Solving Radical Inequalities

Name: ______________________	Date: ______________	Score: _____ / 48

Quick Review

Radical inequalities are radical equations with one extra layer of bookkeeping: the **domain restriction is part of the answer**, not a hidden side condition.

Three things to track. (1) The domain — for square roots, the radicand must be ≥ 0. (2) The sign of the side opposite the radical. The square root is always ≥ 0, so an inequality like $\sqrt{R(x)} < -3$ has *no solution*, while $\sqrt{R(x)} \geq -7$ holds throughout the domain. (3) The squared inequality itself, valid only when both sides are non-negative.

Why squaring needs non-negative sides. The step "square both sides" preserves direction *only* when both sides start out non-negative. With $\sqrt{R} < k$ where $k \geq 0$, squaring gives $R < k^2$ on the same side. With $\sqrt{R} < k$ where $k < 0$, the radical is non-negative and k is negative — so the inequality *has no solution at all*, without squaring.

Standard recipe for $\sqrt{R(x)} \leq k$ (with $k \geq 0$). Domain: $R(x) \geq 0$. Square: $R(x) \leq k^2$. Intersect: the final answer is the set of x in *both*.

Standard recipe for $\sqrt{R(x)} \geq k$. If $k \leq 0$: the inequality holds on the whole domain $R(x) \geq 0$ (any non-negative number is $\geq$ a non-positive one). If $k > 0$: domain plus $R(x) \geq k^2$ — and since $k^2 \geq 0$, the second condition already covers the domain, so just $R(x) \geq k^2$.

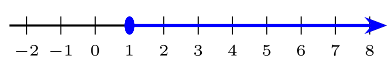
When the right side has a variable. The square root is ≥ 0, so the right side has to be ≥ 0 too for the inequality $\sqrt{R(x)} \leq$ stuff(x) to have any chance. That gives an extra constraint to combine.

Common slips. Dropping the domain from the final answer. Squaring when one side might be negative. Forgetting that $\sqrt{R(x)} \geq 0$ is automatic — comparing it to a negative number is either always true or always false.

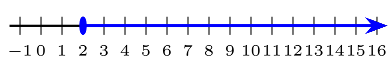
PRACTICE

Track the domain and the sign of the other side; square only when both sides are non-negative.

1. For $\sqrt{x-3} \leq 5$, state the domain restriction the radicand requires. ________
2. Solve $\sqrt{x} \leq 4$. ________
3. For $\sqrt{x-1} > 2$, the number line below shows the *domain* of the radical (where $x - 1 \geq 0$). Use it as a starting point, then solve the inequality. ________



4. Solve $\sqrt{2x+3} < 5$. ________
5. Solve $\sqrt{x+4} \geq 0$. ________
6. Solve $\sqrt{x-2} < -3$. ________
7. For $\sqrt{3x-6} \leq 6$, the number line shows the domain of the radical. Use it, then solve and give the answer in interval notation. ________



8. Mark TRUE or FALSE: Squaring both sides of a radical inequality always preserves the inequality direction. ________
9. Solve $\sqrt{x+2} \leq x$. ________
10. Solve $\sqrt{10-2x} \geq 4$ and write the answer in interval notation. ________
11. Solve $\sqrt{x} > -2$. ________
12. Solve $\sqrt{x+5} \geq 3$. ________
13. Solve $\sqrt{2x+1} < 7$. ________
14. Solve $\sqrt{x-1} \geq -4$. ________

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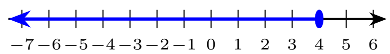
15. For $\sqrt{4-x} < 3$, the number line shows the domain of the radical (where $4-x \geq 0$). Use it, then finish solving. ________



16. Mark TRUE or FALSE: $\sqrt{R(x)} < k$ has no solution when $k < 0$. ________

17. Solve $\sqrt{2x-3} \leq 5$. ________

18. Solve $\sqrt{x+7} > x-1$. ________

19. Solve $\sqrt{x} \geq x-2$. ________

20. Solve $\sqrt{3x+1} < 2$. ________

♦ Word Problems

21. A free-fall time from height h (feet) is $t(h) = \sqrt{h/16}$ seconds. For what heights is the fall at most 3 seconds? ____

22. A safety guideline says: for skid speed at most 50 mph in a curve of radius r (feet), $v = \sqrt{2.5r} \leq 50$. Find all radii that satisfy the guideline. Show the domain restriction explicitly. ____

23. A function $f(x) = \sqrt{x-4}$ models a quantity that must stay below 7 for safety. Find all x values in the model's domain that satisfy $f(x) < 7$. State the answer in interval notation. ____

24. Solve $\sqrt{2x+1} \geq x-3$ and write the answer in interval notation. Show both the domain step and the sign-of-right-side step explicitly. ____

Additional Practice

25. Simplify $\sqrt{72}$. ________

26. Simplify $\sqrt{45}$. ________

27. Simplify $\sqrt[3]{64}$. ________

28. Solve $\sqrt{x+5} = 9$. ________

29. Solve $\sqrt{x} - 3 = 4$. ________

30. Domain of $y = \sqrt{x-6}$. ________

31. Add $3\sqrt{5} + 2\sqrt{5}$. ________

32. Multiply $\sqrt{3} \cdot \sqrt{12}$. ________

33. Rationalize $\frac{4}{\sqrt{2}}$. ________

34. Write $x^{3/2}$ using radicals. ________

35. Simplify $(\sqrt{7})^2$. ________

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36. Solve $\sqrt{x+1} < 4$. ________

37. Simplify $\sqrt{18} + \sqrt{8}$. ________

38. Domain of $y = -\sqrt{x+2} + 1$. ________

39. Is $\sqrt{-9}$ real? ________

40. Simplify $\sqrt{9x^2}$ for $x \geq 0$. ________

41. Simplify $\sqrt{98}$. ________

42. Solve $\sqrt{x+9} = 5$. ________

43. Domain of $y = \sqrt{2x - 10}$. ________

44. Simplify $\sqrt[3]{54}$. ________

45. Rewrite $x^{5/2}$ using radicals. ________

46. Solve $\sqrt{x-4} < 3$. ________

47. Simplify $2\sqrt{12} - \sqrt{27}$. ________

48. Rationalize $\frac{5}{\sqrt{3}}$. ________

Answer Keys

1. $\boxed{x \ge 3}$
2. $\boxed{0 \le x \le 16}$
3. $\boxed{x > 5}$
4. $\boxed{-\frac{3}{2} \le x < 11}$
5. $\boxed{x \ge -4}$
6. no solution
7. $\boxed{[2, 14]}$
8. FALSE
9. $\boxed{x \ge 2}$
10. $\boxed{(-\infty, -3]}$
11. $\boxed{x \ge 0}$
12. $\boxed{x \ge 4}$
13. $\boxed{-\frac{1}{2} \le x < 24}$
14. $\boxed{x \ge 1}$
15. $\boxed{-5 < x \le 4}$
16. TRUE
17. $\boxed{\frac{3}{2} \le x \le 14}$
18. $\boxed{-7 \le x < \frac{3+\sqrt{33}}{2}}$
19. $\boxed{0 \le x \le 4}$
20. $\boxed{-\frac{1}{3} \le x < 1}$
21. $\boxed{0 \le h \le 144 \text{ ft}}$
22. $\boxed{0 \le r \le 1000 \text{ ft}}$
23. $\boxed{[4, 53)}$
24. $\boxed{\left[-\frac{1}{2},\ 4+2\sqrt{2}\right]}$

Additional Practice Answers

25. $\boxed{6\sqrt{2}}$
26. $\boxed{3\sqrt{5}}$
27. $\boxed{4}$
28. $\boxed{x = 76}$
29. $\boxed{x = 49}$
30. $\boxed{x \ge 6}$
31. $\boxed{5\sqrt{5}}$
32. $\boxed{6}$
33. $\boxed{2\sqrt{2}}$
34. $\boxed{\sqrt{x^3}}$
35. $\boxed{7}$
36. $\boxed{-1 \le x < 15}$
37. $\boxed{5\sqrt{2}}$
38. $\boxed{x \ge -2}$
39. no
40. $\boxed{3x}$
41. $\boxed{7\sqrt{2}}$
42. $\boxed{x = 16}$
43. $\boxed{x \ge 5}$
44. $\boxed{3\sqrt[3]{2}}$
45. $\boxed{x^2\sqrt{x}}$
46. $\boxed{4 \le x < 13}$
47. $\boxed{\sqrt{3}}$
48. $\boxed{\frac{5\sqrt{3}}{3}}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

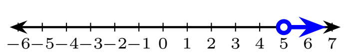
Step-by-Step Explanations

1. For $\sqrt{x-3}$ to be real, the radicand can't be negative, so $x - 3 \ge 0$, giving $x \ge 3$. With inequalities this domain isn't a side note — it gets intersected with the squared result to form the final answer.

2. Domain: $x \ge 0$. Square (both sides non-negative): $x \le 16$. Intersect: $[0, 16]$. (Forgetting the floor $x \ge 0$ is the most common slip on this one.)

3. The shaded ray marks the domain $x \ge 1$ — a prerequisite, not the answer. Square: $x - 1 > 4$, so $x > 5$. That squared condition already lives inside the domain, so the final solution is $(5, \infty)$ — a strictly smaller set than the domain shown.

Answer graph



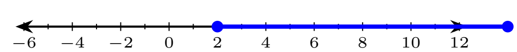
4. First the domain: $2x + 3 \ge 0$ gives $x \ge -\frac{3}{2}$. The right side 5 is positive, so squaring is safe: $2x + 3 < 25$, so $x < 11$. Intersect the two conditions for $-\frac{3}{2} \le x < 11$ — the domain supplies the lower bound the squaring step alone would miss.

5. The principal square root is always ≥ 0 wherever it's defined. So the inequality holds exactly on the domain $x + 4 \ge 0$, i.e., $x \ge -4$.

6. The principal square root is always ≥ 0 wherever defined, so it can never be less than -3. No solution. (Spot this kind of inequality before squaring — squaring it would falsely produce something solvable.)

7. The ray marks the domain $x \ge 2$ (from $3x - 6 \ge 0$). Square: $3x - 6 \le 36$, so $x \le 14$. Intersect the domain with $x \le 14$: $[2, 14]$. The upper cap 14 doesn't show on the domain ray — it comes from the squaring step.

Answer graph



8. Squaring preserves direction only when both sides are non-negative. Otherwise you must handle the sign of the non-radical side first.

9. Domain: $x \ge -2$. The right side must be ≥ 0, so $x \ge 0$. Square: $x + 2 \le x^2$, so $x^2 - x - 2 \ge 0$, giving $(x-2)(x+1) \ge 0$, i.e., $x \le -1$ or $x \ge 2$. Intersect with $x \ge 0$: $x \ge 2$.

10. Domain: $10 - 2x \ge 0$, so $x \le 5$. Square: $10 - 2x \ge 16$, so $-2x \ge 6$ and $x \le -3$ (flip on dividing by -2). This is already inside the domain, so the answer is $(-\infty, -3]$.

11. One steady path is: $\sqrt{x} \ge 0 > -2$ wherever it's defined, so the inequality holds on the whole domain $x \ge 0$. That gives a quick check on the answer.

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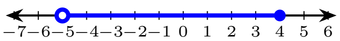
12. Domain: $x+5 \geq 0$, so $x \geq -5$. The right side 3 is positive, so square both sides: $x+5 \geq 9$, giving $x \geq 4$. Since $4 > -5$, this already sits inside the domain, so the answer is simply $x \geq 4$.

13. Domain: $2x+1 \geq 0$, so $x \geq -\frac{1}{2}$. Right side $7 > 0$, so square: $2x+1 < 49$, giving $x < 24$. Intersect the two for $-\frac{1}{2} \leq x < 24$, i.e. $\left[-\frac{1}{2}, 24\right)$.

14. The radical is $\geq 0 \geq -4$ on its entire domain, so the inequality holds whenever the radical is defined: $x \geq 1$.

15. The leftward ray marks the domain $x \leq 4$. Square: $4-x < 9$, so $-x < 5$ and $x > -5$ (flip on dividing by -1). Intersect with the domain: $-5 < x \leq 4$. The lower bound -5 is open and isn't part of the domain ray — it comes from squaring.

Answer graph



16. The principal square root is non-negative, so it can never be less than any negative number. No solution.

17. Domain: $2x-3 \geq 0$, so $x \geq \frac{3}{2}$. Right side $5 \geq 0$, so squaring is valid: $2x-3 \leq 25$, giving $x \leq 14$. Intersect both conditions: $\frac{3}{2} \leq x \leq 14$, or $\left[\frac{3}{2}, 14\right]$.

18. Domain: $x \geq -7$. Case 1: $x-1 < 0$ (i.e., $x < 1$) — the radical is ≥ 0, so the inequality holds automatically. Combined with the domain: $-7 \leq x < 1$. Case 2: $x \geq 1$ — both sides are ≥ 0, so square: $x+7 > (x-1)^2 = x^2-2x+1$, giving $x^2-3x-6 < 0$. Quadratic formula: roots $\frac{3 \pm \sqrt{9+24}}{2} = \frac{3 \pm \sqrt{33}}{2} \approx -1.37, 4.37$. Since the parabola opens up, $x^2-3x-6 < 0$ between the roots. Intersect with $x \geq 1$: $1 \leq x < \frac{3+\sqrt{33}}{2}$. Combine the two cases: $-7 \leq x < \frac{3+\sqrt{33}}{2} \approx 4.37$.

19. Domain: $x \geq 0$. Case 1: $x-2 < 0$ (i.e., $0 \leq x < 2$) — radical $\geq 0 >$ negative, holds. Case 2: $x \geq 2$ — both non-negative, square: $x \geq x^2-4x+4$, so $x^2-5x+4 \leq 0$, giving $(x-1)(x-4) \leq 0$, i.e., $1 \leq x \leq 4$. Intersect with $x \geq 2$: $2 \leq x \leq 4$. Combine: $0 \leq x \leq 4$.

20. Domain: $3x+1 \geq 0$, so $x \geq -\frac{1}{3}$. Right side $2 > 0$, so square: $3x+1 < 4$, giving $x < 1$. Intersect the two for $-\frac{1}{3} \leq x < 1$, i.e. $\left[-\frac{1}{3}, 1\right)$.

21. Set $\sqrt{h/16} \leq 3$. Domain: $h \geq 0$. Square: $h/16 \leq 9$, so $h \leq 144$. Combined: $0 \leq h \leq 144$ ft. **Check the endpoint:** at $h = 144$, $t = \sqrt{9} = 3$ ✓. (Anything taller than 144 ft would take more than 3 seconds to fall.)

22. Domain: $2.5r \geq 0$, so $r \geq 0$. Square: $2.5r \leq 2500$, so $r \leq 1000$. Combined: $0 \leq r \leq 1000$ ft. **Check the cap:** at $r = 1000$, $v = \sqrt{2500} = 50$ ✓. (A radius beyond 1000 ft would invite higher safe speeds; this guideline marks the 50-mph design ceiling.)

23. Domain: $x-4 \geq 0$, so $x \geq 4$. Right side $7 > 0$, square: $x-4 < 49$, so $x < 53$. Intersect: $[4, 53)$. **Check an interior point:** at $x = 20$, $f(20) = \sqrt{16} = 4 < 7$ ✓. **Check the boundary:** at $x = 53$, $f = \sqrt{49} = 7$, not strictly less — so 53 is excluded (open bracket), as the inequality is strict.

24. Domain: $2x+1 \geq 0$, so $x \geq -\frac{1}{2}$. **Case 1:** $x-3 < 0$ (i.e., $x < 3$) — the radical is $\geq 0 >$ the negative right side, so the inequality holds. Combined with domain: $-\frac{1}{2} \leq x < 3$. **Case 2:** $x \geq 3$ — both sides non-negative, square: $2x+1 \geq (x-3)^2 = x^2-6x+9$, so $x^2-8x+8 \leq 0$. Roots: $\frac{8 \pm \sqrt{64-32}}{2} = 4 \pm 2\sqrt{2}$ (approximately 1.17 and 6.83). Inequality holds on $[4-2\sqrt{2}, 4+2\sqrt{2}]$. Intersect with $x \geq 3$: $3 \leq x \leq 4+2\sqrt{2} \approx 6.83$. **Combine** Case 1 and Case 2: $\left[-\frac{1}{2}, 4+2\sqrt{2}\right]$. **Check upper bound $x = 4+2\sqrt{2}$:** left $= \sqrt{2(4+2\sqrt{2})+1} = \sqrt{9+4\sqrt{2}}$; right $= 4+2\sqrt{2}-3 = 1+2\sqrt{2}$. Squaring the right gives $(1+2\sqrt{2})^2 = 1+4\sqrt{2}+8 = 9+4\sqrt{2}$, which matches the radicand — equality at the boundary ✓.

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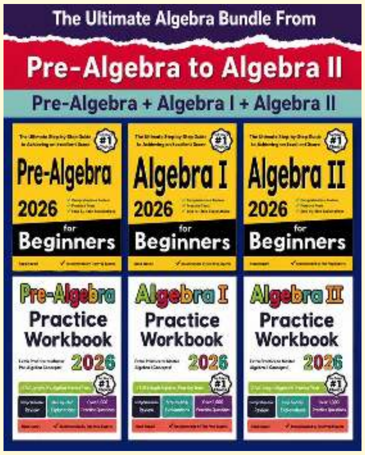

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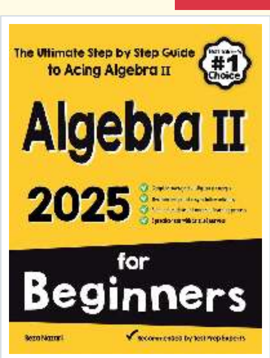

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