Simplifying Radical Expressions

Name: ________ Date: ________ Score: ____ / 36

Quick Review

To **simplify a radical** means to pull out every perfect power that matches the index. The hidden machinery is the **product rule**: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ when the pieces are defined (for even indices we need $a, b \geq 0$; for odd indices any reals are fine). Once you can split the radical, factor out the perfect power and you're done.

The square-root recipe. Find the largest perfect-square factor of the radicand, split, and take its root. Quick check: $\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$. With variables, group the exponent into the biggest even part: $x^5 = x^4 \cdot x$, so $\sqrt{x^5} = x^2\sqrt{x}$ (for $x \geq 0$).

The principal-root rule that trips everyone up. The radical symbol means the *principal* (non-negative) root. So $\sqrt{36} = 6$, not ± 6. The equation $x^2 = 36$ has two solutions, but the radical symbol itself returns only the positive one. Watch for choice-B traps that quietly slip a $\pm$ in.

Absolute value for even roots of even powers. For any real a, $\sqrt{a^2} = |a|$. The bar makes the output non-negative no matter the sign of a. The same rule generalizes: $\sqrt[n]{a^n} = |a|$ when n is even, and $= a$ when n is odd. Many worksheets sidestep this by stating "assume all variables are non-negative." Once you see that assumption, the bars drop.

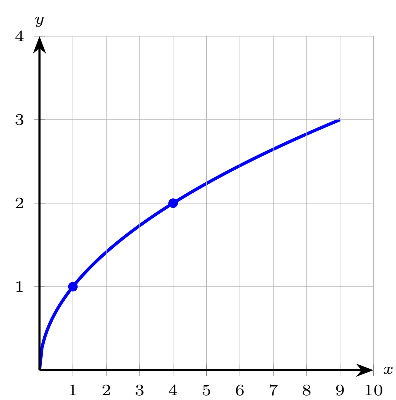
Higher-index radicals. Cube roots accept any real radicand: $\sqrt[3]{-8} = -2$. Fourth roots, sixth roots, and every even index need a non-negative radicand. For $\sqrt[3]{54}$, factor out the largest perfect cube: $54 = 27 \cdot 2$, so the answer is $3\sqrt[3]{2}$.

Common slips. Treating $\sqrt{a+b}$ as $\sqrt{a} + \sqrt{b}$ (wrong — radicals do not distribute over addition). Dropping the absolute value on $\sqrt{m^2}$ when m could be negative. Forgetting that $\sqrt{36}$ is just 6, not ± 6.

PRACTICE

Simplify each radical completely. Pull out perfect powers; keep absolute values when the variable's sign is not pinned down.

1. The curve below is $f(x) = \sqrt{x}$, with two lattice points marked to help you read it. Use the graph to find $f(9)$, the value of $\sqrt{9}$. ________



2. Simplify $\sqrt{50}$. ________
3. Simplify $\sqrt{72x^4}$ for real x. ________
4. Simplify $\sqrt[3]{54}$. ________
5. Simplify $\sqrt{45x^3y^6}$ for $x, y \geq 0$. ________
6. Simplify $\sqrt[4]{16x^8}$ for real x. ________
7. Simplify $\sqrt{200a^5b^3}$ for $a, b \geq 0$. ________
8. Simplify $\sqrt{98m^2n^4}$ for real m, n. ________
9. Simplify $\sqrt[3]{-128p^7q^3}$ for real p, q. ________

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10. Simplify $\sqrt{128}$.

11. Simplify $\sqrt{75}$.

12. Simplify $\sqrt{x^{10}}$ for real x.

13. Simplify $\sqrt[3]{-216}$.

14. Simplify $\sqrt{48x^2y}$ for $x \geq 0,\ y \geq 0$.

15. Simplify $\sqrt[3]{8a^6b^9}$ for real a, b.

16. Mark TRUE or FALSE: $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ for all $a, b \geq 0$.

17. Mark TRUE or FALSE: $\sqrt{x^6} = x^3$ for every real x.

18. Simplify $5\sqrt{20}$.

19. Simplify $\sqrt{300x^4}$ for real x.

20. The table gives values of $g(x) = \sqrt{x}$ at several perfect squares. Following the same pattern, find $g(49)$.

x	1	4	16	25
$g(x)$	1	2	4	5

Word Problems

21. A square plaza has area 288 m^2. Find the exact side length in simplified radical form, then approximate to two decimals.

22. A rectangular garden measures $6\sqrt{5}$ ft by $4\sqrt{15}$ ft. Find the exact area in simplified radical form.

23. The diagonal of a square has length $\sqrt{162}$ cm. Find the side length in simplified radical form. (Hint: for a square, diagonal $=$ side $\cdot\sqrt{2}$.)

24. A cube has volume 1080 in^3. Find the exact edge length in simplified radical form, then approximate to two decimals.

Additional Practice

25. Simplify $\sqrt{72}$.

26. Simplify $\sqrt{45}$.

27. Simplify $\sqrt[3]{64}$.

28. Solve $\sqrt{x+5} = 9$.

29. Solve $\sqrt{x} - 3 = 4$.

30. Domain of $y = \sqrt{x-6}$.

31. Add $3\sqrt{5} + 2\sqrt{5}$.

32. Multiply $\sqrt{3} \cdot \sqrt{12}$.

33. Rationalize $\dfrac{4}{\sqrt{2}}$.

34. Write $x^{3/2}$ using radicals.

35. Simplify $(\sqrt{7})^2$.

36. Solve $\sqrt{x+1} < 4$.

Answer Keys

1. $\boxed{3}$
2. $\boxed{5\sqrt{2}}$
3. $\boxed{6x^2\sqrt{2}}$
4. $\boxed{3\sqrt[3]{2}}$
5. $\boxed{3xy^3\sqrt{5x}}$
6. $\boxed{2x^2}$
7. $\boxed{10a^2b\sqrt{2ab}}$
8. $\boxed{7|m|n^2\sqrt{2}}$
9. $\boxed{-4p^2q\sqrt[3]{2p}}$
10. $\boxed{8\sqrt{2}}$
11. $\boxed{5\sqrt{3}}$
12. $\boxed{x^5 \text{ (if } x \geq 0), \ |x|^5 \text{ in general}}$
13. $\boxed{-6}$
14. $\boxed{4x\sqrt{3y}}$
15. $\boxed{2a^2b^3}$
16. FALSE
17. FALSE
18. $\boxed{10\sqrt{5}}$
19. $\boxed{10x^2\sqrt{3}}$
20. $\boxed{7}$
21. $\boxed{12\sqrt{2} \text{ m} \approx 16.97 \text{ m}}$
22. $\boxed{120\sqrt{3} \text{ ft}^2}$
23. $\boxed{9 \text{ cm}}$
24. $\boxed{3\sqrt[3]{40} \text{ in} \approx 10.26 \text{ in}}$

Additional Practice Answers

25. $\boxed{6\sqrt{2}}$
26. $\boxed{3\sqrt{5}}$
27. $\boxed{4}$
28. $\boxed{x = 76}$
29. $\boxed{x = 49}$
30. $\boxed{x \geq 6}$
31. $\boxed{5\sqrt{5}}$
32. $\boxed{6}$
33. $\boxed{2\sqrt{2}}$
34. $\boxed{\sqrt{x^3}}$
35. $\boxed{7}$
36. $\boxed{-1 \leq x < 15}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Trace up from $x = 9$ to the curve, then across: the height is 3. Algebraically $\sqrt{9} = 3$ because $3^2 = 9$. The marked points $(1, 1)$ and $(4, 2)$ just calibrate the square-root shape — the curve rises, but ever more slowly.

2. Keep the rule visible: $50 = 25 \cdot 2$, so $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$. Always reach for the *largest* perfect square — using $50 = 4 \cdot 12.5$ gets you nowhere. That gives a quick check on the answer.

3. Split the radical into its perfect-square pieces: $72 = 36 \cdot 2$ and $x^4 = (x^2)^2$, so $\sqrt{72x^4} = \sqrt{36}\,\sqrt{x^4}\,\sqrt{2} = 6x^2\sqrt{2}$. No absolute-value bar is needed on x^2 because $x^2 \geq 0$ for every real x — it already comes out non-negative.

4. Largest perfect cube in 54 is 27: $54 = 27 \cdot 2$. $\sqrt[3]{27} = 3$, so the answer is $3\sqrt[3]{2}$. (Powers of cubes to keep handy: $1, 8, 27, 64, 125$.)

5. Break each factor into its biggest perfect-square part: $45 = 9 \cdot 5$, $x^3 = x^2 \cdot x$, $y^6 = (y^3)^2$. The square roots of the perfect parts step outside — 3, x, and y^3 — while the leftovers 5 and x stay under, giving $3xy^3\sqrt{5x}$. With $x, y \geq 0$ stated, no absolute-value bars are needed.

6. Keep the rule visible: $16 = 2^4$ and $x^8 = (x^2)^4$, so $\sqrt[4]{16x^8} = 2x^2$. No absolute-value bar needed because $x^2 \geq 0$ already. That gives a quick check on the answer.

7. Pull out the biggest perfect-square factor of each piece: $200 = 100 \cdot 2$, $a^5 = a^4 \cdot a$, $b^3 = b^2 \cdot b$. Their roots $10, a^2, b$ come outside; the unpaired factors $2, a, b$ combine under one radical as $2ab$, giving $10a^2b\sqrt{2ab}$. The $a, b \geq 0$ condition lets every variable root drop its absolute-value bars.

8. Start with the key idea: $98 = 49 \cdot 2$. $\sqrt{m^2} = |m|$ because m could be negative, and the principal root must come out non-negative. $\sqrt{n^4} = n^2$ (already non-negative, no bars). Dropping the $|\cdot|$ here is the classic trap. That gives a quick check on the answer.

9. Look for perfect-cube factors this time, since the index is 3: $-128 = -64 \cdot 2$, $p^7 = (p^2)^3 \cdot p$, and $q^3 = q^3$. The cube roots -4, p^2, q step outside, leaving $2p$ under, so the answer is $-4p^2q\sqrt[3]{2p}$. Cube roots need no absolute-value bars — an odd index keeps the sign of its input.

10. The largest perfect square dividing 128 is 64, so $128 = 64 \cdot 2$ and $\sqrt{128} = \sqrt{64}\sqrt{2} = 8\sqrt{2}$. Reaching for 64 rather than a smaller square like 4 finishes the job in one step.

11. Factor out the largest perfect square: $75 = 25 \cdot 3$, so $\sqrt{75} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$. The 3 is square-free, so it stays under the radical and you're done.

12. Start with the key idea: $x^{10} = (x^5)^2$, so $\sqrt{x^{10}} = |x^5| = |x|^5$. When $x \geq 0$ it's just x^5; otherwise the absolute value matters because x^5 flips sign with x. That gives a quick check on the answer.

13. Ask what number cubed gives -216: since $6^3 = 216$, $(-6)^3 = -216$, so $\sqrt[3]{-216} = -6$. Cube roots accept negative radicands and return a single real value — an odd index keeps the sign, so no absolute value is involved.

14. Separate the perfect-square parts: $48 = 16 \cdot 3$ and x^2 is already a square, while y has no pair. The roots 4 and x step outside and $3y$ stays under, giving $4x\sqrt{3y}$. With $x \geq 0$ the bar on $\sqrt{x^2} = |x|$ drops to just x.

15. Every factor is already a perfect cube: $8 = 2^3$, $a^6 = (a^2)^3$, $b^9 = (b^3)^3$. The cube root divides each exponent by 3, so the whole thing comes out clean as $2a^2b^3$ with nothing left inside. (Odd index, so no absolute-value bars even though a, b may be negative.)

16. Radicals don't split over addition. A counterexample is $\sqrt{9 + 16} = \sqrt{25} = 5$, but $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$. Different numbers. This is the most-cited radical mistake in the book.

17. A careful way to see it: $\sqrt{x^6} = |x^3| = |x|^3$. When $x < 0$, $x^3 < 0$ but the principal square root is ≥ 0, so they disagree. True only when $x \geq 0$. That gives a quick check on the answer.

18. First simplify inside: $20 = 4 \cdot 5$, so $\sqrt{20} = 2\sqrt{5}$. Then multiply by the outside coefficient: $5 \cdot 2\sqrt{5} = 10\sqrt{5}$. Coefficients multiply together; the radical part rides along unchanged.

19. Pull out the perfect squares: $300 = 100 \cdot 3$ and $x^4 = (x^2)^2$. The roots 10 and x^2 step outside, leaving the square-free 3 under, so $\sqrt{300x^4} = 10x^2\sqrt{3}$. No bar on x^2 since it's never negative.

20. Each listed x is a perfect square and g returns its root: $g(25) = 5$ because $5^2 = 25$. Since $49 = 7^2$, the next value is $g(49) = 7$. (The table deliberately skips $x = 49$ — read the pattern, don't look it up.)

21. Side $= \sqrt{288}$. Factor: $288 = 144 \cdot 2$, so $\sqrt{288} = 12\sqrt{2}$ m exactly. Approximation: $12 \cdot 1.4142 \approx 16.97$ m. (Always present the exact radical first — the decimal is a follow-up, not a replacement.)

22. Multiply lengths: $(6\sqrt{5})(4\sqrt{15}) = 24\sqrt{75}$. Then $\sqrt{75} = 5\sqrt{3}$, so the area is $24 \cdot 5\sqrt{3} = 120\sqrt{3}$ ft^2. (Multiply the radicands first; simplify the final radical at the

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end — that order keeps the arithmetic clean.)

23. Side $= \frac{\sqrt{162}}{\sqrt{2}} = \sqrt{\frac{162}{2}} = \sqrt{81} = 9$ cm. (Combining the radicals first turns the ugly $\sqrt{162}$ into a clean integer — worth reaching for whenever a quotient of radicals hides a perfect square.)

24. Edge $= \sqrt[3]{1080}$. Factor: $1080 = 27 \cdot 40$, so $\sqrt[3]{1080} = 3\sqrt[3]{40}$ in exactly. Approximation: $\sqrt[3]{40} \approx 3.420$, so the edge is roughly $3 \cdot 3.420 = 10.26$ in. (Sanity: $10^3 = 1000$ and $11^3 = 1331$, so the edge should sit just above 10. Matches.)