

Solving Rational Inequalities

Name: _____ Date: _____ Score: _____ / 29

Q Quick Review

A **rational inequality** compares a rational expression to zero (or to a number). The safe method is the same every time: *move everything to one side, factor, find critical points, sign-chart*.

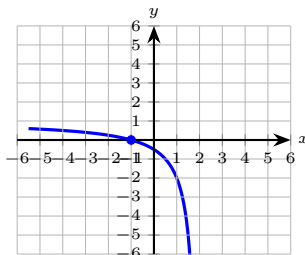
Why not just clear denominators? Multiplying both sides by an expression with unknown sign can flip the inequality. For numerical equations that's fine; for inequalities it's a disaster. Never multiply both sides by an expression that could be positive or negative.

The recipe. (1) Move every term to one side – get 0 on the other. (2) Combine into a single fraction $\frac{p(x)}{q(x)}$. (3) Find *critical points*: zeros of p (numerator) and zeros of q (denominator). (4) Mark them on a number line; test the sign in each interval. (5) Include numerator zeros for non-strict $\leq 0, \geq 0$; always exclude denominator zeros (undefined).

Sign chart shortcut. Pick one test point per interval and plug into the factored expression. The signs alternate at *simple zeros*; double roots don't flip the sign.

Interpreting endpoints. Solid dot \bullet at $x = a$ means a is included (numerator zero, non-strict inequality). Open circle \circ means excluded (denominator zero – always – or a strict inequality at a numerator zero).

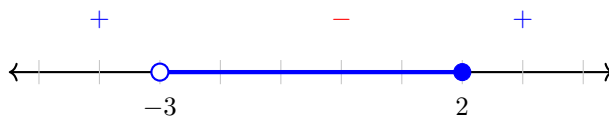
Common slips. Multiplying through by a denominator with unknown sign. Including a denominator zero in the solution (always wrong – the expression is undefined there). Mixing up strict and non-strict endpoints. **Graph check.** A rational inequality changes sign at zeros and at excluded values. A quick coordinate graph helps students see the vertical break before they shade intervals.



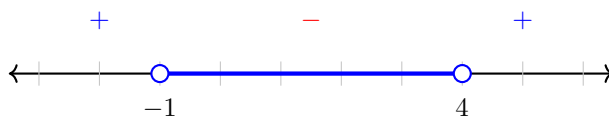
PRACTICE

Move everything to one side, combine, find critical points, sign-chart, and write the solution in interval notation.

- For $\frac{p(x)}{q(x)} \leq 0$, the critical points are zeros of: _____
- Solve $\frac{1}{x} > 0$. _____
- Solve $\frac{x-2}{x+3} \leq 0$. The sign chart below organizes the analysis. _____

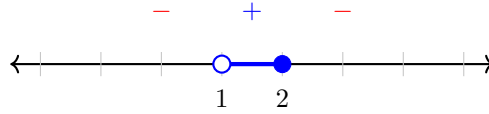


- Solve $\frac{x+1}{x-4} < 0$. Use the sign chart to read the interval. _____



5. Solve $\frac{x-5}{x+2} \geq 0$. _____

6. Solve $\frac{x}{x-1} \geq 2$. _____

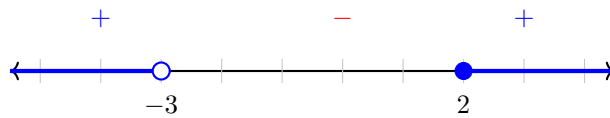


7. Solve $\frac{(x-1)(x+4)}{x-3} > 0$ and write in interval notation. _____

8. Solve $\frac{x+2}{x-5} < 1$. _____

9. Solve $\frac{x-4}{x+1} \leq 2$ and write in interval notation. _____

10. Solve $\frac{x-2}{x+3} > 0$. The sign chart confirms. _____



11. Solve $\frac{3}{x-1} \leq 2$. _____

12. Solve $\frac{x+1}{x^2-9} < 0$. _____

13. Mark TRUE or FALSE: To solve $\frac{p(x)}{q(x)} < 0$, you may multiply both sides by $q(x)$. _____

14. Solve $\frac{1}{x-2} < 3$. _____

15. Solve $\frac{2x+1}{x-3} \geq 0$. _____

16. Solve $\frac{x^2-4}{x+3} > 0$. _____

17. Solve $\frac{x-1}{x+1} \geq \frac{1}{2}$. _____

18. Solve $\frac{x}{(x-2)^2} > 0$. _____

19. Solve $\frac{(x-1)^2}{x+3} \leq 0$. _____

20. Solve $\frac{2}{x+1} > \frac{1}{x-1}$. _____



◆ Word Problems

21. A profit function is $P(x) = \frac{200x - 2000}{x + 10}$ thousand dollars, where x is units produced. For which $x \geq 0$ is the profit nonnegative? Show the sign-chart reasoning. _____

22. A medicine's concentration in the blood is $C(t) = \frac{4t}{t^2 + 4}$ mg/L, with t in hours after a dose. Find all times $t \geq 0$ for which $C(t) \geq 0.8$ mg/L. _____

23. For what real values of x is the rate $r(x) = \frac{x + 5}{x - 2}$ between 1 and 3? Solve the compound inequality $1 < \frac{x + 5}{x - 2} < 3$ and give the answer in interval notation. _____

24. A student claims the solution to $\frac{3}{x - 1} \leq 2$ is $\frac{5}{2} \leq x$ because they multiplied both sides by $x - 1$. Find the correct solution using a sign chart, and explain why the student's move was illegal. _____

Additional Practice

25. Simplify $\frac{x^2 - 9}{x - 3}$. _____

26. Excluded value of $\frac{1}{x + 4}$. _____

27. Domain of $f(x) = \frac{x}{x - 5}$. _____

28. Multiply $\frac{x}{3} \cdot \frac{6}{x}$. _____

29. Divide $\frac{x^2}{5} \div \frac{x}{10}$. _____



Answer Keys

<p>1. both $p(x)$ and $q(x)$</p> <p>2. $x > 0$</p> <p>3. $(-3, 2]$</p> <p>4. $-1 < x < 4$</p> <p>5. $x < -2$ or $x \geq 5$</p> <p>6. $1 < x \leq 2$</p> <p>7. $(-4, 1) \cup (3, \infty)$</p> <p>8. $x < 5$</p> <p>9. $(-\infty, -6] \cup (-1, \infty)$</p> <p>10. $(-\infty, -3) \cup (2, \infty)$</p> <p>11. $x < 1$ or $x \geq \frac{5}{2}$</p> <p>12. $(-\infty, -3) \cup (-1, 3)$</p> <p>13. FALSE</p> <p>Additional Practice Answers</p> <p>25. $x + 3, x \neq 3$</p> <p>26. $x = -4$</p> <p>27. $x \neq 5$</p>	<p>14. $x < 2$ or $x > \frac{7}{3}$</p> <p>15. $x \leq -\frac{1}{2}$ or $x > 3$</p> <p>16. $(-3, -2) \cup (2, \infty)$</p> <p>17. $x < -1$ or $x \geq 3$</p> <p>18. $x > 0, x \neq 2$</p> <p>19. $x < -3$ or $x = 1$</p> <p>20. $(-1, 1) \cup (3, \infty)$</p> <p>21. $x \geq 10$</p> <p>22. $1 \leq t \leq 4$ hours</p> <p>23. $x > \frac{11}{2}$</p> <p>24. $(-\infty, 1) \cup \left[\frac{5}{2}, \infty\right)$</p> <p>28. 2</p> <p>29. $2x$</p>
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Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Zeros of p are where the expression might equal 0; zeros of q are where it's undefined. Both kinds split the number line into test intervals.

2. Keep the rule visible: $\frac{1}{x}$ is positive iff x is positive. $x = 0$ is excluded (undefined). That gives a quick check on the answer.

3. Critical points $x = -3$ (denom zero, open) and $x = 2$ (num zero, closed for \leq). Sign chart: $+$ $-$ $+$. Negative interval is $(-3, 2]$.

4. Critical points -1 and 4 . Sign chart: $+$ $-$ $+$. Strict < 0 : open at both endpoints. Solution: $(-1, 4)$.

5. Critical points 5 (closed, \geq) and -2 (open, denom). Sign chart: $+$ $-$ $+$. Solution: $(-\infty, -2) \cup [5, \infty)$.

6. Move to one side: $\frac{x}{x-1} - 2 = \frac{x-2(x-1)}{x-1} = \frac{-x+2}{x-1} = \frac{2-x}{x-1} \geq 0$. Critical points 1 (open) and 2 (closed). Sign chart confirms positive only on $(1, 2]$.

7. Critical points $-4, 1, 3$. Sign chart with three factors: at $x < -4$: $(-)(-)/(-) = -$. At $-4 < x < 1$: $(-)(+)/(-) = +$. At $1 < x < 3$: $(+)(+)/(-) = -$. At $x > 3$: $(+)(+)/(+)$ = $+$. Strict > 0 : $(-4, 1) \cup (3, \infty)$.

8. Move: $\frac{x+2}{x-5} - 1 = \frac{x+2-(x-5)}{x-5} = \frac{7}{x-5} < 0$. Numerator $7 > 0$ always; quotient negative iff $x-5 < 0$, i.e., $x < 5$. ($x = 5$ excluded.)

9. Move: $\frac{x-4}{x+1} - 2 = \frac{x-4-2(x+1)}{x+1} = \frac{-x-6}{x+1} \leq 0$. Multiply by -1 (flip): $\frac{x+6}{x+1} \geq 0$. Critical points -6 (closed) and -1 (open). Sign chart: $+$ $-$ $+$.

10. Same critical points as item 3, but the inequality is strict > 0 : positive intervals, both endpoints excluded.

11. Move: $\frac{3}{x-1} - 2 = \frac{3-2(x-1)}{x-1} = \frac{5-2x}{x-1} \leq 0$. Critical points $\frac{5}{2}$ (num zero, closed) and 1 (denom, open). Sign chart: $-$ $+$ $-$. Solution: $(-\infty, 1) \cup [\frac{5}{2}, \infty)$.

12. Start with the key idea: $x^2 - 9 = (x-3)(x+3)$. Critical points $-3, -1, 3$. Sign chart: at $x < -3$: $(-)/(+) = -$. At $-3 < x < -1$: $(-)/(-) = +$. At $-1 < x < 3$: $(+)(-)$ = $-$. At $x > 3$: $(+)(+)$ = $+$. Strict < 0 : $(-\infty, -3) \cup (-1, 3)$. That gives a quick check on the answer.

13. Only if you know the sign of $q(x)$ on the interval – otherwise the inequality direction could flip. The safe method is to move terms to one side and sign-chart.

14. Move: $\frac{1-3(x-2)}{x-2} = \frac{7-3x}{x-2} < 0$. Critical points $\frac{7}{3}$ (open, strict) and 2 (open, denom). Sign chart: $-$ $+$ $-$. Solution: $(-\infty, 2) \cup (\frac{7}{3}, \infty)$.

15. Critical points $-\frac{1}{2}$ (closed) and 3 (open). Sign chart: $+$ $-$ $+$. Solution: $(-\infty, -\frac{1}{2}] \cup (3, \infty)$.

16. Factor: $\frac{x+3}{(x-2)(x+2)} > 0$. Critical points $-3, -2, 2$. Sign chart: $-$, $+$, $-$, $+$. Strict > 0 : $(-3, -2) \cup (2, \infty)$.

17. Move: $\frac{x-1}{x+1} - \frac{1}{2} = \frac{2(x-1)-(x+1)}{2(x+1)} = \frac{x-3}{2(x+1)} \geq 0$. Critical points 3 (closed) and -1 (open). Sign chart: $+$ $-$ $+$. Solution: $(-\infty, -1) \cup [3, \infty)$.

18. Keep the rule visible: $(x-2)^2 \geq 0$ always; equals zero at $x = 2$. So the sign of the quotient matches the sign of x . Solution: $x > 0$ but $x \neq 2$ (denominator). In interval notation: $(0, 2) \cup (2, \infty)$. That gives a quick check on the answer.

19. One steady path is: $(x-1)^2 \geq 0$ always; equals zero at $x = 1$. Quotient ≤ 0 iff numerator is 0 (at $x = 1$, valid) or denominator is negative ($x < -3$). Solution: $(-\infty, -3) \cup \{1\}$. That gives a quick check on the answer.

20. Move to one side: $\frac{2(x-1)-(x+1)}{(x+1)(x-1)} = \frac{x-3}{(x+1)(x-1)} > 0$. Critical points $-1, 1, 3$ (all open: $-1, 1$ are denom zeros; 3 is the strict-inequality numerator zero). Sign chart by test points – at $x = -2$: $\frac{-5}{(-)(-)} = \frac{-5}{+} < 0$. At

$x = 0$: $\frac{-3}{(+)(-)} = \frac{-3}{-} > 0$. At $x = 2$: $\frac{-1}{(+)(+)} < 0$. At $x = 4$: $\frac{1}{(+)(+)} > 0$.

Positive intervals: $(-1, 1) \cup (3, \infty)$.

21. Solve $\frac{200x-2000}{x+10} \geq 0$. Numerator zero at $x = 10$; denominator zero at $x = -10$. Critical points -10 (excluded – denominator) and 10 (included – numerator zero, \geq). Sign chart over $x \geq 0$: at $0 \leq x < 10$, $\frac{\text{neg}}{\text{pos}} < 0$. At $x > 10$, $\frac{\text{pos}}{\text{pos}} > 0$. So $P(x) \geq 0$ iff $x \geq 10$. In context: produce at least 10 units to break even.

22. Solve $\frac{4t}{t^2+4} \geq 0.8$, i.e., $\frac{4t}{t^2+4} - \frac{4}{5} \geq 0$. Combine: $\frac{20t-4(t^2+4)}{5(t^2+4)} =$



$$\frac{-4t^2 + 20t - 16}{5(t^2 + 4)} = \frac{-4(t^2 - 5t + 4)}{5(t^2 + 4)} = \frac{-4(t-1)(t-4)}{5(t^2 + 4)} \geq 0.$$

Denominator $t^2 + 4 > 0$ for all real t . Need $-4(t-1)(t-4) \geq 0$, i.e., $(t-1)(t-4) \leq 0$. Critical points 1 and 4, both included. Solution on $t \geq 0$: $1 \leq t \leq 4$ hours. Outside that window, the concentration is below the 0.8 mg/L threshold.

23. Solve in two parts. **Part 1:** $\frac{x+5}{x-2} > 1$. $\frac{x+5-(x-2)}{x-2} = \frac{7}{x-2} > 0$, so

$x > 2$. **Part 2:** $\frac{x+5}{x-2} < 3$. $\frac{x+5-3(x-2)}{x-2} = \frac{-2x+11}{x-2} < 0$. Multiply by -1 (flip): $\frac{2x-11}{x-2} > 0$. Critical points $\frac{11}{2}$ and 2. Sign chart: $+ - +$. Positive

intervals: $(-\infty, 2) \cup (\frac{11}{2}, \infty)$. **Intersect with** $x > 2$: $x > \frac{11}{2}$. Final answer:

$x > \frac{11}{2}$, or $(\frac{11}{2}, \infty)$. (Spot check at $x = 10$: $\frac{15}{8} = 1.875$, between 1 and 3.

✓.)

24. The student's error: multiplying by $x-1$ without knowing its sign. When $x-1 < 0$, the inequality direction flips – but the student assumed it stayed the

same. **Correct method:** move to one side. $\frac{3}{x-1} - 2 = \frac{3-2(x-1)}{x-1} =$

$\frac{5-2x}{x-1} \leq 0$. Critical points $x = \frac{5}{2}$ (numer zero, closed) and $x = 1$ (denom,

open). Sign chart: at $x < 1$, $\frac{+}{-} < 0$ ✓. At $1 < x < \frac{5}{2}$, $\frac{+}{+} > 0$ ✗. At $x > \frac{5}{2}$,

$\frac{-}{+} < 0$ ✓. Solution: $(-\infty, 1) \cup [\frac{5}{2}, \infty)$. The student missed the entire $x < 1$

branch. **Spot check:** at $x = 0$, $\frac{3}{-1} = -3 \leq 2$ ✓ – included in the correct answer but not the student's.



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