Sigma Notation

Name: _____	Date: _____	Score: _____ / 29

Quick Review

Sigma notation packs a long sum into a compact symbol. $\sum_{k=m}^{n} a_k$ means: plug $k = m, m+1, m+2, \ldots, n$ into a_k and add the results. Number of terms: $n - m + 1$ (count both endpoints).

Three rules you'll use constantly:

(1) Constant sum. $\sum_{k=1}^{n} c = c \cdot n$. You're adding c to itself n times.

(2) Linearity (distributive). $\sum_{k}(a_k + b_k) = \sum_{k} a_k + \sum_{k} b_k$ and $\sum_{k} c \cdot a_k = c \sum_{k} a_k$. Constants slide out front; sums split across addition.

(3) Closed forms. $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$. These let you compute big sums without expanding.

Switching the dummy variable. $\sum_{k=1}^{n} a_k$ and $\sum_{j=1}^{n} a_j$ are the same sum – the letter is a placeholder.

Re-indexing. $\sum_{k=2}^{6}(k-1)$ rewrites cleanly with $j = k - 1$ (so $j = 1$ to 5): $\sum_{j=1}^{5} j = 15$. The trick: shift the index, shift the formula to match, and the sum doesn't change.

Common slips. Off-by-one term counts: $\sum_{k=0}^{5}$ has 6 terms, not 5. Trying to split a sum of products ($\sum a_k b_k$ is *not* $(\sum a_k)(\sum b_k)$ in general). Forgetting to apply the closed-form formulas start at $k = 1$ – a sum starting at $k = 0$ may need a separate term added or a shift.

PRACTICE

Evaluate each sum. Use linearity and the closed-form formulas to avoid grinding through every term.

1. $\sum_{k=1}^{4} k$ _____

2. $\sum_{k=1}^{5} 7$ _____

3. Use the term table to evaluate $\sum_{k=1}^{5}(2k+1)$. _____

k	1	2	3	4	5
$2k+1$	3	5	7	9	11

4. $\sum_{k=1}^{10} k$ _____

5. Use the table to evaluate $\sum_{k=1}^{4} k^2$. _____

k	1	2	3	4
k^2	1	4	9	16

6. $\sum_{k=3}^{5}(k-2)$ _____

Scan Me

7. Use the term table to evaluate $\sum_{k=1}^{6}(3k+1)$. _____

k	1	2	3	4	5	6
$3k+1$	4	7	10	13	16	19

8. Which sigma expression equals $4+7+10+13+16+19$? _____

9. $\sum_{j=2}^{6}(3j-4)$ _____

10. $\sum_{k=1}^{8}(4k-3)$ _____

11. True or False: $\sum_{k=1}^{n} k = \frac{n(n-1)}{2}$. _____

12. $\sum_{k=0}^{4} k$ _____

13. $\sum_{k=1}^{6} k^2$ _____

14. $\sum_{k=1}^{5}(k^2-k)$ _____

15. $\sum_{k=1}^{20} 5$ _____

16. Number of terms in $\sum_{k=3}^{17} a_k$. _____

17. $\sum_{k=1}^{4}(2^k)$ _____

18. $\sum_{k=1}^{n}(2k-1)$ in closed form. _____

19. $\sum_{k=2}^{6}(k+3)$ _____

20. $\sum_{k=1}^{100} k$ _____

Scan Me

Word Problems

21. A theater hosts a different concert each night for 30 nights. On night k, ticket revenue is $\$(50k + 200)$. Express the total 30-night revenue in sigma notation and compute it. ____

22. A pizzeria sells pizzas for $\$12$ on Monday, $\$13$ on Tuesday, and so on, raising the price by $\$1$ each day for 7 days. Write the week's revenue as a sigma sum and compute it, assuming exactly one pizza is sold each day. ____

23. A robot collects k^2 samples on day k, for $k = 1, 2, \ldots, 5$. How many samples total? ____

24. A coach times 10 workouts in seconds: workout k lasts $60 + 5(k - 1)$ seconds. Express the total workout time as a sigma sum and find it in minutes. ____

Additional Practice

25. Find the next term: $4, 9, 14, 19, \ldots$. _____

26. Find a_{10} if $a_1 = 3$ and $d = 5$. _____

27. Find the next term: $2, 6, 18, 54, \ldots$. _____

28. Find a_6 if $a_1 = 5$ and $r = 2$. _____

29. Sum $1 + 2 + 3 + \cdots + 20$. _____

Scan Me

Answer Keys

1. $\boxed{10}$
2. $\boxed{35}$
3. $\boxed{35}$
4. $\boxed{55}$
5. $\boxed{30}$
6. $\boxed{6}$
7. $\boxed{69}$
8. $\boxed{\sum_{k=1}^{6}(3k+1)}$
9. $\boxed{40}$
10. $\boxed{120}$
11. $\boxed{\text{False}}$
12. $\boxed{10}$
13. $\boxed{91}$
14. $\boxed{40}$
15. $\boxed{100}$
16. $\boxed{15}$
17. $\boxed{30}$
18. $\boxed{n^2}$
19. $\boxed{35}$
20. $\boxed{5050}$
21. $\boxed{\sum_{k=1}^{30}(50k+200) = \$29{,}250}$
22. $\boxed{\sum_{k=1}^{7}(11+k) = \$105}$
23. $\boxed{55 \text{ samples}}$
24. $\boxed{\sum_{k=1}^{10}(60+5(k-1)) = 825 \text{ s} \approx 13.75 \text{ min}}$

Additional Practice Answers

25. $\boxed{24}$
26. $\boxed{48}$
27. $\boxed{162}$
28. $\boxed{160}$
29. $\boxed{210}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Add $1+2+3+4=10$. (Closed form: $\frac{4\cdot 5}{2}=10$.) This is the part to check before moving on, because it keeps the answer tied to the original question.

2. The summand is the constant 7, with k running 1 to 5 – five terms. The constant-sum rule says $\sum_{k=1}^{n} c = c\cdot n$, so this is $7\cdot 5 = 35$.

3. One steady path is: Add the listed terms: $3+5+7+9+11=35$. (Or split: $2\sum k+\sum 1 = 2(15)+5 = 35$.) That gives a quick check on the answer.

4. Start with the key idea: $\frac{10\cdot 11}{2}=55$. (Or pair $(1+10)+(2+9)+\cdots+(5+6)$: five pairs of 11.) That gives a quick check on the answer.

5. A careful way to see it: Add the squares from the table: $1+4+9+16=30$. (Closed form: $\frac{4\cdot 5\cdot 9}{6}=30$.) That gives a quick check on the answer.

6. Keep the rule visible: Plug in $k=3,4,5$: $1+2+3=6$. (Or re-index $j=k-2$: $\sum_{j=1}^{3} j = 6$.) That gives a quick check on the answer.

7. Add the listed terms: $4+7+10+13+16+19=69$. (Or arithmetic series with $a_1=4$, $a_6=19$: $\frac{6}{2}(4+19)=69$.)

8. The terms form an arithmetic sequence with $a_1=4$, $d=3$. Explicit: $a_k = 3k+1$. So $\sum_{k=1}^{6}(3k+1)$.

9. Plug in $j=2,3,4,5,6$: $2,5,8,11,14$. Sum: $2+5+8+11+14=40$. (Or arithmetic series with $a_1=2$, $n=5$, $a_5=14$: $\frac{5}{2}(16)=40$.)

10. Split: $4\sum_{k=1}^{8} k - 3\sum_{k=1}^{8} 1 = 4\cdot 36 - 3\cdot 8 = 144-24 = 120$. (Or arithmetic: $a_1=1$, $a_8=29$, sum $=\frac{8}{2}(30)=120$.)

11. The correct formula uses $n+1$, not $n-1$: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$. (Quick check at $n=3$: $1+2+3=6$, and $\frac{3\cdot 4}{2}=6$ ✓.)

12. Five terms ($k=0,1,2,3,4$): $0+1+2+3+4=10$. (Including $k=0$ adds nothing – so it equals $\sum_{k=1}^{4} k$.)

13. A careful way to see it: $1+4+9+16+25+36=91$. (Closed form: $\frac{6\cdot 7\cdot 13}{6}=91$.) This is the part to check before moving on, because it keeps the answer tied to the original question.

14. Keep the rule visible: Split: $\sum k^2 - \sum k = \frac{5\cdot 6\cdot 11}{6} - \frac{5\cdot 6}{2} = 55-15 = 40$. This is the part to check before moving on, because it keeps the answer tied to the original question.

15. The summand is the constant 5 and k runs 1 to 20, so there are 20 terms. By the constant-sum rule $\sum_{k=1}^{n} c = c\cdot n$, the total is $5\cdot 20 = 100$.

16. Start with the key idea: $17-3+1=15$. (Count both endpoints. Off-by-one trap: $17-3=14$ would skip one endpoint.) That gives a quick check on the answer.

17. A careful way to see it: $2+4+8+16=30$. (Or geometric series with $a_1=2$, $r=2$, $n=4$: $\frac{2(1-16)}{1-2}=30$.) That gives a quick check on the answer.

18. Split: $2\sum k - \sum 1 = 2\cdot\frac{n(n+1)}{2} - n = n(n+1)-n = n^2+n-n = n^2$. (The classic identity: sum of first n odd numbers is n^2.)

19. Plug in: $5+6+7+8+9=35$. (Or arithmetic series: $a_1=5$, $a_5=9$, sum $=\frac{5}{2}(14)=35$.)

20. Start with the key idea: $\frac{100\cdot 101}{2}=5050$. (Gauss's classic.) This is the part to check before moving on, because it keeps the answer tied to the original question.

21. Total $=\sum_{k=1}^{30}(50k+200)$. Split: $50\sum_{k=1}^{30} k + 200\sum_{k=1}^{30} 1 = 50\cdot\frac{30\cdot 31}{2} + 200\cdot 30 = 50\cdot 465 + 6000 = 23{,}250 + 6{,}000 = 29{,}250$, so \$29,250. (Reality check: average nightly revenue is $\$29{,}250/30 = \$975$ – right in the middle of night-1's \$250 and night-30's \$1700.)

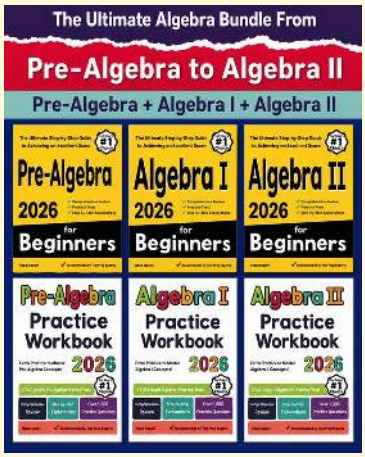
22. On day k, the price is $\$(11+k)$ (so day 1 is \$12). Total: $\sum_{k=1}^{7}(11+k) = \sum_{k=1}^{7} 11 + \sum_{k=1}^{7} k = 77+28 = 105$, so \$105 for the week. (Sanity check: $12+13+14+15+16+17+18 = 105$ ✓.)

23. One steady path is: $\sum_{k=1}^{5} k^2 = 1+4+9+16+25 = 55$ samples. (Closed form: $\frac{5\cdot 6\cdot 11}{6}=55$ ✓.) Reality check: a positive integer total, increasing daily counts – both fit. That gives a quick check on the answer.

24. Workout k lasts $60+5(k-1) = 55+5k$ seconds. Total seconds: $\sum_{k=1}^{10}(55+5k) = 10(55)+5\sum_{k=1}^{10} k = 550+5(55) = 550+275 = 825$ seconds. Convert: $825/60 = 13.75$ minutes. (Reality check: workout 1 is 60 seconds, workout 10 is 105 seconds, average is 82.5 seconds – times 10 is 825 ✓.)

Scan Me

Build Algebra Confidence From Pre-Algebra Through Algebra II



The Complete Algebra Success Bundle

Pre-Algebra, Algebra I, and Algebra II in one clear path

Friendly lessons, focused practice, and full-review support for every stage.

Scan Me

Scan for the Bundle

6 Books
3 Courses
1 Path

Bundle Value: Six coordinated books help students review missing skills, learn new algebra topics, and practice until the steps feel natural.

Complete Course Path

- ✓ Starts with Pre-Algebra foundations
- ✓ Moves smoothly into Algebra I skills
- ✓ Extends learning through Algebra II topics
- ✓ Great for review, tutoring, and summer study

One bundle, one steady path.

Step-by-Step Lessons

- ✓ Plain-English explanations students can follow
- ✓ Worked examples that show every important step
- ✓ Common mistakes called out before they stick
- ✓ Skill-building practice after each lesson
- ✓ Helpful for independent study or class support

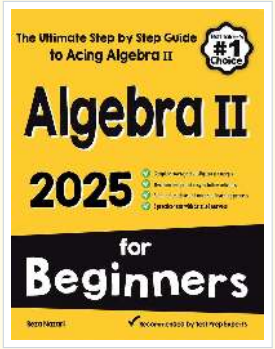
Less guessing. More understanding.

Practice That Sticks

- ✓ Matching practice workbooks for extra repetition
- ✓ Review sets to keep older skills fresh
- ✓ Answer explanations for checking thinking
- ✓ Strong support before tests and final exams
- ✓ Designed to build fluency and confidence

Practice today. Remember tomorrow.

STUDENT FAVORITE • Master Algebra II From the Ground Up



✓ 100% Guaranteed
✓ Lifetime Support
✓ Trusted by Teachers

Start Your Algebra Journey Today! →

Algebra II for Beginners

Written by a top math teacher & aligned with national and state Algebra II courses. From polynomial functions to logarithms, trigonometry, and rational expressions — explained the easy way.

- ✓ **Complete coverage** of every Algebra II concept — perfect companion to these worksheets
- ✓ **Step-by-step explanations** with worked examples on every topic
- ✓ **QR codes in every chapter** for free video lessons & bonus practice
- ✓ **2 full-length practice tests** with detailed answer keys

★ STUDENT'S #1 CHOICE ★

Teacher-recommended • 12,000+ Happy Students

↓ PDF EDITION

Scan Me

Instant download • any device

PAPERBACK

Scan Me

Paperback on Amazon

Hold it in your hands

Pair these free worksheets with *Algebra II for Beginners* and you have a complete self-paced course — concept lessons, daily practice, and full exam-style reviews, all in one path. → **EffortlessMath.com/product/algebra-ii-for-beginners**