, \frac{5\pi}{6}$ , and  $\sin x = -1$  gives  $x = \frac{3\pi}{2}$ .

**21.** Set  $0.5 \sin t = 0.25$ , so  $\sin t = \frac{1}{2}$ . Solutions in  $[0, 2\pi)$ :  $t = \frac{\pi}{6}$  and  $t = \frac{5\pi}{6}$ . (The pendulum passes through 0.25 rad twice in one full cycle – once going each way.)

**22.** Keep the rule visible:  $70 + 10 \cos t = 65 \Rightarrow \cos t = -\frac{1}{2}$ . Cosine is  $-\frac{1}{2}$  in

Q2 and Q3, with reference  $\frac{\pi}{3}$ :  $t = \frac{2\pi}{3}$  and  $t = \frac{4\pi}{3}$ . That gives a quick check on the answer.

**23.** One steady path is:  $2 \sin t = 2 \Rightarrow \sin t = 1$ , which happens only at  $t = \frac{\pi}{2} + 2\pi k$ . In  $[0, 4\pi)$  that's  $t = \frac{\pi}{2}$  and  $t = \frac{5\pi}{2}$  – two times. (One per full period of length  $2\pi$ , and the interval is two periods long.) That gives a quick check on the answer.

**24.** Start with the key idea:  $120 \sin t = -60\sqrt{3} \Rightarrow \sin t = -\frac{\sqrt{3}}{2}$ . Sine is negative in Q3 and Q4; reference angle  $\frac{\pi}{3}$ . So  $t = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$  (Q3) and  $t = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$  (Q4). That gives a quick check on the answer.



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