Solving Logarithmic Equations

Name: ____________ Date: ____________ Score: ______ / 33

Quick Review

Solving log equations boils down to one of two moves, plus a mandatory domain check at the end.

Move 1: single log = constant. Get the equation into the form $\log_b(\text{stuff}) = c$, then rewrite in exponential form stuff $= b^c$ and solve.

Move 2: log = log, same base. If both sides reduce to the form $\log_b(A) = \log_b(B)$, then $A = B$. Logs are one-to-one.

Combining first. When you see multiple log terms, use the product/quotient/power rules to combine them into one log. Once everything is a single log on each side, apply Move 1 or Move 2.

The extraneous-solutions trap — always check. Solving log equations often leads to a polynomial (often a quadratic). Some of its roots may force the original log arguments to be zero or negative, which is undefined for real logs. *Discard those.* Quick check: $\log_2(x) + \log_2(x-6) = 4$ becomes $x(x-6) = 16$, so $x^2 - 6x - 16 = 0$, giving $x = 8$ or $x = -2$. Throw out $x = -2$ because the original $\log_2(x)$ needs $x > 0$. The valid answer is $x = 8$.

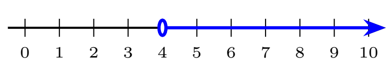
The check itself is a step — not a verification. Even if the algebra is clean, write down "Check: argument > 0? Yes / No" for each original log. If a candidate fails, cross it off; if all fail, the equation has *no solution*.

Common slips. Forgetting the check (most common). Combining logs that don't share a base. Using $\log(A) + \log(B) = \log(A+B)$ (wrong — it's $\log(AB)$). Skipping the change-of-base when bases differ on the two sides.

PRACTICE

Combine, exponentiate, solve, then verify the domain. List extraneous solutions explicitly.

1. Solve $\log_3(x) = 4$. ________
2. Solve $\log_2(x+1) = 5$. ________
3. Solve $\log_2(x) + \log_2(x-6) = 4$ and discard extraneous roots. ________
4. Solve $\log(x+5) = \log(2x-3)$. ________
5. Solve $\log_3(2x+1) - \log_3(x-1) = 1$. ________
6. Solve $\log(x) + \log(x-3) = 1$ and identify any extraneous roots. ________
7. Solve $\ln(x+4) - \ln(x) = \ln(3)$. ________
8. Which steps below are VALID when solving log equations? Combine same-base logs; convert $\log_b(x) = c$ to $x = b^c$; check domain; equate arguments when $\log_b(A) = \log_b(B)$. ________
9. Solve $\log_5(x) + \log_5(x-4) = 1$. The number line below shades the valid domain for the original equation — use it to test each candidate root. ________



10. Solve $\log_2(x-1) + \log_2(x-5) = 3$. State extraneous solutions. ________
11. Solve $\log(2x) = 3$. ________
12. Solve $\log_4(x) = \frac{3}{2}$. ________
13. Solve $\ln(3x-2) = \ln(x+8)$. ________
14. Solve $\log(x^2) = \log(4x-3)$. ________
15. Solve $\log_2(x) + \log_2(x+2) = 3$ and discard extraneous roots. ________

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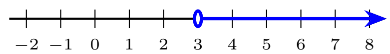
16. Solve $\log_3(x+3) + \log_3(x-3) = 3$. The valid domain is shown on the number line below; use it to decide which root to keep.



17. Solve $\log(x+2) + \log(x-1) = \log(4x-2)$.

18. Identify extraneous candidates: $\log(x-2) + \log(x+5) = \log(8)$ gives $x = 3$ or $x = -6$.

19. Solve $\log_2(8x) = 5$. The table of values for $\log_2(x)$ below may help you read off the step where $\log_2(x) = 2$.

x	1	2	8	16	32
$\log_2(x)$	0	1	3	4	5

20. Solve $\log_6(x) + \log_6(x-5) = 2$.

♦ Word Problems

21. Solve $\log_4(x) + \log_4(x-6) = 2$. Show both roots of the resulting quadratic and explain which one is extraneous and why.

22. Solve $\log(x+3) + \log(x) = \log(28)$. State any extraneous roots explicitly. Then back-substitute to verify.

23. Solve $\log_3(x-2) + \log_3(x+6) = 2$. List the extraneous root and explain which domain restriction kills it.

24. A geology student is told that the equation $\log_2(x+1) + \log_2(x-3) = 4$ describes a calibration step in a sensor reading. Solve for x and explain the extraneous candidate in terms of the log's domain.

Additional Practice

25. Evaluate $\log_2 32$.

26. Evaluate $\log_5 125$.

27. Rewrite $\log_3 81 = 4$ exponentially.

28. Solve $\log_4 x = 3$.

29. Domain of $y = \log(x-7)$.

30. Expand $\log(ab)$.

31. Expand $\log(x^3)$.

32. Condense $\log x + \log 5$.

33. Condense $2 \log x$.

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Answer Keys

1. $x = 81$
2. $x = 31$
3. $x = 8$ (extraneous: $x = -2$)
4. $x = 8$
5. $x = 4$
6. $x = 5$ (extraneous: $x = -2$)
7. $x = 2$
8. all four
9. $x = 5$ (extraneous: $x = -1$)
10. $x = 3 + 2\sqrt{3}$ (extraneous: $x = 3 - 2\sqrt{3}$)
11. $x = 500$
12. $x = 8$
13. $x = 5$
14. $x = 1$ or $x = 3$
15. $x = 2$ (extraneous: $x = -4$)
16. $x = 6$ (extraneous: $x = -6$)
17. $x = 3$ (extraneous: $x = 0$)
18. $x = 3$ (extraneous: $x = -6$)
19. $x = 4$
20. $x = 9$ (extraneous: $x = -4$)
21. $x = 8$ (extraneous: $x = -2$)
22. $x = 4$ (extraneous: $x = -7$)
23. $x = 3$ (extraneous: $x = -7$)
24. $x = 1 + 2\sqrt{5}$ (extraneous: $x = 1 - 2\sqrt{5}$)

Additional Practice Answers

25. 5
26. 3
27. $3^4 = 81$
28. $x = 64$
29. $x > 7$
30. $\log a + \log b$
31. $3 \log x$
32. $\log(5x)$
33. $\log(x^2)$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. This is a single log equal to a constant, so rewrite it in exponential form: $x = 3^4 = 81$. Domain check: the argument $x = 81 > 0$, so the solution is valid.

2. Single log equals a constant, so go to exponential form: $x + 1 = 2^5 = 32$. Subtract 1 to get $x = 31$. Domain check: the argument $x + 1 = 32 > 0$, so it is valid.

3. Combine: $\log_2(x(x-6)) = 4$, so $x(x-6) = 16$, $x^2 - 6x - 16 = 0$, $(x-8)(x+2) = 0$. $x = -2$ makes the original $\log_2(x)$ undefined — toss it. Only $x = 8$ survives.

4. Same base, equate arguments: $x + 5 = 2x - 3$, so $x = 8$. Check both arguments at $x = 8$: $13 > 0$ and $13 > 0$ ✓.

5. Quotient: $\log_3\left(\frac{2x+1}{x-1}\right) = 1$. Exponential: $\frac{2x+1}{x-1} = 3$. Cross-multiply: $2x + 1 = 3x - 3$, so $x = 4$. Check: $2x + 1 = 9 > 0$, $x - 1 = 3 > 0$ ✓.

6. Combine: $\log(x(x-3)) = 1$, so $x(x-3) = 10$, $x^2 - 3x - 10 = 0$, $(x-5)(x+2) = 0$. $x = -2$ makes $\log(x)$ undefined; discard. Only $x = 5$ is valid.

7. Quotient: $\ln\left(\frac{x+4}{x}\right) = \ln 3$. Equate arguments: $\frac{x+4}{x} = 3$, so $x + 4 = 3x$ and $x = 2$. Check: $x + 4 = 6 > 0$, $x = 2 > 0$ ✓.

8. These are the four legitimate moves. The two illegitimate ones to avoid: $\log A + \log B \neq \log(A+B)$, and $\log A \cdot \log B \neq \log(AB)$.

9. Combine: $\log_5(x(x-4)) = 1$, so $x(x-4) = 5$, $x^2 - 4x - 5 = 0$, $(x-5)(x+1) = 0$. Original requires $x > 0$ and $x - 4 > 0$, i.e., $x > 4$ — exactly the ray shown. So $x = -1$ fails (left of the open circle); $x = 5$ survives.

10. Combine: $\log_2((x-1)(x-5)) = 3$, so $(x-1)(x-5) = 8$, $x^2 - 6x - 3 = 0$. Quadratic formula: $x = \frac{6 \pm \sqrt{36+12}}{2} = 3 \pm 2\sqrt{3}$. The domain needs $x > 5$. Now $3 - 2\sqrt{3} \approx 0.54$ fails (< 5). $3 + 2\sqrt{3} \approx 6.46$ passes. Only the $+$ root survives.

11. No base is written, so it is base 10. Rewrite in exponential form: $2x = 10^3 = 1000$, then divide by 2 to get $x = 500$. Domain check: the argument $2x = 1000 > 0$, valid.

12. Rewrite in exponential form: $x = 4^{3/2}$. Read the fractional exponent as root-then-power: $4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$. Domain check: $x = 8 > 0$, valid.

13. Equate arguments: $3x - 2 = x + 8$, so $2x = 10$ and $x = 5$. Check: $3(5) - 2 = 13 > 0$ and $5 + 8 = 13 > 0$ ✓.

14. Equate arguments: $x^2 = 4x - 3$, so $x^2 - 4x + 3 = 0$ and $(x-1)(x-3) = 0$. Check: at $x = 1$, both sides give $\log(1) = 0$ ✓. At $x = 3$, both sides give $\log(9)$ ✓. Both valid.

15. Combine: $\log_2(x(x+2)) = 3$, so $x(x+2) = 8$, $x^2 + 2x - 8 = 0$, $(x-2)(x+4) = 0$. Original requires $x > 0$, so $x = -4$ is extraneous; $x = 2$ is the answer.

16. Combine: $\log_3(x^2 - 9) = 3$, so $x^2 - 9 = 27$, $x^2 = 36$, $x = \pm 6$. Domain needs $x + 3 > 0$ and $x - 3 > 0$, i.e., $x > 3$ — the ray on the line. So $x = -6$ falls outside it and fails; $x = 6$ wins.

17. Combine LHS: $\log((x+2)(x-1)) = \log(4x-2)$. Equate: $(x+2)(x-1) = 4x - 2$, $x^2 + x - 2 = 4x - 2$, $x^2 - 3x = 0$, $x(x-3) = 0$. So $x = 0$ or $x = 3$. At $x = 0$: $\log(2) + \log(-1)$ — the second log is undefined. At $x = 3$: $\log(5) + \log(2) = \log(10)$ and RHS $= \log(10)$ ✓. The valid solution is $x = 3$.

18. Keep the rule visible: At $x = -6$: $\log(-8)$ and $\log(-1)$ are both undefined. At $x = 3$: $\log(1) + \log(8) = 0 + \log 8 = \log 8$ ✓. That gives a quick check on the answer.

19. One steady path is: $8x = 2^5 = 32$, so $x = 4$. (Or expand first: $3 + \log_2(x) = 5$, so $\log_2(x) = 2$. The table jumps from $\log_2(2) = 1$ to $\log_2(8) = 3$, so $\log_2(x) = 2$ sits between, at $x = 4$. Either way.) That gives a quick check on the answer.

20. Combine: $\log_6(x(x-5)) = 2$, $x(x-5) = 36$, $x^2 - 5x - 36 = 0$, $(x-9)(x+4) = 0$. Domain: $x > 5$. So $x = -4$ fails, $x = 9$ wins.

21. Combine using the product rule: $\log_4(x(x-6)) = 2$. Convert to exponential form: $x(x-6) = 4^2 = 16$. Expand: $x^2 - 6x - 16 = 0$, which factors as $(x-8)(x+2) = 0$. So the candidates are $x = 8$ and $x = -2$. Now the domain check: the original equation has $\log_4(x)$ and $\log_4(x-6)$, so we need both $x > 0$ and $x - 6 > 0$, i.e., $x > 6$. $x = -2$ fails on both counts. $x = 8$ passes ($8 > 6$). So $x = -2$ is extraneous — it would make $\log_4(-2)$ and $\log_4(-8)$, both undefined. The valid solution is $x = 8$. Verify: $\log_4(8) + \log_4(2) = \log_4(16) = 2$ ✓.

22. Combine the LHS by the product property: $\log(x(x+3)) = \log(28)$. Equate arguments: $x(x+3) = 28$, so $x^2 + 3x - 28 = 0$. Factor: $(x+7)(x-4) = 0$. Candidates: $x = -7$ or $x = 4$. Domain check on the original: $\log(x+3)$ needs $x > -3$, and $\log(x)$ needs $x > 0$, so altogether $x > 0$. $x = -7$ fails ($-7 < 0$, would give $\log(-7)$ and $\log(-4)$ — both undefined). $x = 4$ passes. Back-substitute $x = 4$: $\log(7) + \log(4) = \log(28)$ ✓ by the product rule. (Always do this final plug-back step. It catches arithmetic slips, too.)

23. Combine: $\log_3((x-2)(x+6)) = 2$. Exponential form: $(x-2)(x+6) = 9$.

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Expand: $x^2 + 4x - 12 = 9$, so $x^2 + 4x - 21 = 0$ and $(x + 7)(x - 3) = 0$. Candidates: $x = -7$ or $x = 3$. Domain: $x - 2 > 0$ requires $x > 2$, and $x + 6 > 0$ requires $x > -6$. Both must hold, so $x > 2$. $x = -7$ fails the stricter condition $x > 2$ (and also makes $x - 2 = -9$, i.e., $\log_3(-9)$ undefined). $x = 3$ passes $(3 > 2)$. Verify: $\log_3(1) + \log_3(9) = 0 + 2 = 2$ ✓.

24. Combine: $\log_2((x+1)(x-3)) = 4$. Exponential form: $(x+1)(x-3) = 2^4 = 16$. Expand: $x^2 - 2x - 3 = 16$, so $x^2 - 2x - 19 = 0$. Use the quadratic formula: $x = \dfrac{2 \pm \sqrt{4 + 76}}{2} = \dfrac{2 \pm \sqrt{80}}{2} = 1 \pm 2\sqrt{5}$. Numerically: $1 + 2\sqrt{5} \approx 5.47$ and $1 - 2\sqrt{5} \approx -3.47$. Domain: $x + 1 > 0$ and $x - 3 > 0$, so $x > 3$. The positive root ≈ 5.47 passes; the negative root fails. So $x = 1 + 2\sqrt{5} \approx 5.47$ is the valid solution; $1 - 2\sqrt{5} \approx -3.47$ is extraneous. (The reading $x \approx 5.5$ is the physically meaningful one; the negative candidate is an arithmetic shadow the algebra produces but the log forbids.)

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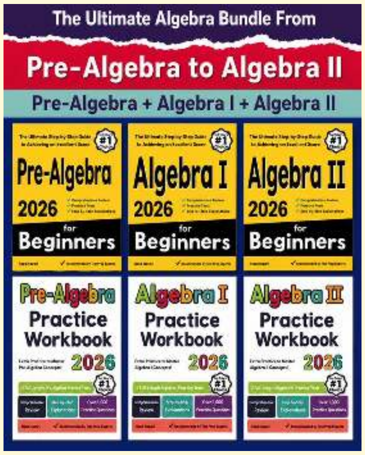

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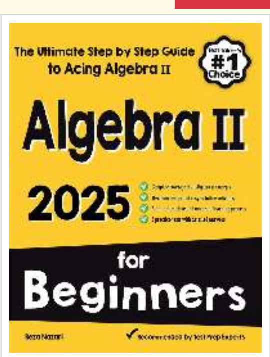

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