

Simplifying Rational Expressions

Name: _____ Date: _____ Score: _____ / 30

Q Quick Review

A **rational expression** is a fraction $\frac{P(x)}{Q(x)}$ with $Q(x) \neq 0$. Simplifying one is the same move you make with numeric fractions: factor, cancel common factors, state what the denominator is not allowed to be.

Step 1 – factor everything. Don't even think about canceling until both top and bottom are fully factored. Common patterns: GCF (e.g. $2x^2 - 8 = 2(x - 2)(x + 2)$), difference of squares ($a^2 - b^2 = (a - b)(a + b)$), and trinomial factoring ($x^2 + 5x + 6 = (x + 2)(x + 3)$).

Step 2 – cancel factors, not terms. You can cross out $(x - 3)$ when it appears in both numerator and denominator. You *cannot* cross out the x in $\frac{x + 5}{x - 3}$ – that x is part of a sum, not a factor. This is the single biggest rational-expression error students make.

Step 3 – state restrictions. Every value of x that makes the *original* denominator zero stays excluded, even if the offending factor cancels. Quick check: $\frac{x^2 - 9}{x - 3} = \frac{(x - 3)(x + 3)}{x - 3} = x + 3$ – but the answer still requires $x \neq 3$. (At $x = 3$ the original expression is undefined; the simplified version isn't, so they only agree off that one point.)

Sign-flipped factors. $a - b = -(b - a)$. So $\frac{2 - x}{x - 2} = \frac{-(x - 2)}{x - 2} = -1$ (for $x \neq 2$). When you see almost-matching factors that disagree in sign, factor out the -1 .

Common slips. Canceling across addition (treating $\frac{x + 5}{x}$ as 5). Dropping the restriction once the factor cancels. Forgetting that $\frac{0}{x}$ is undefined at $x = 0$, even though it's 0 everywhere else.

PRACTICE

Factor numerator and denominator, cancel common factors, and state every restriction from the original denominator.

1. Simplify $\frac{4x}{8x^2}$. Use the table to check your simplified form against the original at a few inputs. _____

x	1	2	4
$\frac{4x}{8x^2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

2. Simplify $\frac{x^2 - 9}{x - 3}$. The table evaluates the expression away from the excluded value. _____

x	4	5	6
$\frac{x^2 - 9}{x - 3}$	7	8	9

3. Simplify $\frac{x^2 - 5x + 6}{x - 2}$. The table gives a few sample values to test against your answer. _____

x	0	1	4
$\frac{x^2 - 5x + 6}{x - 2}$	-3	-2	1

4. $\frac{2 - x}{x - 2}$ _____

5. $\frac{6x^2y}{9xy^3}$ _____

6. $\frac{x^2 + x - 6}{x^2 - 4}$ _____

7. $\frac{2x^2 - 8}{x^2 + 2x - 8}$ _____

8. $\frac{x^2 - 7x + 12}{x^2 - 9}$ _____

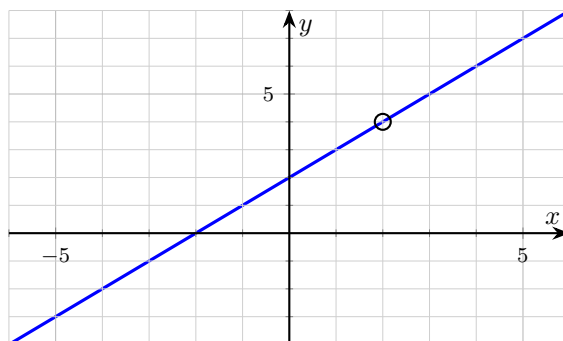
9. $\frac{3x - 6}{4 - 2x}$ _____



10. $\frac{x^2 + x - 12}{x^2 - 16}$ _____
11. $\frac{x^2 - 25}{x + 5}$ _____
12. $\frac{x^3 - x}{x^2 + x}$ _____
13. $\frac{x^2 - 1}{x^2 - 2x + 1}$ _____
14. $\frac{4x^2 - 9}{2x + 3}$ _____
15. $\frac{5 - x}{x^2 - 25}$ _____
16. $\frac{x^2 + 6x + 9}{x^2 - 9}$ _____
17. $\frac{2x^2 + 5x + 3}{x + 1}$ _____
18. $\frac{x^2 - 4x}{x^2 - 16}$ _____
19. $\frac{x^2 - x - 12}{x^2 - 2x - 8}$ _____
20. $\frac{9 - x^2}{x^2 - 3x}$ _____

◆ Word Problems

21. A rectangle has area $x^2 + 5x + 6$ square inches and width $x + 2$ inches. Find the length, in simplest form, and state the values of x for which the answer makes physical sense. _____
22. A student writes $\frac{x + 3}{x} = 3$ by “canceling the x .” Show what’s wrong by plugging in $x = 2$ on both sides, and write the correct simplified form (or explain why none is possible). _____
23. The expression $f(x) = \frac{x^2 - 4}{x - 2}$ is graphed below. Simplify $f(x)$ algebraically, state any restriction, and identify the hole shown in the graph. _____



24. A rate problem leads to the expression $\frac{2x^2 + 7x + 3}{x^2 - 9}$. Simplify it and state every restriction. (The expression came from a real context where $x > 3$, so the algebraic restrictions can be checked against the context.) _____



Additional Practice

25. Simplify $\frac{x^2 - 9}{x - 3}$. _____

26. Excluded value of $\frac{1}{x + 4}$. _____

27. Domain of $f(x) = \frac{x}{x - 5}$. _____

28. Multiply $\frac{x}{3} \cdot \frac{6}{x}$. _____

29. Divide $\frac{x^2}{5} \div \frac{x}{10}$. _____

30. Add $\frac{3}{x} + \frac{5}{x}$. _____



Answer Keys

1. $\frac{1}{2x}, x \neq 0$	13. $\frac{x+1}{x-1}, x \neq 1$
2. $x+3, x \neq 3$	14. $2x-3, x \neq -\frac{3}{2}$
3. $x-3, x \neq 2$	15. $-\frac{1}{x+5}, x \neq \pm 5$
4. $-1, x \neq 2$	16. $\frac{x+3}{x-3}, x \neq \pm 3$
5. $\frac{2x}{3y^2}, x \neq 0, y \neq 0$	17. $2x+3, x \neq -1$
6. $\frac{x+3}{x+2}, x \neq \pm 2$	18. $\frac{x}{x+4}, x \neq \pm 4$
7. $\frac{2(x+2)}{x+4}, x \neq -4, 2$	19. $\frac{x+3}{x+2}, x \neq -2, 4$
8. $\frac{x-4}{x+3}, x \neq \pm 3$	20. $-\frac{x+3}{x}, x \neq 0, 3$
9. $-\frac{3}{2}, x \neq 2$	21. $x+3 \text{ in}, x > 0$
10. $\frac{x-3}{x-4}, x \neq \pm 4$	22. Invalid cancel; $f(2) = \frac{5}{2} \neq 3$
11. $x-5, x \neq -5$	23. $f(x) = x+2, x \neq 2$; hole at (2, 4)
12. $x-1, x \neq 0, -1$	24. $\frac{2x+1}{x-3}, x \neq \pm 3$
Additional Practice Answers	
25. $x+3, x \neq 3$	28. 2
26. $x = -4$	29. $\frac{2x}{x}$
27. $x \neq 5$	30. $\frac{8}{x}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

- GCF on top and bottom is $4x$. Divide: $\frac{4x}{8x^2} = \frac{1}{2x}$. The original denominator is zero at $x = 0$, so $x \neq 0$. (The table values match $\frac{1}{2x}$ at $x = 1, 2, 4$ - a quick correctness check.)
- Top is a difference of squares: $(x-3)(x+3)$. Cancel $x-3$. The restriction $x \neq 3$ stays even though the factor cancels - that's the whole point. (Notice the table skips $x = 3$, where the original is undefined.)
- Factor the trinomial: $x^2 - 5x + 6 = (x-2)(x-3)$. Cancel $x-2$; the simplified form is $x-3$, valid for $x \neq 2$. (The table values agree with $x-3$ at $x = 0, 1, 4$.)
- Start with the key idea: $2-x = -(x-2)$. Cancel to get -1 . (Sign-flipped factors always collapse to -1 .) That gives a quick check on the answer.
- Coefficients: $\frac{6}{9} = \frac{2}{3}$. Powers: $\frac{x^2}{x} = x, \frac{y}{y^3} = \frac{1}{y^2}$. Combine: $\frac{2x}{3y^2}$.
- Top: $(x+3)(x-2)$. Bottom (difference of squares): $(x-2)(x+2)$. Cancel $x-2$. Both ± 2 remain excluded because they sat in the original denominator.
- Top: $2(x^2-4) = 2(x-2)(x+2)$. Bottom: $(x-2)(x+4)$. Cancel $x-2$. The factor pulls a 2 along for the ride.
- Start with the key idea: Top: $(x-3)(x-4)$. Bottom: $(x-3)(x+3)$. Cancel $x-3$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Top: $3(x-2)$. Bottom: $-2(x-2)$ (factor out the sign-flipper). Cancel $(x-2)$: $\frac{3}{-2} = -\frac{3}{2}$.
- Keep the rule visible: Top: $(x+4)(x-3)$. Bottom: $(x-4)(x+4)$. Cancel $x+4$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- One steady path is: Top: $(x-5)(x+5)$. Cancel $x+5$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Top: $x(x^2-1) = x(x-1)(x+1)$. Bottom: $x(x+1)$. Cancel the x and the $(x+1)$. Both $x = 0$ and $x = -1$ stay excluded.
- Top: $(x-1)(x+1)$. Bottom: $(x-1)^2$. Cancel one copy of $x-1$. (One $x-1$ remains in the denominator - still need $x \neq 1$.)

- Top is a difference of squares: $(2x-3)(2x+3)$. Cancel $2x+3$. The restriction $2x+3 \neq 0$ becomes $x \neq -\frac{3}{2}$.
- One steady path is: Bottom: $(x-5)(x+5)$. Top: $-(x-5)$. Cancel $(x-5)$: $\frac{-1}{x+5}$. That gives a quick check on the answer.
- Start with the key idea: Top: $(x+3)^2$. Bottom: $(x-3)(x+3)$. Cancel one $(x+3)$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- A careful way to see it: Top: $(2x+3)(x+1)$. Cancel $x+1$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Keep the rule visible: Top: $x(x-4)$. Bottom: $(x-4)(x+4)$. Cancel $x-4$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Top: $(x-4)(x+3)$. Bottom: $(x-4)(x+2)$. Cancel $x-4$. (Both original denominator factors $-x = -2$ and $x = 4$ remain excluded.)
- Top: $9-x^2 = -(x^2-9) = -(x-3)(x+3)$. Bottom: $x(x-3)$. Cancel $(x-3)$: $\frac{-(x+3)}{x} = -\frac{x+3}{x}$.
- Length = $\frac{\text{area}}{\text{width}} = \frac{x^2+5x+6}{x+2}$. Factor the top: $(x+2)(x+3)$. Cancel $x+2$: length = $x+3$ in. The restriction from the algebra is $x \neq -2$, but physically the rectangle's dimensions must be positive, so we also need $x > 0$ (which automatically rules out $x = -2$). At $x = 4$, for example, area = 42 in^2 , width = 6 in , length = 7 in - consistent with the simplified formula.
- Plug in $x = 2$: the left side is $\frac{2+3}{2} = \frac{5}{2} = 2.5$, not 3. So the "cancel the x " move is wrong. The reason: x on top is part of a sum $(x+3)$, not a factor. Only factors cancel. The expression $\frac{x+3}{x} = 1 + \frac{3}{x}$ is as simplified as it gets without canceling a common factor (and no such factor exists). This trap is one of the most common errors in Algebra 2 - it's worth keeping the numeric check in your back pocket.



23. Factor the numerator: $x^2 - 4 = (x - 2)(x + 2)$. So $f(x) = \frac{(x - 2)(x + 2)}{x - 2} = x + 2$ when $x \neq 2$. At $x = 2$ the original expression is $\frac{0}{0}$ – undefined – but the simplified version gives $2 + 2 = 4$. That mismatch is exactly what produces a *hole* at $(2, 4)$ on the graph: a single missing point on an otherwise straight line $y = x + 2$. (The open circle on the plot is the visual signature of a canceled factor – a removable discontinuity.)

24. Factor: top = $(2x + 1)(x + 3)$ (split $7x$ as $6x + x$; group $2x^2 + 6x = 2x(x + 3)$ and $x + 3$); bottom = $(x - 3)(x + 3)$. Cancel $x + 3$: $\frac{2x + 1}{x - 3}$. Restrictions: $x \neq \pm 3$ (both from the original denominator). The context $x > 3$ rules out both algebraic problem values automatically – the simplified rate is safe to use. Sanity check at $x = 5$: original = $\frac{50 + 35 + 3}{25 - 9} = \frac{88}{16} = \frac{11}{2}$. Simplified = $\frac{11}{2}$. Match.



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