Simplifying Radicals with Fractions

Name: ____________________	Date: ______________	Score: _____ / 34

Quick Review

Two new tools join the radical kit when fractions appear: the **quotient rule** and **rationalizing** a denominator.

Quotient rule. For $a \geq 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$. So $\sqrt{\frac{9}{16}} = \frac{3}{4}$ and $\sqrt{\frac{50}{9}} = \frac{5\sqrt{2}}{3}$. (Same rule going backward: $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$, which often collapses an ugly quotient into a perfect square.)

Why rationalize? A simplified-radical answer never leaves a radical in the denominator. It's a tidiness convention — it makes answers easier to compare and decimals easier to compute by hand. To clear a single $\sqrt{c}$ from a denominator, multiply numerator and denominator by $\sqrt{c}$: $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.

Binomial denominators — use the conjugate. The *conjugate* of $a + \sqrt{c}$ is $a - \sqrt{c}$ (and vice versa). Multiplying by the conjugate triggers the difference-of-squares pattern $(a+b)(a-b) = a^2 - b^2$, which kills the radical: $\frac{1}{\sqrt{5}-2} \cdot \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2$.

Order matters in the sign step. When the denominator after the conjugate step is negative (e.g. $4 - 7 = -3$ in $\frac{3}{2+\sqrt{7}} \cdot \frac{2-\sqrt{7}}{2-\sqrt{7}}$), the minus sign flips both terms in the numerator. Don't lose it.

Common slips. Leaving $\sqrt{\ }$ in the denominator. Forgetting that the conjugate only flips the sign on the radical term, not the rational one. Writing $\sqrt{\frac{a}{b}} = \sqrt{a} - \sqrt{b}$ (wrong — radicals don't split over addition or subtraction).

PRACTICE

Simplify and rationalize. The final form should never have a radical in the denominator.

1. Simplify $\sqrt{\frac{9}{16}}$. ________
2. Rationalize $\frac{1}{\sqrt{2}}$. ________
3. Simplify $\sqrt{\frac{50}{9}}$. ________
4. Rationalize $\frac{6}{\sqrt{3}}$. ________
5. Rationalize $\frac{1}{\sqrt{5}-2}$. ________
6. Simplify $\frac{\sqrt{12x^3}}{\sqrt{3x}}$ for $x > 0$. ________

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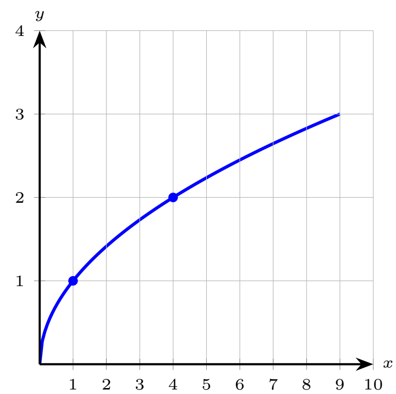
7. First simplify $\frac{\sqrt{45}}{\sqrt{5}}$ to a single number N. Then use the graph of $f(x) = \sqrt{x}$ to read off $f(N)$. ________



8. Mark TRUE or FALSE: A radical fraction is fully simplified even if a radical remains in the denominator. ________

9. Rationalize $\frac{3}{2+\sqrt{7}}$. ________

10. Simplify $\frac{\sqrt{80r^5}}{\sqrt{5r}}$ for $r > 0$. ________

11. Simplify $\sqrt{\frac{8}{25}}$. ________

12. Rationalize $\frac{4}{\sqrt{6}}$. ________

13. Rationalize $\frac{2}{\sqrt{3}+1}$. ________

14. Simplify $\sqrt{\frac{49a^2}{25}}$ for $a \geq 0$. ________

15. Mark TRUE or FALSE: $\sqrt{\frac{a}{b}} = \sqrt{a} - \sqrt{b}$ for $a, b \geq 0$. ________

16. Rationalize $\frac{5}{\sqrt{10}}$. ________

17. The table gives values of $g(x) = \sqrt{x}$ at several perfect squares. Using the pattern, find $g(64)$. ________

x	4	9	25	36
$g(x)$	2	3	5	6

18. Rationalize $\frac{\sqrt{2}}{\sqrt{3}}$. ________

19. Rationalize $\frac{\sqrt{2}}{\sqrt{5}-\sqrt{3}}$. ________

20. Simplify $\frac{\sqrt{27}}{\sqrt{75}}$. ________

Word Problems

21. A right triangle has legs of length 1 and $\sqrt{3}$ units. Find the exact ratio (hypotenuse) ÷ (shorter leg), in simplified radical form with no radical in the denominator. ____

22. A track shaped like a rectangle has area $\sqrt{72}$ square units and width $\sqrt{8}$ units. Find the exact length in simplified form, with no radical in the denominator. ____

23. Free-fall from a tall building takes time $t = \sqrt{\dfrac{2h}{g}}$ seconds, with h in meters and $g = 10$ m/s^2. For $h = 45$ m, give t exactly in simplified radical form, then approximate to two decimals. ____

24. The golden-ratio expression $\dfrac{1+\sqrt{5}}{2}$ appears in many places. Find its reciprocal in the form $a + b\sqrt{5}$ with no radical in the denominator. ____

Additional Practice

25. Simplify $\sqrt{72}$. ________

26. Simplify $\sqrt{45}$. ________

27. Simplify $\sqrt[3]{64}$. ________

28. Solve $\sqrt{x+5} = 9$. ________

29. Solve $\sqrt{x} - 3 = 4$. ________

30. Domain of $y = \sqrt{x-6}$. ________

31. Add $3\sqrt{5} + 2\sqrt{5}$. ________

32. Multiply $\sqrt{3} \cdot \sqrt{12}$. ________

33. Rationalize $\dfrac{4}{\sqrt{2}}$. ________

34. Write $x^{3/2}$ using radicals. ________

Answer Keys

1. $\boxed{\frac{3}{4}}$

2. $\boxed{\frac{\sqrt{2}}{2}}$

3. $\boxed{\frac{5\sqrt{2}}{3}}$

4. $\boxed{2\sqrt{3}}$

5. $\boxed{\sqrt{5}+2}$

6. $\boxed{2x}$

7. $\boxed{f(9)=3}$

8. FALSE

9. $\boxed{\sqrt{7}-2}$

10. $\boxed{4r^2}$

11. $\boxed{\frac{2\sqrt{2}}{5}}$

12. $\boxed{\frac{2\sqrt{6}}{3}}$

13. $\boxed{\sqrt{3}-1}$

14. $\boxed{\frac{7a}{5}}$

15. FALSE

16. $\boxed{\frac{\sqrt{10}}{2}}$

17. $\boxed{8}$

18. $\boxed{\frac{\sqrt{6}}{3}}$

19. $\boxed{\frac{\sqrt{10}+\sqrt{6}}{2}}$

20. $\boxed{\frac{3}{5}}$

21. $\boxed{2}$

22. $\boxed{3}$

23. 3 s (exact), ≈ 3.00 s

24. $\boxed{-\frac{1}{2}+\frac{1}{2}\sqrt{5}}$

Additional Practice Answers

25. $\boxed{6\sqrt{2}}$

26. $\boxed{3\sqrt{5}}$

27. $\boxed{4}$

28. $\boxed{x=76}$

29. $\boxed{x=49}$

30. $\boxed{x\ge 6}$

31. $\boxed{5\sqrt{5}}$

32. $\boxed{6}$

33. $\boxed{2\sqrt{2}}$

34. $\boxed{\sqrt{x^3}}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Quotient rule: $\frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$. Both numerator and denominator are perfect squares, so nothing's left under the radical.

2. Multiply top and bottom by $\sqrt{2}$: $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$. The denominator becomes $(\sqrt{2})^2 = 2$.

3. Split: $\frac{\sqrt{50}}{3}$. Then $\sqrt{50} = 5\sqrt{2}$, so the answer is $\frac{5\sqrt{2}}{3}$. The denominator is already a perfect square — no further rationalizing needed.

4. Start with the key idea: $\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$. Reducing $\frac{6}{3}$ at the end gives a clean integer coefficient. That gives a quick check on the answer.

5. Conjugate of $\sqrt{5}-2$ is $\sqrt{5}+2$. Multiply: $\frac{1}{\sqrt{5}-2} \cdot \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2$. The difference-of-squares pattern wipes the radical from below.

6. Use the quotient rule backward to merge the two radicals into one: $\frac{\sqrt{12x^3}}{\sqrt{3x}} = \sqrt{\frac{12x^3}{3x}} = \sqrt{4x^2} = 2|x|$. Since $x > 0$ is given, $|x| = x$, so the answer is $2x$. Combining first turns a messy quotient into a clean perfect square.

7. Combine first: $\frac{\sqrt{45}}{\sqrt{5}} = \sqrt{\frac{45}{5}} = \sqrt{9} = 3$, so $N = 9$. Now trace the curve at $x = 9$: the height is 3, i.e. $f(9) = \sqrt{9} = 3$. (Combining the radicals first turns an ugly quotient into a clean integer.)

8. Standard simplified form clears every radical out of the denominator. (Calculators don't care, but the convention makes hand-comparison easy.)

9. Conjugate $= 2-\sqrt{7}$. Multiply: $\frac{3(2-\sqrt{7})}{4-7} = \frac{3(2-\sqrt{7})}{-3} = -(2-\sqrt{7}) = \sqrt{7}-2$. The negative denominator flips both numerator terms — the sign step is where this problem hides.

10. Merge under one radical with the quotient rule: $\frac{\sqrt{80r^5}}{\sqrt{5r}} = \sqrt{\frac{80r^5}{5r}} = \sqrt{16r^4}$. Both 16 and r^4 are perfect squares, so this is $4r^2$. The $r > 0$ tag keeps the denominator nonzero and lets the root come out without bars.

11. Split with the quotient rule: $\sqrt{\frac{8}{25}} = \frac{\sqrt{8}}{\sqrt{25}} = \frac{\sqrt{8}}{5}$. The denominator $\sqrt{25} = 5$ is already rational, so just simplify the top: $\sqrt{8} = 2\sqrt{2}$, giving $\frac{2\sqrt{2}}{5}$. No conjugate needed here.

12. Start with the key idea: $\frac{4}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$. Always reduce the fraction at the end. That gives a quick check on the answer.

13. The conjugate of $\sqrt{3}+1$ is $\sqrt{3}-1$. Multiply top and bottom by it; the denominator becomes the difference of squares $(\sqrt{3})^2 - 1^2 = 3 - 1 = 2$: $\frac{2(\sqrt{3}-1)}{2} = \sqrt{3}-1$. The radical vanishes from below, exactly what the conjugate is for.

14. Apply the quotient rule, then root each perfect square: $\sqrt{\frac{49a^2}{25}} = \frac{\sqrt{49a^2}}{\sqrt{25}} = \frac{7|a|}{5}$. Since $a \ge 0$ is given, $|a| = a$, so the answer is $\frac{7a}{5}$ — already radical-free below.

15. Radicals don't turn quotients into differences. The quotient rule says $\sqrt{a/b} = \sqrt{a}/\sqrt{b}$ — a *quotient* of radicals, not a difference.

16. Multiply top and bottom by $\sqrt{10}$ to clear the root: $\frac{5}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{5\sqrt{10}}{10}$. Then reduce $\frac{5}{10}$ to $\frac{1}{2}$, leaving $\frac{\sqrt{10}}{2}$. Always reduce the fraction after rationalizing.

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17. Every entry is a perfect square mapped to its root: $g(36) = 6$ since $6^2 = 36$. As $64 = 8^2$, the pattern continues to $g(64) = 8$. (The table omits $x = 64$ on purpose — extend the pattern rather than reading it off.)

18. Multiply top and bottom by $\sqrt{3}$ to clear the denominator: $\frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{2}\cdot\sqrt{3}}{3} = \frac{\sqrt{6}}{3}$. The two top radicals merge by the product rule into $\sqrt{6}$, and the bottom becomes the rational 3.

19. The conjugate of $\sqrt{5} - \sqrt{3}$ is $\sqrt{5} + \sqrt{3}$. Multiply top and bottom by it; the denominator becomes the difference of squares $(\sqrt{5})^2 - (\sqrt{3})^2 = 5 - 3 = 2$. The numerator distributes: $\sqrt{2}\cdot\sqrt{5} + \sqrt{2}\cdot\sqrt{3} = \sqrt{10} + \sqrt{6}$, giving $\frac{\sqrt{10}+\sqrt{6}}{2}$.

20. Combine under one radical first: $\frac{\sqrt{27}}{\sqrt{75}} = \sqrt{\frac{27}{75}}$. Reduce the fraction $\frac{27}{75} = \frac{9}{25}$, which exposes a perfect square on top and bottom, so $\sqrt{\frac{9}{25}} = \frac{3}{5}$. Reducing before rooting saves you from simplifying two ugly radicals.

21. By the Pythagorean theorem, hypotenuse $= \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$. The ratio is $\frac{2}{1} = 2$, already radical-free. (This is the 30-60-90 triangle in disguise — worth recognizing.)

22. Length $= \frac{\sqrt{72}}{\sqrt{8}} = \sqrt{\frac{72}{8}} = \sqrt{9} = 3$. Combine the radicals before simplifying — the quotient under the radical reduces to a perfect square. (Try it the long way: $\sqrt{72} = 6\sqrt{2}$, $\sqrt{8} = 2\sqrt{2}$, ratio $= 6\sqrt{2}/(2\sqrt{2}) = 3$. Same answer, more steps.)

23. Plug in: $t = \sqrt{\frac{2(45)}{10}} = \sqrt{9} = 3$ s exactly. Decimal: 3.00 s. (The numbers were chosen to give a clean integer — when the radicand reduces this smoothly, double-check the problem expects an integer answer.)

24. Reciprocal $= \frac{2}{1+\sqrt{5}}$. Multiply by the conjugate $1 - \sqrt{5}$: $\frac{2(1-\sqrt{5})}{(1)^2 - (\sqrt{5})^2} = \frac{2(1-\sqrt{5})}{1-5} = \frac{2(1-\sqrt{5})}{-4} = \frac{\sqrt{5}-1}{2} = -\frac{1}{2} + \frac{1}{2}\sqrt{5}$. (Cute fact: this is exactly $\varphi - 1$, where $\varphi = (1+\sqrt{5})/2$ is the golden ratio. Its reciprocal differs from φ by exactly 1.)