Sampling Distributions and Margin of Error

Name: ________________	Date: ____________	Score: _____ / 35

Quick Review

A **sampling distribution** is the distribution of a statistic across all possible samples of the same size from a population. It's a distribution *of statistics*, not of raw data.

Why it matters. If you knew the exact sampling distribution of $\bar{X}$, you'd know how much sample-to-sample variation to expect. The CLT gives you that knowledge for large samples: $\bar{X}$ is approximately $N(\mu, \sigma/\sqrt{n})$.

Confidence interval for a mean (large n). $\bar{x} \pm z^* \dfrac{\sigma}{\sqrt{n}}$, where z^* is the critical value (for 95%: $z^* \approx 1.96$; for 90%: $z^* \approx 1.645$; for 99%: $z^* \approx 2.576$).

Confidence interval for a proportion. $\hat{p} \pm z^* \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$. The thing after the $\pm$ is the **margin of error** (MOE).

What "95% confidence" really means. If you repeated the sampling process many times and built a CI each time, about 95% of those intervals would contain the true parameter. It does *not* mean "there is a 95% chance the parameter is in this particular interval" – the parameter is fixed; the interval varies.

Bigger sample, smaller MOE. Standard error shrinks like $1/\sqrt{n}$, so quadrupling n halves the MOE. To go from $\pm 4\%$ down to $\pm 2\%$, the sample size has to *quadruple*, not double.

Higher confidence, larger MOE. Going from 95% to 99% confidence uses a bigger z^*, which makes the interval wider. There's no free lunch.

Common slips. Saying "95% probability the parameter is in this interval" (wrong interpretation). Doubling n and expecting the MOE to halve (it needs to quadruple). Forgetting to divide by $\sqrt{n}$ when computing standard error. Confusing the sample size with the margin of error directly.

PRACTICE

Compute margins of error, intervals, and required sample sizes.

1. Sampling distribution of a statistic $=?$ ________
2. MOE for a proportion at 95% ________
3. Increase $n \Rightarrow$ MOE ________
4. $\hat{p} = 0.5,\ n = 400$. MOE at 95% ________
5. $\hat{p} = 0.6,\ n = 100$. MOE at 95% ________
6. Critical value for a 95% CI ________
7. Critical value for a 90% CI ________
8. Critical value for a 99% CI ________
9. Use sample 1 from the table to build a 95% CI for μ. ________

Sample	$\bar{x}$	σ	n
1	50	10	25
2	80	12	36
3	100	20	400

10. To halve MOE, multiply n by ________
11. $\hat{p} = 0.5$; want MOE ≤ 0.03 at 95%. Find min n. ________
12. True or false: "95% CI" means there's a 95% chance the parameter is in this CI ________
13. If confidence goes up from 95% to 99%, MOE ________

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14. Use sample 2 from the table above to build a 95% CI. ________

Sample	$\bar{x}$	σ	n
1	50	10	25
2	80	12	36
3	100	20	400

15. If MOE is small, the estimate is ________

16. $\hat{p} = 0.7,\ n = 200$. MOE at 95% ________

17. Use sample 3 from the table above to build a 95% CI. ________

Sample	$\bar{x}$	σ	n
1	50	10	25
2	80	12	36
3	100	20	400

18. Same setup but $n = 1600$. MOE ________

19. Center of the sampling distribution of $\bar{X}$ ________

20. Standard deviation of $\bar{X}$ ________

♦ Word Problems

21. A polling organization surveys $1{,}000$ randomly chosen voters and finds that $\hat{p} = 0.52$ support candidate A. Construct a 95% confidence interval for the true population proportion. ____

22. A polling organization wants a margin of error of at most $\pm 2\%$ at the 95% confidence level, using $\hat{p} = 0.5$ as a worst-case planning value. What is the minimum required sample size? ____

23. A quality engineer samples $n = 49$ widgets and finds a mean weight of $\bar{x} = 200$ g with $\sigma = 7$ g (known from past data). Construct a 95% confidence interval for the true mean widget weight. ____

24. A researcher reports a 95% confidence interval of $[42, 58]$ for the mean of a population. A student says: "So there's a 95% chance the population mean is between 42 and 58." What's the correct interpretation, and what makes the student's phrasing wrong? ____

Additional Practice

25. Find the mean of $4, 6, 8, 10$. ________

26. Find the median of $3, 9, 4, 10, 7$. ________

27. Find the range of $12, 5, 9, 20$. ________

28. Find the mode of $2, 3, 3, 5, 8$. ________

29. Find z for $x = 72$, mean 60, standard deviation 6. ________

30. Interpret $z = -1.5$. ________

31. Predicted y for $\hat{y} = 2x + 5$ at $x = 6$. ________

32. Residual if actual $= 20$ and predicted $= 17$. ________

33. Positive association: slope sign? ________

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34. Margin of error $= 3\%$ around 58%. ________

35. Sample or census: survey every student. ________

Answer Keys

1. distribution over all size-n samples	13. increases
2. $1.96\sqrt{\hat{p}(1-\hat{p})/n}$	14. $[76.08,\ 83.92]$
3. decreases	15. precise
4. 0.049	16. ≈ 0.064
5. ≈ 0.096	17. $[98.04,\ 101.96]$
6. 1.96	18. 0.98
7. 1.645	19. μ
8. 2.576	20. $\sigma/\sqrt{n}$
9. $[46.08,\ 53.92]$	21. $[0.489,\ 0.551]$
10. 4	22. $n = 2401$
11. ≈ 1068	23. $[198.04,\ 201.96]$ g
12. False	24. about 95% of such CIs capture μ

Additional Practice Answers

25. 7	31. 17
26. 7	32. 3
27. 15	33. positive
28. 3	34. 55% to 61%
29. 2	35. census
30. 1.5 SD below mean	

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. The distribution of the statistic's values across every possible sample of the same size – not the distribution of one sample's raw data.
2. Keep the rule visible: Critical value 1.96 times the standard error of the sample proportion. That gives a quick check on the answer.
3. One steady path is: Standard error is $1/\sqrt{n}$, so larger samples mean smaller MOE. That gives a quick check on the answer.
4. Start with the key idea: $1.96\sqrt{\frac{0.5(0.5)}{400}} = 1.96\sqrt{0.000625} = 1.96(0.025) = 0.049$. This is the part to check before moving on, because it keeps the answer tied to the original question.
5. A careful way to see it: $1.96\sqrt{\frac{0.6(0.4)}{100}} = 1.96\sqrt{0.0024} \approx 1.96(0.049) \approx 0.096$. This is the part to check before moving on, because it keeps the answer tied to the original question.
6. Keep the rule visible: Standard z^* for 95% confidence. This is the part to check before moving on, because it keeps the answer tied to the original question.
7. One steady path is: $z^* \approx 1.645$ from the standard normal table. This is the part to check before moving on, because it keeps the answer tied to the original question.
8. Start with the key idea: More confidence ⇒ bigger critical value ⇒ wider interval. This is the part to check before moving on, because it keeps the answer tied to the original question.
9. A careful way to see it: Sample 1: SE $= 10/\sqrt{25} = 2$. MOE $= 1.96(2) = 3.92$. CI: $50 \pm 3.92 = [46.08,\ 53.92]$. That gives a quick check on the answer.
10. Keep the rule visible: MOE $\propto 1/\sqrt{n}$. To cut MOE in half, $\sqrt{n}$ must double, so n must quadruple. That gives a quick check on the answer.
11. One steady path is: $1.96\sqrt{\frac{0.25}{n}} \le 0.03 \Rightarrow \sqrt{\frac{0.25}{n}} \le \frac{0.03}{1.96} \Rightarrow n \ge \frac{0.25 \cdot 1.96^2}{0.03^2} \approx 1067.1$, so use $n = 1068$. That gives a quick check on the answer.
12. The parameter is fixed; the interval is the random thing. 95% of intervals built from repeated sampling would capture the parameter.
13. A careful way to see it: Bigger z^* multiplier ⇒ wider interval. More confidence costs precision. That gives a quick check on the answer.
14. Keep the rule visible: Sample 2: SE $= 12/\sqrt{36} = 2$. MOE $= 1.96(2) = 3.92$. CI: $80 \pm 3.92 = [76.08,\ 83.92]$. That gives a quick check on the answer.
15. Small MOE = tight interval = precise estimate. Doesn't necessarily mean *accurate* – precision can be misleading if there's bias.
16. Start with the key idea: $1.96\sqrt{\frac{0.7(0.3)}{200}} = 1.96\sqrt{0.00105} \approx 1.96(0.0324) \approx 0.064$. This is the part to check before moving on, because it keeps the answer tied to the original question.
17. Sample 3: SE $= 20/\sqrt{400} = 1$. MOE $= 1.96(1) = 1.96$. CI: $100 \pm 1.96 = [98.04,\ 101.96]$.
18. Keep the rule visible: SE $= 20/\sqrt{1600} = 0.5$. MOE $= 1.96(0.5) = 0.98$. Quadrupling n halved the MOE. That gives a quick check on the answer.
19. One steady path is: $E[\bar{X}] = \mu$. Repeated sampling averages out to the true mean. That gives a quick check on the answer.
20. Start with the key idea: Standard error – shrinks as $\sqrt{n}$ grows. This is the part to check before moving on, because it keeps the answer tied to the original question.
21. Standard error: $\sqrt{\frac{0.52(0.48)}{1000}} = \sqrt{0.0002496} \approx 0.0158$. MOE: $1.96(0.0158) \approx 0.031$. CI: $0.52 \pm 0.031 = [0.489,\ 0.551]$. Notice the interval includes 0.50, so we can't confidently say candidate A is ahead – the race is a statistical dead heat at this sample size.
22. Set up MOE ≤ 0.02: $1.96\sqrt{\frac{0.5(0.5)}{n}} \le 0.02$. Solve: $\sqrt{\frac{0.25}{n}} \le \frac{0.02}{1.96} \approx 0.01020$. Square: $\frac{0.25}{n} \le 0.000104$. So $n \ge \frac{0.25}{0.000104} \approx 2401$. Use $n = 2401$. (This is why polling firms typically sample around $2{,}500$ for $\pm 2\%$ precision.)
23. SE $= 7/\sqrt{49} = 1$. MOE $= 1.96(1) = 1.96$. CI: 200 ± 1.96, i.e., $[198.04, 201.96]$ grams. So we're 95% confident the true mean widget weight is in that interval – comfortably consistent with a target of 200 g.
24. The parameter μ is fixed – it either is or isn't in $[42, 58]$. The interval is the random thing. The correct interpretation: "In the long run, about 95% of intervals built this way capture the true μ." The student's phrasing makes it sound like the parameter has a 95% probability of landing in our specific interval – a common but misleading way to talk about confidence.

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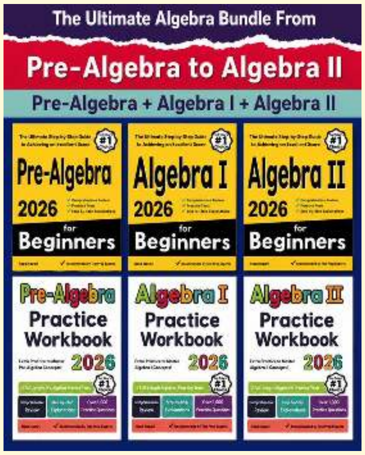

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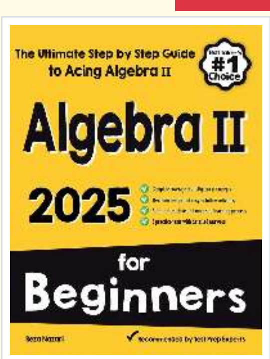

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