Solving Quadratic Inequalities

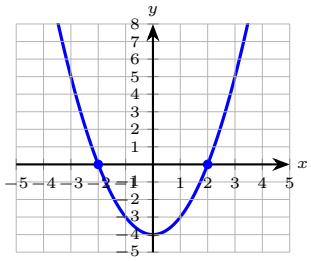
Name: _____	Date: _____	Score: _____ / 31

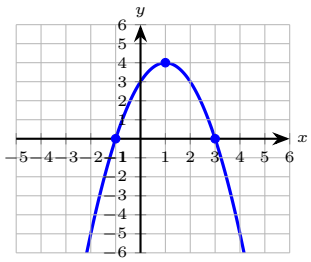
Quick Review

A **quadratic inequality** like $ax^2 + bx + c < 0$ asks where the parabola sits below (or above) the x-axis. Three steps work every time. (1) Move everything to one side so the right side is 0. (2) Solve the related *equation* $ax^2 + bx + c = 0$ to find the boundary values (the x-intercepts of the parabola). (3) Use the parabola's direction to pick the correct region.

For a parabola opening **up** ($a > 0$): the expression is *negative between the roots* and *positive outside*. For one opening **down** ($a < 0$): negative outside and positive between. Endpoints come along for the ride only when the inequality is non-strict ($\leq, \geq$); strict inequalities ($<, >$) exclude them.

Three traps. (a) When multiplying both sides by a negative, flip the inequality — same rule as with linear ones. (b) If the parabola never touches the x-axis (negative discriminant), the expression is one sign everywhere, so the solution is either all reals or empty. (c) Interval notation: open dot = parenthesis, closed dot = bracket; infinity always uses a parenthesis.**Graph check.** For a quadratic inequality, the zeros split the number line into intervals. A quick graph shows where the parabola is above or below the x-axis.





PRACTICE

Solve each inequality. Give the solution in interval form when asked.

1. Solve $x^2 - 4 < 0$ and show the interval on the number line. _____

2. Solve $x^2 \geq 9$ and show the two-ray solution. _____

3. $x^2 - 5x + 6 < 0$ _____

4. $x^2 + 6x + 8 \geq 0$ _____

5. $-x^2 + 4 \geq 0$ _____

6. $x^2 - x - 6 > 0$ _____

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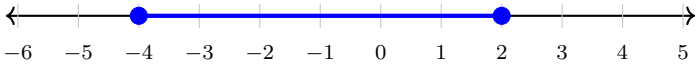
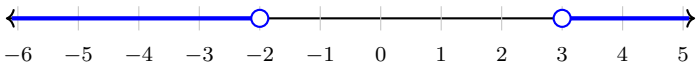
7. Solve $x^2 + 2x \leq 8$. Sketch the solution on the number line. _____



8. Solve $(x + 2)(x - 3) > 0$. Sketch the solution set. _____



9. $-t^2 + 6t - 5 \geq 0$ _____

10. $2x^2 - 7x - 4 \leq 0$ _____

11. $x^2 + 1 > 0$ _____

12. $x^2 + 4 < 0$ _____

13. $x^2 - 2x - 3 \geq 0$ _____

14. $3x^2 - 12 < 0$ _____

15. $(x - 1)^2 \leq 9$ _____

16. $x^2 \leq 0$ _____

17. $x^2 > 0$ _____

18. $x^2 + 5x \leq 0$ _____

19. $-2x^2 + 8 > 0$ _____

20. $x^2 - 6x + 8 < 0$ _____

♦ Word Problems

21. A model rocket's height (in meters) is $h(t) = -5t^2 + 30t$, where t is seconds. During what time interval is the rocket at least 25 meters above the ground? ____

22. A factory's daily profit (in dollars) is $P(x) = -x^2 + 40x - 300$, where x is the number of widgets produced. For what number of widgets is the profit positive? ____

23. A rectangular pen with one side along a barn uses 40 feet of fence for the other three sides. If the pen has width w, the area is $A(w) = w(40 - 2w)$. For what widths is the area at least 150 ft^2? ____

24. A diver's path follows $y(t) = -16t^2 + 12t + 4$ feet, with t in seconds. During what time interval is the diver above the water (i.e., $y > 0$)? ____

Additional Practice

25. Solve $x^2 - 5x + 6 = 0$. _____

26. Solve $x^2 = 49$. _____

27. Find the vertex of $y = (x - 3)^2 - 4$. _____

28. Find the axis of symmetry of $y = x^2 + 6x + 1$. _____

29. Factor $x^2 + 7x + 10$. _____

30. Find the discriminant of $x^2 - 4x + 8 = 0$. _____

31. Solve $2x^2 - 8 = 0$. _____

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Answer Keys

1. $-2 < x < 2$
2. $x \leq -3$ or $x \geq 3$
3. $2 < x < 3$
4. $x \leq -4$ or $x \geq -2$
5. $-2 \leq x \leq 2$
6. $x < -2$ or $x > 3$
7. $[-4, 2]$
8. $x < -2$ or $x > 3$
9. $1 \leq t \leq 5$
10. $\left[-\frac{1}{2},\ 4\right]$
11. all real numbers
12. $\varnothing$
13. $x \leq -1$ or $x \geq 3$
14. $-2 < x < 2$
15. $-2 \leq x \leq 4$
16. $x = 0$
17. $x \neq 0$
18. $-5 \leq x \leq 0$
19. $-2 < x < 2$
20. $2 < x < 4$
21. $1 \leq t \leq 5$ seconds
22. $10 < x < 30$
23. $5 \leq w \leq 15$ feet
24. $0 \leq t < 1$ second

Additional Practice Answers

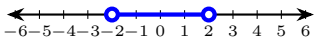
25. $x = 2,\ 3$
26. $x = -7,\ 7$
27. $(3, -4)$
28. $x = -3$
29. $(x + 5)(x + 2)$
30. -16
31. $x = -2,\ 2$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Factor: $(x-2)(x+2) < 0$. Roots ± 2. Parabola opens up $\Rightarrow$ negative between the roots. Strict, so no endpoints.

Answer graph



2. Rewrite: $x^2 - 9 \geq 0$. Factor: $(x - 3)(x + 3) \geq 0$. Roots ± 3. Opens up $\Rightarrow$ nonnegative outside the roots. $\geq$, so include the endpoints.

3. Factor to find the boundary roots: $(x - 2)(x - 3) = 0$ gives $x = 2, 3$. The parabola opens up ($a > 0$), so it dips below zero only between the roots: $2 < x < 3$. Strict, so the endpoints are excluded.

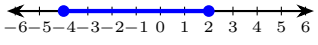
4. Factor: $(x+4)(x+2) = 0$ gives roots $-4, -2$. Opens up, so the expression is nonnegative *outside* the roots. With $\geq$, include the endpoints: $x \leq -4$ or $x \geq -2$.

5. Multiply by -1 and *flip* the inequality: $x^2 - 4 \leq 0$. Factor: $(x - 2)(x + 2) \leq 0$, roots ± 2. Opens up, so it's negative between the roots: $-2 \leq x \leq 2$, endpoints included.

6. Factor: $(x - 3)(x + 2) = 0$ gives roots $-2, 3$. Opens up, so the expression is positive *outside* the roots. Strict $>$, so exclude endpoints: $x < -2$ or $x > 3$.

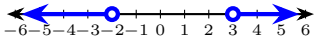
7. Move all to one side: $x^2 + 2x - 8 \leq 0$. Factor: $(x + 4)(x - 2) \leq 0$. Roots $-4, 2$. Solution in interval notation: $[-4, 2]$ (closed because $\leq$).

Answer graph



8. Roots $-2, 3$. Product positive outside the roots. Strict: no endpoints. Solution: $(-\infty, -2) \cup (3, \infty)$.

Answer graph



9. Multiply by -1 and flip the sign: $t^2 - 6t + 5 \leq 0$. Factor: $(t - 1)(t - 5) \leq 0$, roots 1 and 5. Now the parabola opens up, so it's negative between the roots: $1 \leq t \leq 5$, endpoints included.

10. Factor: $(2x + 1)(x - 4) \leq 0$. Roots $-\frac{1}{2}, 4$. Opens up: negative between, include endpoints. Interval: $[-\frac{1}{2}, 4]$.

11. One steady path is: $\Delta = -4 < 0$, so the parabola never touches the x-axis. Since it opens up, the expression is positive everywhere. Solution: $(-\infty, \infty)$. That gives a quick check on the answer.

12. Same parabola (shifted): positive everywhere, never less than 0. No solution.

13. Factor: $(x - 3)(x + 1) = 0$ gives roots $-1, 3$. Opens up, so the expression is nonnegative outside the roots. With $\geq$, include the endpoints: $x \leq -1$ or $x \geq 3$.

14. Divide by 3 first — it's positive, so the inequality does *not* flip: $x^2 - 4 < 0$. Factor: $(x - 2)(x + 2) < 0$, roots ± 2. Negative between: $-2 < x < 2$.

15. Square roots: $|x - 1| \leq 3$, so $-3 \leq x - 1 \leq 3$, giving $-2 \leq x \leq 4$. (Equivalent to factoring $(x - 1 - 3)(x - 1 + 3) = (x - 4)(x + 2) \leq 0$.)

16. A real square is never negative. $x^2 \leq 0$ holds only when $x^2 = 0$, i.e. $x = 0$. (Single-point solution.)

17. True for every x except where $x^2 = 0$. Solution: $(-\infty, 0) \cup (0, \infty)$, i.e. all reals except 0.

18. Factor out the common x: $x(x + 5) \leq 0$, giving roots 0 and -5. Opens up, so the product is negative between the roots: $-5 \leq x \leq 0$, endpoints included.

19. Divide by -2 and *flip* the inequality: $x^2 - 4 < 0$. Factor: $(x - 2)(x + 2) < 0$, roots ± 2. Negative between the roots: $-2 < x < 2$.

20. Factor: $(x - 2)(x - 4) < 0$. Roots $2, 4$. The parabola opens up, so the product is negative between the roots: $2 < x < 4$.

21. Set $h(t) \geq 25$: $-5t^2 + 30t \geq 25$, i.e. $-5t^2 + 30t - 25 \geq 0$. Divide by -5 and flip: $t^2 - 6t + 5 \leq 0$. Factor: $(t - 1)(t - 5) \leq 0$. So $1 \leq t \leq 5$ seconds.

22. Solve $-x^2 + 40x - 300 > 0$. Multiply by -1 and flip: $x^2 - 40x + 300 < 0$. Factor: $(x-10)(x-30) < 0$. So $10 < x < 30$. Profit is positive when production is strictly between 10 and 30 widgets.

23. Solve $w(40 - 2w) \geq 150$: $40w - 2w^2 \geq 150$, i.e. $-2w^2 + 40w - 150 \geq 0$. Divide by -2 (flip): $w^2 - 20w + 75 \leq 0$. Factor: $(w - 5)(w - 15) \leq 0$. So $5 \leq w \leq 15$ feet.

24. Solve $-16t^2 + 12t + 4 = 0$: divide by -4, $4t^2 - 3t - 1 = 0$. Factor: $(4t + 1)(t - 1) = 0$, giving $t = -\frac{1}{4}$ or $t = 1$. So the diver is above water for $-\frac{1}{4} < t < 1$. Physically $t \geq 0$, so the relevant interval is $0 \leq t < 1$ second. (The diver enters the water exactly at $t = 1$.)

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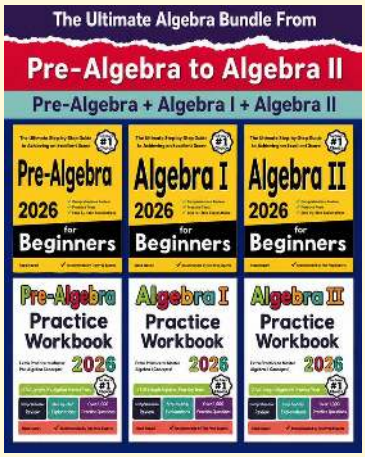

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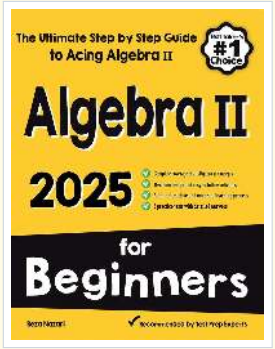

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