Solving Systems with Matrix Equations

Name: ______________________ | Date: ______________ | Score: _____ / 31

Quick Review

Any linear system can be rewritten as a single matrix equation $A\vec{x} = \vec{b}$, where A is the **coefficient matrix** (one row per equation, one column per variable), $\vec{x}$ is the column vector of unknowns, and $\vec{b}$ is the column vector of constants. The system $\begin{cases} 2x + 3y = 8 \\ x - y = 1 \end{cases}$ becomes $\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$.

Solving by inverse. If A is invertible, left-multiply both sides by A^{-1}: $A^{-1}A\vec{x} = A^{-1}\vec{b} \Rightarrow I\vec{x} = A^{-1}\vec{b} \Rightarrow \vec{x} = A^{-1}\vec{b}$. The *left* side matters — matrix multiplication is non-commutative, so $\vec{b}A^{-1}$ is wrong (and often undefined).

Three cases by determinant. (1) $\det(A) \neq 0 \Rightarrow$ unique solution $\vec{x} = A^{-1}\vec{b}$. (2) $\det(A) = 0$ and the system is consistent $\Rightarrow$ infinitely many solutions (rows are dependent and $\vec{b}$ lines up). (3) $\det(A) = 0$ and the system is inconsistent $\Rightarrow$ no solution. The matrix tells you which case before you even compute.

Always verify by plugging back into the original system. A clean check catches arithmetic slips in the inverse and the multiplication. And remember — you can always cross-check by elimination or substitution; the matrix route is one tool among several.

PRACTICE

Solve each system using the matrix equation $A\vec{x} = \vec{b}$. State "no unique solution" when $\det(A) = 0$.

1. Write the matrix equation $A\vec{x} = \vec{b}$ for the system $\begin{cases} 2x + 3y = 8 \\ x - y = 1 \end{cases}$. ________

2. State the formula for the unique solution of $A\vec{x} = \vec{b}$ when A is invertible. ________

3. Solve the system $\begin{cases} x + 2y = 5 \\ 3x + 4y = 11 \end{cases}$. The coefficient matrix and constants are laid out below. ________

Eq.	Coef. x	Coef. y	Constant
1	1	2	5
2	3	4	11

4. Solve $\begin{cases} 3x + 2y = 7 \\ 5x + 4y = 13 \end{cases}$ ________

5. Solve $\begin{cases} 2x + y = 4 \\ x + 3y = 7 \end{cases}$ ________

6. Solve $\begin{cases} 2x + 4y = 6 \\ 3x + 6y = 9 \end{cases}$ ________

7. Solve $\begin{cases} 4x + y = 9 \\ 2x + 3y = 17 \end{cases}$ ________

8. Solve $\begin{cases} x + y = 10 \\ 2x + 3y = 24 \end{cases}$ ________

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9. A coffee shop sells lattes at \$4 and cappuccinos at \$5. On a day they sold 50 drinks total for \$215. Write the matrix equation for the system (let ℓ be lattes, c be cappuccinos). ______

Drink	Price (\$)
Latte	4
Cappuccino	5
Total drinks	50
Total revenue	215

10. Solve the lattes/cappuccinos system above ______

11. For what value of k does the system $\begin{cases} kx + 2y = 5 \\ 3x + y = 2 \end{cases}$ have no unique solution? ______

12. Solve $\begin{cases} 3x - 2y = 4 \\ x + y = 3 \end{cases}$ ______

13. Solve $\begin{cases} 5x + y = 12 \\ 2x - y = 2 \end{cases}$ ______

14. What is the dimension requirement on A for A^{-1} to even exist? ______

15. Solve $\begin{cases} x + y = 7 \\ x - y = 1 \end{cases}$ ______

16. True or false: if $\det(A) = 0$, the system $A\vec{x} = \vec{b}$ always has no solutions. ______

17. Solve $\begin{cases} x + 2y = 5 \\ 2x + 5y = 11 \end{cases}$ ______

18. A store sells notebooks (\$3) and binders (\$5). One order has 18 items totaling \$70. Find the number of notebooks n and binders b. ______

Item	Price (\$)
Notebook	3
Binder	5
Total items	18
Total cost	70

19. Solve $\begin{cases} 2x + y = 9 \\ x - 2y = -3 \end{cases}$ ______

20. Solve $\begin{cases} 3x + y = 8 \\ 6x + 2y = 15 \end{cases}$ ______

Word Problems

21. A pet shelter knows the total of cats and dogs is 45. Each cat eats 4 cups of food per day and each dog eats 7 cups, for a daily total of 237 cups. Set up and solve the matrix equation to find the number of cats and dogs. ____

22. A concert sells general-admission tickets at \$12 and reserved-seating tickets at \$20. The venue sold a total of 260 tickets and collected \$4080 in revenue. Use a matrix equation to find the number of each type of ticket. ____

23. A chemistry student mixes a 10% saline solution with a 40% saline solution to make 60 liters of a 25% saline solution. Set up and solve a matrix equation to find how many liters of each she needs. ____

24. Two roommates split a phone bill and an electric bill. This month they paid \$84 total, with the phone bill twice the electric bill. Use a matrix equation to find each bill amount. ____

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Additional Practice

25. Solve: $2x + y = 7$ and $x - y = 2$. ________

26. Solve: $x + y = 10$ and $x - y = 4$. ________

27. Solve: $3x - 2y = 4$ and $x + y = 6$. ________

28. Classify: $2x + 4y = 8$ and $x + 2y = 4$. ________

29. Classify: $y = 3x + 1$ and $y = 3x - 5$. ________

30. Solve: $4x + y = 1$ and $2x - y = 5$. ________

31. Eliminate y: $x + y = 8$ and $2x - y = 7$. ________

Answer Keys

1. $\begin{bmatrix}2 & 3\\1 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}8\\1\end{bmatrix}$
2. $\vec{x}=A^{-1}\vec{b}$
3. $(x,y)=(1,2)$
4. $(x,y)=(1,2)$
5. $(x,y)=(1,2)$
6. no unique solution
7. $(x,y)=(1,5)$
8. $(x,y)=(6,4)$
9. $\begin{bmatrix}1 & 1\\4 & 5\end{bmatrix}\begin{bmatrix}\ell\\c\end{bmatrix}=\begin{bmatrix}50\\215\end{bmatrix}$
10. $(\ell,c)=(35,15)$
11. $k=6$
12. $(x,y)=(2,1)$
13. $(x,y)=(2,2)$
14. A must be square
15. $(x,y)=(4,3)$
16. false
17. $(x,y)=(3,1)$
18. $(n,b)=(10,8)$
19. $(x,y)=(3,3)$
20. no solution (inconsistent)
21. 26 cats, 19 dogs
22. 140 general, 120 reserved
23. 30 L of 10%, 30 L of 40%
24. electric \$28, phone \$56

Additional Practice Answers

25. $(3,1)$
26. $(7,3)$
27. $(\frac{16}{5},\frac{14}{5})$
28. infinitely many
29. no solution
30. $(1,-3)$
31. $x=5$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Row 1 of A holds the coefficients of the first equation: $(2,3)$. Row 2 holds the second equation's coefficients: $(1,-1)$. The variable column is (x,y) and constants $(8,1)$. Order matters — swap and the system changes.

2. Left-multiply both sides by A^{-1}: $A^{-1}A\vec{x}=A^{-1}\vec{b}\Rightarrow I\vec{x}=A^{-1}\vec{b}$, giving $\vec{x}=A^{-1}\vec{b}$. The *left* side is essential — $\vec{b}A^{-1}$ is usually undefined and never correct.

3. One steady path is: $A=[1,2;3,4]$, $\det(A)=-2$, $A^{-1}=\frac{1}{-2}[4,-2;-3,1]$. Multiply: $A^{-1}\vec{b}=\frac{1}{-2}[(4)(5)+(-2)(11);(-3)(5)+(1)(11)]=\frac{1}{-2}[-2;-4]=[1;2]$. Verify: $1+4=5$ and $3+8=11$ ✓. That gives a quick check on the answer.

4. Start with the key idea: $A=[3,2;5,4]$, $\det=2$, $A^{-1}=\frac{1}{2}[4,-2;-5,3]$. Multiply: $A^{-1}[7;13]=\frac{1}{2}[28-26;-35+39]=[1;2]$. Verify: $3+4=7$, $5+8=13$ ✓. That gives a quick check on the answer.

5. Quick elimination: multiply equation 1 by 3, subtract equation 2: $6x+3y-(x+3y)=12-7$, so $5x=5$ and $x=1$. Then $2(1)+y=4$, giving $y=2$. (Matrix method works too: $\det=5$, $A^{-1}=\frac{1}{5}[3,-1;-1,2]$, then $\vec{x}=\frac{1}{5}[12-7;-4+14]=[1;2]$.)

6. Keep the rule visible: $\det(A)=(2)(6)-(4)(3)=0$. Equation 2 is exactly $1.5\times$ equation 1, so they're the same line — infinitely many solutions. Singular coefficient matrix means no unique solution. That gives a quick check on the answer.

7. Eliminate y: multiply equation 1 by 3 to get $12x+3y=27$, then subtract equation 2 ($2x+3y=17$): $10x=10$, so $x=1$. Back-substitute into equation 1: $4(1)+y=9$, giving $y=5$. Verify in equation 2: $2(1)+3(5)=17$ ✓.

8. Substitution: $y=10-x$, plug into the second: $2x+3(10-x)=24$, so $30-x=24$, giving $x=6$ and $y=4$. (Matrix method: $\det=1$, easy inverse.)

9. The count equation: $\ell+c=50$, giving coefficients $(1,1)$. The revenue equation: $4\ell+5c=215$, giving coefficients $(4,5)$. Stack into the matrix. (Always match the row order to the equation order.)

10. Keep the rule visible: $\det(A)=(1)(5)-(1)(4)=1$. $A^{-1}=[5,-1;-4,1]$. Multiply: $\vec{x}=[(5)(50)+(-1)(215);(-4)(50)+(1)(215)]=[35;15]$. So 35 lattes and 15 cappuccinos. Verify: $35+15=50$ and $4(35)+5(15)=140+75=215$ ✓. That gives a quick check on the answer.

11. Set $\det(A)=k(1)-(2)(3)=k-6=0$, so $k=6$. At $k=6$, rows $(6,2)$ and $(3,1)$ are proportional (factor 2) — coefficient matrix is singular.

12. Start with the key idea: $A=[3,-2;1,1]$, $\det=3-(-2)=5$. $A^{-1}=\frac{1}{5}[1,2;-1,3]$. Multiply by $\vec{b}=(4,3)$: $\frac{1}{5}[4+6;-4+9]=\frac{1}{5}[10;5]=[2;1]$. Verify: $6-2=4$, $2+1=3$ ✓. That gives a quick check on the answer.

13. Add the two equations to eliminate y: $7x=14$, so $x=2$. Then $5(2)+y=12$, giving $y=2$. (Matrix route works too: $\det=-7$, manageable.) Verify: $10+2=12$ and $4-2=2$ ✓.

14. Inverses only exist for square matrices. A 2×3 matrix has no inverse no matter what its entries are. (Square is necessary but not sufficient — you *also* need $\det\neq 0$.)

15. Add: $2x=8$, $x=4$. Then $4+y=7$, $y=3$. (Or via matrix: $A=[1,1;1,-1]$, $\det=-2$, $A^{-1}=\frac{1}{-2}[-1,-1;-1,1]$, then $\vec{x}=\frac{1}{-2}[-7-1;-7+1]=\frac{1}{-2}[-8;-6]=[4;3]$.) Verify: $4+3=7$ and $4-3=1$ ✓.

16. Start with the key idea: $\det(A)=0$ rules out a *unique* solution. The system might have no solutions (inconsistent) *or* infinitely many (consistent, dependent). Which case depends on whether $\vec{b}$ lies in the column space of A. That gives a quick check on the answer.

17. Multiply equation 1 by 2 then subtract equation 2: $(2x+4y)-(2x+5y)=10-11$, so $-y=-1$ and $y=1$. Then $x+2=5$ gives $x=3$. (Matrix: $\det=1$, easy.) Verify: $3+2=5$ ✓, $6+5=11$ ✓.

18. System: $n+b=18$ and $3n+5b=70$. Substitute $n=18-b$: $3(18-b)+5b=70\Rightarrow 54+2b=70\Rightarrow b=8$. Then $n=10$. Verify: $10+8=18$ and $30+40=70$ ✓.

19. From the second: $x=2y-3$. Plug into the first: $2(2y-3)+y=9\Rightarrow 5y=15\Rightarrow y=3$. Then $x=2(3)-3=3$. Verify: $6+3=9$ and $3-6=-3$ ✓.

20. Start with the key idea: $\det(A)=(3)(2)-(1)(6)=0$, so no unique solution. Equation 2 is $2\times$ equation 1 for the left-hand side ($6x+2y=2(3x+y)$), which should equal $2(8)=16$ — but it equals 15. Inconsistent: no solution. That gives a quick check on the answer.

21. Let c = cats and d = dogs. System: $c+d=45$ and $4c+7d=237$. Matrix form: $A=[1,1;4,7]$, $\vec{b}=[45;237]$. $\det(A)=7-4=3$, $A^{-1}=\frac{1}{3}[7,-1;-4,1]$. Multiply: $\vec{x}=A^{-1}\vec{b}=\frac{1}{3}[(7)(45)+(-1)(237);(-4)(45)+(1)(237)]=\frac{1}{3}[78;57]=[26;19]$. So 26 cats and 19 dogs. Verify: $26+19=45$ ✓and $4(26)+7(19)=104+133=237$ ✓.

22. Let g = general, r = reserved. System: $g+r=260$ and $12g+20r=4080$. Solve by substitution: $g=260-r$, plug in: $12(260-r)+20r=4080\Rightarrow 3120+8r=4080\Rightarrow 8r=960\Rightarrow r=120$. Then $g=140$. Verify: $140+120=260$ and $12(140)+20(120)=1680+2400=4080$ ✓.

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23. Let x = liters of 10% and y = liters of 40%. Volume: $x + y = 60$. Saline content (as decimals): $0.10x + 0.40y = 0.25(60) = 15$. Multiply that second equation by 10 to clear decimals: $x + 4y = 150$. Subtract the first: $3y = 90$, so $y = 30$ and $x = 30$. Both 30 L, a sensible $50/50$ mix to land on the midpoint concentration.

24. Let p = phone, e = electric. System: $p + e = 84$ and $p = 2e$. Substitute: $2e + e = 84 \Rightarrow 3e = 84 \Rightarrow e = 28$. Then $p = 56$. Verify: $56 + 28 = 84$ and $56 = 2(28)$ ✓. (Whole-dollar answers — the problem was designed that way.)

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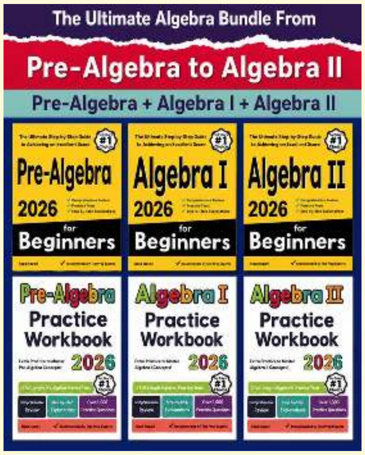

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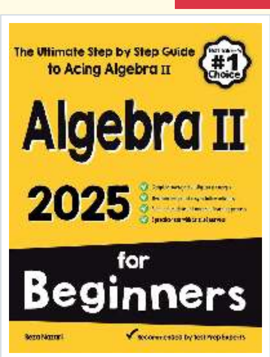

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