Simplifying Complex Fractions

Name: _____	Date: _____	Score: _____ / 34

Quick Review

A **complex fraction** has fractions inside its numerator, its denominator, or both. Two reliable methods turn one into a single, clean rational expression.

Method 1 – combine, then divide. Combine the numerator into a single fraction. Combine the denominator into a single fraction. Then divide (multiply by the reciprocal). Slow but foolproof.

Method 2 – multiply by the LCD of all the small fractions. This is the fast method. Find the LCD of every small fraction (top *and* bottom). Multiply the entire complex fraction (top and bottom of the big bar) by that LCD. The little fractions all clear in one move. Quick check: $\frac{\frac{1}{x}+1}{\frac{1}{x}-1}\cdot\frac{x}{x}=\frac{1+x}{1-x}$.

Restrictions come from two places. (a) Every small denominator (those values would make the inner fractions undefined). (b) The overall denominator of the simplified complex fraction (those values would make the whole thing undefined).

Common slips. Forgetting to add restrictions from the small denominators after simplifying makes them disappear. Canceling *terms* (not factors) inside the small fractions. Multiplying the top by the LCD but not the bottom (you have to multiply *both*, otherwise you change the value).

PRACTICE

Multiply numerator and denominator by the LCD of all inner fractions, simplify, and state every restriction.

1. $\dfrac{\frac{2}{3}}{\frac{4}{9}}$ _____

2. Simplify $\dfrac{\frac{1}{x}+1}{\frac{1}{x}-1}$. The table evaluates the complex fraction at a few inputs. _____

x	2	3	4
value	-3	-2	$-\frac{5}{3}$

3. Simplify $\dfrac{\frac{1}{2}-\frac{1}{x}}{\frac{1}{2x}}$. The table samples the complex fraction at several inputs. _____

x	1	3	4
value	-1	1	2

4. $\dfrac{\frac{a}{b}}{\frac{c}{d}}$ _____

5. Simplify $\dfrac{1-\frac{1}{x}}{1+\frac{1}{x}}$. The table evaluates the complex fraction at a few inputs. _____

x	2	3	4
value	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{5}$

6. $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{xy}}$ _____

7. $\dfrac{\frac{1}{x+2}+\frac{1}{x-2}}{\frac{1}{x^2-4}}$ _____

8. $\dfrac{1+\frac{2}{x}}{3-\frac{1}{x}}$ _____

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9. $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x+y}}$ _____

10. $\dfrac{2-\frac{1}{x}}{x-\frac{1}{2}}$ _____

11. $\dfrac{\frac{x+1}{x}}{\frac{x-1}{x^2}}$ _____

12. $\dfrac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x}-\frac{1}{y}}$ _____

13. $\dfrac{x+\frac{1}{x}}{x-\frac{1}{x}}$ _____

14. $\dfrac{\frac{2}{x+3}}{\frac{4}{x^2-9}}$ _____

15. $\dfrac{1-\frac{4}{x^2}}{1-\frac{2}{x}}$ _____

16. $\dfrac{\frac{3}{x-1}}{\frac{6}{x+1}}$ _____

17. $\dfrac{\frac{a-b}{a+b}}{\frac{a^2-b^2}{(a+b)^2}}$ _____

18. Mark TRUE or FALSE: To simplify a complex fraction by the LCD method, multiply only the numerator by the LCD. _____

19. $\dfrac{\frac{x}{x+1}+1}{\frac{1}{x+1}}$ _____

20. $\dfrac{\frac{1}{x-1}-\frac{1}{x+1}}{\frac{2}{x^2-1}}$ _____

♦ Word Problems

21. In a parallel-resistor formula, the total resistance can be written as $R=\dfrac{1}{\frac{1}{r_1}+\frac{1}{r_2}}$. Simplify this complex fraction to a single rational expression in r_1 and r_2. _____

22. A student writes $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{2}-\frac{1}{3}}=\dfrac{1+1}{1-1}$ by "canceling the 2s and 3s." Find the correct value and explain the error. _____

23. Simplify the slope expression $m=\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$ to a clean single rational form. _____

24. In a harmonic-mean problem, the result is $H=\dfrac{2}{\frac{1}{a}+\frac{1}{b}}$. Simplify to a clean rational form, and find H when $a=4$ and $b=12$. _____

Additional Practice

25. Add $(3+2i)+(5-i)$. _____

26. Subtract $(4-i)-(1+6i)$. _____

27. Multiply $(2+3i)(1-i)$. _____

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28. Simplify i^{17}. _____

29. Simplify i^{22}. _____

30. Find the conjugate of $6 - 5i$. _____

31. Find $|3 + 4i|$. _____

32. Write $\sqrt{-36}$ in simplest form. _____

33. Divide $\frac{8 + 6i}{2}$. _____

34. Multiply $(5 + i)(5 - i)$. _____

Answer Keys

1. $\boxed{\frac{3}{2}}$
2. $\boxed{\frac{1+x}{1-x},\ x \neq 0,\ 1}$
3. $\boxed{x-2,\ x \neq 0}$
4. $\boxed{\frac{ad}{bc},\ b,c,d \neq 0}$
5. $\boxed{\frac{x-1}{x+1},\ x \neq 0,\ -1}$
6. $\boxed{y-x,\ x,y \neq 0}$
7. $\boxed{2x,\ x \neq \pm 2}$
8. $\boxed{\frac{x+2}{3x-1},\ x \neq 0,\ \frac{1}{3}}$
9. $\boxed{\frac{(x+y)^2}{xy},\ x,y \neq 0,\ x+y \neq 0}$
10. $\boxed{\frac{2}{x},\ x \neq 0,\ \frac{1}{2}}$
11. $\boxed{\frac{x(x+1)}{x-1},\ x \neq 0,\ 1}$
12. $\boxed{\frac{x+y}{xy},\ x,y \neq 0,\ x \neq y}$
13. $\boxed{\frac{x^2+1}{x^2-1},\ x \neq 0,\ \pm 1}$
14. $\boxed{\frac{x-3}{2},\ x \neq \pm 3}$
15. $\boxed{\frac{x+2}{x},\ x \neq 0,\ 2}$
16. $\boxed{\frac{x+1}{2(x-1)},\ x \neq \pm 1}$
17. $\boxed{1,\ a \neq \pm b}$
18. $\boxed{\text{FALSE}}$
19. $\boxed{2x+1,\ x \neq -1}$
20. $\boxed{1,\ x \neq \pm 1}$
21. $\boxed{R=\frac{r_1 r_2}{r_1+r_2},\ r_1,r_2 \neq 0,\ r_1+r_2 \neq 0}$
22. $\boxed{5}$
23. $\boxed{m=-\frac{1}{x(x+h)},\ x \neq 0,\ x+h \neq 0,\ h \neq 0}$
24. $\boxed{H=\frac{2ab}{a+b};\ H=6 \text{ when } a=4,\ b=12}$

Additional Practice Answers

25. $\boxed{8+i}$
26. $\boxed{3-7i}$
27. $\boxed{5+i}$
28. $\boxed{i}$
29. $\boxed{-1}$
30. $\boxed{6+5i}$
31. $\boxed{5}$
32. $\boxed{6i}$
33. $\boxed{4+3i}$
34. $\boxed{26}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Divide by the reciprocal: $\frac{2}{3} \cdot \frac{9}{4} = \frac{18}{12} = \frac{3}{2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

2. LCD of small fractions $= x$. Multiply top and bottom: $\frac{1+x}{1-x}$. Restrictions: $x \neq 0$ (small denom), $x \neq 1$ (overall denom). (The table matches $\frac{1+x}{1-x}$ at $x = 2, 3, 4$.)

3. LCD $= 2x$. Multiply top and bottom: $\frac{x-2}{1} = x-2$. (The table values match $x-2$ at $x = 1, 3, 4$.)

4. Start with the key idea: Multiply by reciprocal: $\frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

5. A careful way to see it: LCD $= x$. Multiply: $\frac{x-1}{x+1}$. (The table matches $\frac{x-1}{x+1}$ at $x = 2, 3, 4$.) That gives a quick check on the answer.

6. Keep the rule visible: LCD $= xy$. Top: $y-x$; bottom: 1. Result: $y-x$. This is the part to check before moving on, because it keeps the answer tied to the original question.

7. One steady path is: $x^2-4 = (x+2)(x-2)$. LCD $= (x+2)(x-2)$. Top: $(x-2)+(x+2) = 2x$. Bottom: 1. Result: $2x$. That gives a quick check on the answer.

8. Start with the key idea: LCD $= x$. Top: $x+2$; bottom: $3x-1$. This is the part to check before moving on, because it keeps the answer tied to the original question.

9. A careful way to see it: Top: $\frac{x+y}{xy}$. Divide by $\frac{1}{x+y}$ (i.e., multiply by $x+y$): $\frac{(x+y)^2}{xy}$. That gives a quick check on the answer.

10. LCD $= 2x$. Top: $4x-2 = 2(2x-1)$. Bottom: $2x^2-x = x(2x-1)$. Ratio: $\frac{2(2x-1)}{x(2x-1)} = \frac{2}{x}$.

11. One steady path is: Multiply by reciprocal: $\frac{x+1}{x} \cdot \frac{x^2}{x-1} = \frac{x(x+1)}{x-1}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

12. Top: $\frac{y^2-x^2}{x^2y^2} = \frac{(y-x)(y+x)}{x^2y^2}$. Bottom: $\frac{y-x}{xy}$. Divide: $\frac{(y-x)(y+x)}{x^2y^2} \cdot \frac{xy}{y-x} = \frac{x+y}{xy}$.

13. A careful way to see it: LCD $= x$. Top: x^2+1. Bottom: x^2-1. This is the part to check before moving on, because it keeps the answer tied to the original question.

14. Keep the rule visible: Multiply by reciprocal: $\frac{2}{x+3} \cdot \frac{x^2-9}{4} = \frac{2(x-3)(x+3)}{4(x+3)} = \frac{x-3}{2}$. That gives a quick check on the answer.

15. LCD $= x^2$. Top: $x^2-4 = (x-2)(x+2)$. Bottom: $x^2-2x = x(x-2)$. Ratio: $\frac{(x-2)(x+2)}{x(x-2)} = \frac{x+2}{x}$.

16. Start with the key idea: Multiply by reciprocal: $\frac{3}{x-1} \cdot \frac{x+1}{6} = \frac{x+1}{2(x-1)}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

17. A careful way to see it: $a^2-b^2 = (a-b)(a+b)$. Multiply by reciprocal:

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$\frac{a-b}{a+b} \cdot \frac{(a+b)^2}{(a-b)(a+b)} = 1$. That gives a quick check on the answer.

18. Multiply *both* numerator and denominator by the LCD. Multiplying just one side would change the value.

19. One steady path is: LCD $= x+1$. Top: $x + (x+1) = 2x+1$. Bottom: 1. Result: $2x+1$. That gives a quick check on the answer.

20. Start with the key idea: $x^2 - 1 = (x-1)(x+1)$. Top: $\frac{(x+1)-(x-1)}{(x-1)(x+1)} = \frac{2}{x^2-1}$. Bottom: $\frac{2}{x^2-1}$. Ratio: 1. That gives a quick check on the answer.

21. Bottom: $\frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1+r_2}{r_1 r_2}$. Then $R = \frac{1}{(r_1+r_2)/(r_1 r_2)} = \frac{r_1 r_2}{r_1+r_2}$. This is the familiar product-over-sum form for two parallel resistors. Sanity check with $r_1 = r_2 = 10$: $R = \frac{100}{20} = 5$ ohms (half of each individual value, as expected for two identical parallel resistors).

22. You cannot cancel digits inside fractions like that – canceling is for matching *factors*, not numerals. Use the LCD: top $= \frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = \frac{5}{6}$; bottom $= \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$. Divide: $\frac{5/6}{1/6} = 5$. (The student's "answer" $\frac{2}{0}$ is undefined – a tell that the cancellation move was illegal.)

23. Top: $\frac{1}{x+h} - \frac{1}{x} = \frac{x-(x+h)}{x(x+h)} = \frac{-h}{x(x+h)}$. Divide by h: $m = \frac{-h}{x(x+h)} \cdot \frac{1}{h} = -\frac{1}{x(x+h)}$. (This is the difference-quotient setup that becomes $f'(x) = -\frac{1}{x^2}$ for $f(x) = \frac{1}{x}$ as $h \to 0$. The simplification is exactly what makes the derivative computation manageable.)

24. Bottom of the big fraction: $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$. So $H = \frac{2}{(a+b)/(ab)} = \frac{2ab}{a+b}$. At $a = 4, b = 12$: $H = \frac{96}{16} = 6$. **Verify directly:** $\frac{1}{4} + \frac{1}{12} = \frac{3}{12} + \frac{1}{12} = \frac{4}{12} = \frac{1}{3}$, so $H = \frac{2}{1/3} = 6$. ✓. (Harmonic mean is always less than or equal to the arithmetic mean $\frac{a+b}{2} = 8$, and the value 6 confirms that ordering.)

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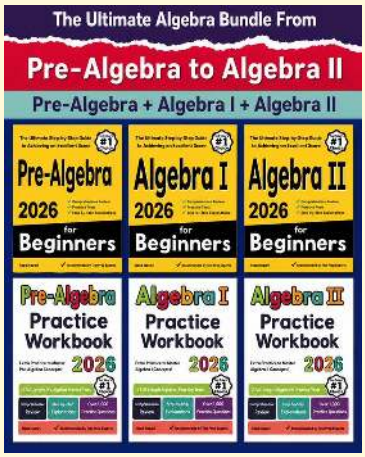

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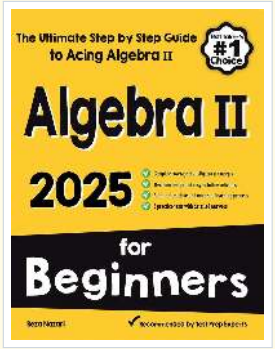

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