

# Solving Absolute Value Equations

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 28

## Q Quick Review

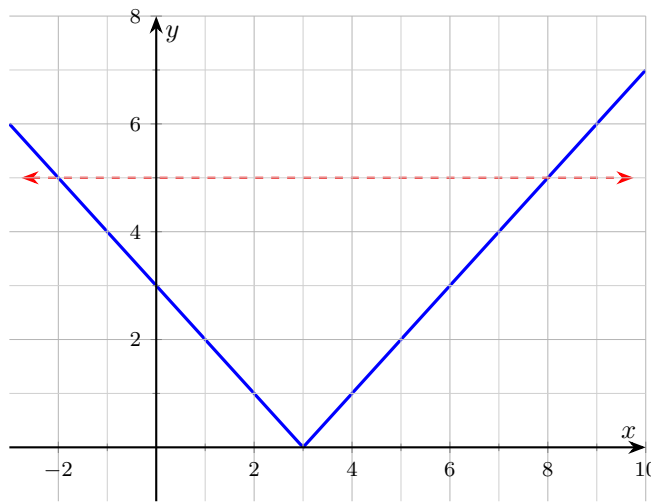
The **absolute value**  $|x|$  is the distance from  $x$  to 0 on the number line — never negative. To solve  $|E| = k$  where  $k > 0$ , split into two cases:  $E = k$  or  $E = -k$ . (Two distances from 0 to  $k$ , one on each side.) If  $k = 0$ , there's a single case:  $E = 0$ . And if  $k < 0$  — no solution. Distances can't be negative. The classic trap: forgetting the negative case. **Always isolate the absolute-value expression first.** For  $3|2x - 1| + 4 = 13$ , subtract 4 to get  $3|2x - 1| = 9$ , divide by 3 to get  $|2x - 1| = 3$ , then split into  $2x - 1 = 3$  and  $2x - 1 = -3$ . The graph of  $y = |x|$  is a V-shape with vertex at the origin: two rays meeting at a sharp corner. Shifts work like with any parent function:  $|x - h| + k$  moves the vertex to  $(h, k)$ . **Distance table.** Absolute value equations are distance questions. The center is the number inside the absolute value, and the distance gives the two possible directions.

Equation	Meaning	Solutions
$ x - 3  = 5$	5 units from 3	$x = -2, 8$
$ 2x + 1  = 7$	$2x + 1 = 7$ or $-7$	$x = 3, -4$

## PRACTICE

Solve each equation. List all real solutions.

- $|x| = 7$  \_\_\_\_\_
- Solve  $|x - 3| = 5$  and verify graphically. \_\_\_\_\_

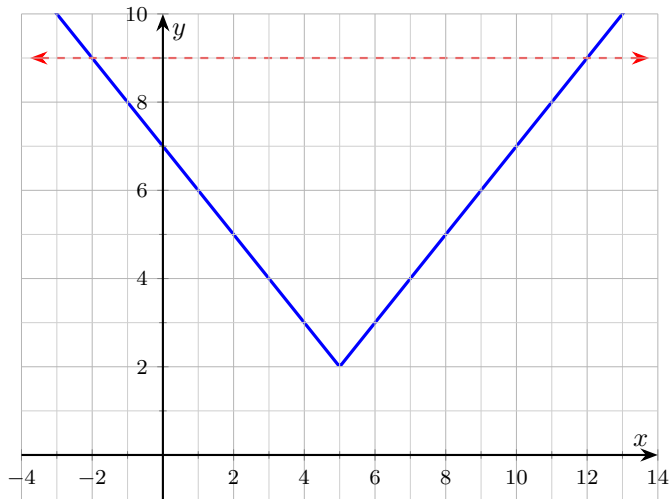


- $|2x + 1| = 9$  \_\_\_\_\_
- $3|x| - 4 = 11$  \_\_\_\_\_
- $|x + 4| = 0$  \_\_\_\_\_
- $|3x - 6| = -2$  \_\_\_\_\_



7. Solve  $|x - 5| + 2 = 9$  and verify graphically.

\_\_\_\_\_



8.  $2|x + 1| = 10$

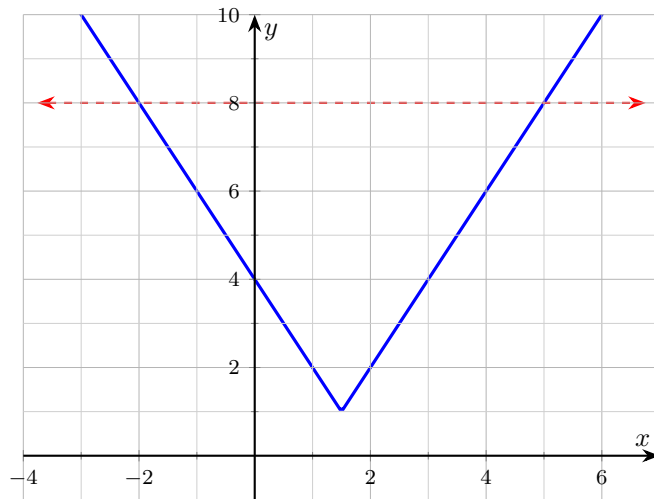
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9.  $|4x| = 12$

\_\_\_\_\_

10. Solve  $|2x - 3| + 1 = 8$  and verify graphically.

\_\_\_\_\_



11.  $|5 - x| = 3$

\_\_\_\_\_

12.  $4|x - 2| + 3 = 19$

\_\_\_\_\_

13.  $|x + 7| = 7$

\_\_\_\_\_

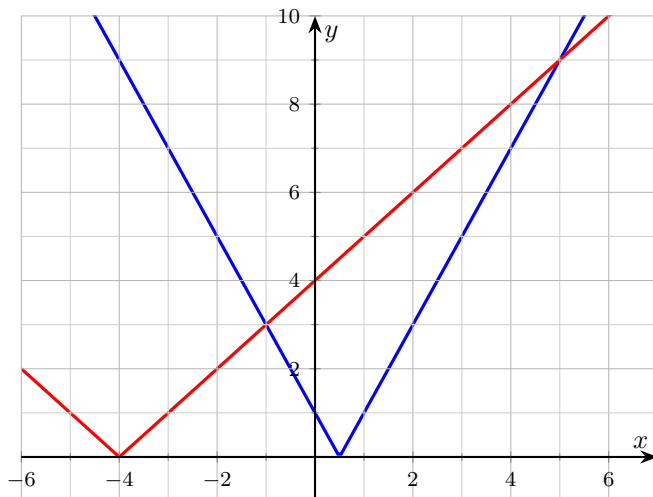
14.  $-3|x| + 12 = 0$

\_\_\_\_\_



15. Solve  $|2x - 1| = |x + 4|$  and verify graphically.

\_\_\_\_\_



16.  $|\frac{x}{2} + 1| = 4$

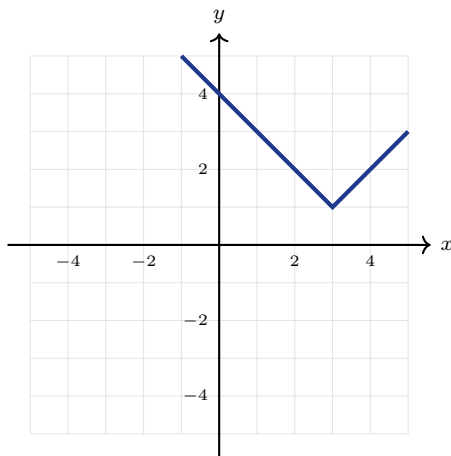
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17.  $|3x + 2| - 1 = 9$

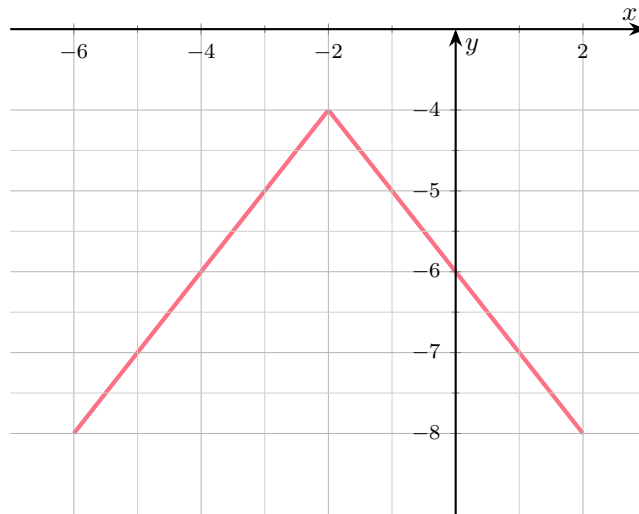
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18. Read the vertex of the V-graph below.

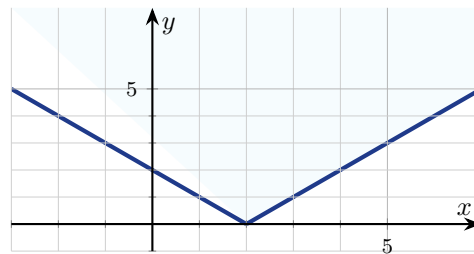
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19. Read the vertex of the downward V-graph below. \_\_\_\_\_



20. Use the table idea and the graph of  $y = |x - 2|$  to find  $y$  when  $x = -3, 0, 2, 5$ . \_\_\_\_\_



◆ Word Problems

- 21. A factory's machine produces bolts with a target length of 5.0 cm. Bolts whose lengths differ from the target by more than 0.1 cm are rejected. Write the equation that gives the boundary lengths — the two lengths that are exactly 0.1 cm off. \_\_\_\_\_
- 22. A submarine descends and ascends along a path whose depth  $d$  satisfies  $|d + 200| = 50$ , where depth is measured in feet (negative below sea level). Find the two depths. \_\_\_\_\_
- 23. A thermostat is set to 68°F. The actual room temperature satisfies  $|T - 68| = 2$ . What are the two extreme temperatures? \_\_\_\_\_
- 24. A long-jumper's distance  $d$  (in meters) satisfies  $|d - 7.5| = 0.3$  for the two practice jumps that qualified for the final. Find both distances. \_\_\_\_\_

Additional Practice

- 25. Solve  $|x - 4| = 9$ . \_\_\_\_\_
- 26. Solve  $|2x + 1| < 7$ . \_\_\_\_\_
- 27. Solve  $|x + 3| \geq 5$ . \_\_\_\_\_
- 28. Find the vertex of  $y = |x - 2| - 3$ . \_\_\_\_\_



## Answer Keys

1.  $x = 7$  or  $x = -7$

2.  $x = 8$  or  $x = -2$

3.  $x = 4$  or  $x = -5$

4.  $x = 5$  or  $x = -5$

5.  $x = -4$

6. no solution

7.  $x = 12$  or  $x = -2$

8.  $x = 4$  or  $x = -6$

9.  $x = 3$  or  $x = -3$

10.  $x = 5$  or  $x = -2$

11.  $x = 2$  or  $x = 8$

12.  $x = 6$  or  $x = -2$

## Additional Practice Answers

25.  $x = -5, 13$

26.  $-4 < x < 3$

13.  $x = 0$  or  $x = -14$

14.  $x = 4$  or  $x = -4$

15.  $x = 5$  or  $x = -1$

16.  $x = 6$  or  $x = -10$

17.  $x = \frac{8}{3}$  or  $x = -4$

18.  $(3, 1)$

19.  $(-2, -4)$

20.  $y = 5, 2, 0, 3$

21.  $x = 5.1$  or  $x = 4.9$

22.  $d = -150$  ft or  $d = -250$  ft

23.  $T = 70$  or  $T = 66$

24.  $d = 7.8$  or  $d = 7.2$

27.  $x \leq -8$  or  $x \geq 2$

28.  $(2, -3)$

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

- Two points are distance 7 from 0: 7 and  $-7$ . (Always remember the negative case.)
- The V opens up with vertex at  $(3, 0)$ . Where  $|x - 3| = 5$  the V crosses  $y = 5$ , which happens twice at  $x = -2$  and  $x = 8$ . Algebraically:  $x - 3 = 5 \Rightarrow x = 8$ ;  $x - 3 = -5 \Rightarrow x = -2$ .
- The inside can be 9 or  $-9$ , so split into two cases. Case 1:  $2x + 1 = 9 \Rightarrow 2x = 8 \Rightarrow x = 4$ . Case 2:  $2x + 1 = -9 \Rightarrow 2x = -10 \Rightarrow x = -5$ . Don't forget the negative case.
- Start with the key idea: Isolate first:  $3|x| = 15$ , so  $|x| = 5$ . Then split:  $x = \pm 5$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
- Distance is zero, so  $x + 4 = 0$ . Only one solution:  $x = -4$ . (Zero is the one case with a single solution.)
- Keep the rule visible: Absolute value can't equal a negative number. Empty solution set. That gives a quick check on the answer.
- Subtract 2 first:  $|x - 5| = 7$ . The V has vertex  $(5, 2)$ ; the dashed line  $y = 9$  meets it twice. Algebraically  $x - 5 = 7 \Rightarrow x = 12$ ;  $x - 5 = -7 \Rightarrow x = -2$ .
- Start with the key idea: Divide by 2:  $|x + 1| = 5$ . Split:  $x + 1 = 5 \Rightarrow x = 4$ ;  $x + 1 = -5 \Rightarrow x = -6$ . That gives a quick check on the answer.
- Split into the two cases:  $4x = 12 \Rightarrow x = 3$ , and  $4x = -12 \Rightarrow x = -3$ . Both make  $|4x|$  equal 12, so there are two solutions.
- Subtract 1:  $|2x - 3| = 7$ . The V has vertex  $(\frac{3}{2}, 1)$  and meets  $y = 8$  twice. Algebraically  $2x - 3 = 7 \Rightarrow x = 5$ ;  $2x - 3 = -7 \Rightarrow x = -2$ .
- Two cases. Case 1:  $5 - x = 3 \Rightarrow -x = -2 \Rightarrow x = 2$ . Case 2:  $5 - x = -3 \Rightarrow -x = -8 \Rightarrow x = 8$ . (Watch the sign on  $-x$  when you solve each one.)
- Start with the key idea: Isolate:  $4|x - 2| = 16$ , so  $|x - 2| = 4$ . Split:  $x = 6$  or  $x = -2$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
- Split:  $x + 7 = 7 \Rightarrow x = 0$ , or  $x + 7 = -7 \Rightarrow x = -14$ . Even though one answer is 0, both are valid — the negative case still produces a real solution.

- Keep the rule visible: Move:  $-3|x| = -12$ , so  $|x| = 4$ . Split:  $x = \pm 4$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
- When two absolute values are equal, the expressions are equal or opposites. The two V graphs meet exactly at  $x = -1$  and  $x = 5$ . Algebraically:  $2x - 1 = x + 4 \Rightarrow x = 5$ ;  $2x - 1 = -(x + 4) \Rightarrow 3x = -3 \Rightarrow x = -1$ .
- Start with the key idea:  $\frac{x}{2} + 1 = 4 \Rightarrow x = 6$ ;  $\frac{x}{2} + 1 = -4 \Rightarrow \frac{x}{2} = -5 \Rightarrow x = -10$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
- A careful way to see it: Add 1:  $|3x + 2| = 10$ . Split:  $3x + 2 = 10 \Rightarrow x = \frac{8}{3}$ ;  $3x + 2 = -10 \Rightarrow x = -4$ . That gives a quick check on the answer.
- Find the lowest point of the V: it sits at  $(3, 1)$ . (The two arms have slope  $\pm 1$ , matching  $y = |x - 3| + 1$ .)
- For a downward-opening V, the vertex is the *highest* point (the tip). Reading the graph, that peak sits at  $(-2, -4)$  — matching  $y = -|x + 2| - 4$ , where the  $+2$  inside shifts left 2 and the  $-4$  shifts down 4.
- Plug in each  $x$ :  $|-3 - 2| = 5$ ,  $|0 - 2| = 2$ ,  $|2 - 2| = 0$ ,  $|5 - 2| = 3$ . (Notice  $y = 0$  at the vertex  $x = 2$ .)
- A careful way to see it:  $|x - 5.0| = 0.1$  gives the boundary:  $x - 5.0 = 0.1 \Rightarrow x = 5.1$  or  $x - 5.0 = -0.1 \Rightarrow x = 4.9$ . Anything strictly farther than 0.1 from the target gets rejected. That gives a quick check on the answer.
- Keep the rule visible:  $d + 200 = 50 \Rightarrow d = -150$  (closer to surface);  $d + 200 = -50 \Rightarrow d = -250$  (deeper). Both are below sea level (negative). That gives a quick check on the answer.
- One steady path is:  $T - 68 = 2 \Rightarrow T = 70$ ;  $T - 68 = -2 \Rightarrow T = 66$ . The room varies between  $66^\circ$  and  $70^\circ$ F. That gives a quick check on the answer.
- Start with the key idea:  $d - 7.5 = 0.3 \Rightarrow d = 7.8$ ;  $d - 7.5 = -0.3 \Rightarrow d = 7.2$ . Both jumps were within 0.3 m of the target 7.5 m. That gives a quick check on the answer.



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