Remainder and Factor Theorems

Name: ______________________	Date: ______________	Score: _____ / 32

Quick Review

The **Remainder Theorem** is a shortcut for finding the remainder when a polynomial $f(x)$ is divided by $(x-c)$: just evaluate $f(c)$. No long division needed.

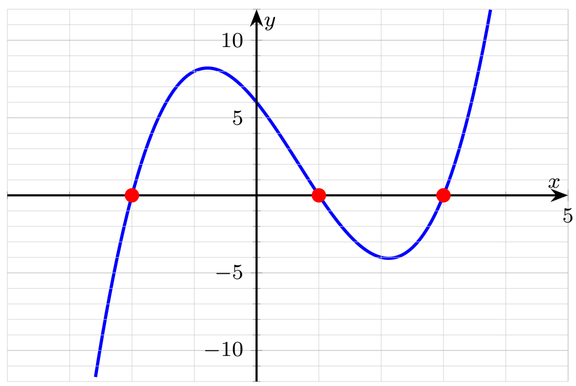
Why does this work? If $f(x) = (x-c)q(x) + r$, plugging in $x = c$ gives $f(c) = (c-c)q(c) + r = 0 + r = r$. The divisor vanishes, leaving only the remainder.

The **Factor Theorem** is the zero-remainder case: $(x-c)$ is a factor of $f(x)$ *if and only if* $f(c) = 0$. So checking whether a number is a root is the same as checking whether the matching linear binomial is a factor.

Sign convention to memorize: for division by $(x+a)$, write it as $(x-(-a))$, so $c = -a$. Quick check: dividing by $(x+3)$ means $c = -3$. Plug in -3, not 3. This sign flip is the single most common mistake.

The Remainder/Factor Theorems together give a quick way to test candidate rational roots: just plug them in. Whichever values make $f(c) = 0$ are zeros, and each one corresponds to a linear factor.

The graph below shows $f(x) = x^3 - 2x^2 - 5x + 6$. By the Factor Theorem, the x-intercepts (red dots) at $-2, 1, 3$ correspond to factors $(x+2)(x-1)(x-3)$.



PRACTICE

Use the Remainder or Factor Theorem to answer each question.

1. The table gives some values of $f(x) = x^2 + 3x - 4$. Use the Remainder Theorem to find the remainder when f is divided by $(x-2)$. ________

x	-1	0	1	3
$f(x)$	-6	-4	0	14

2. Is $(x+3)$ a factor of $f(x)$ when $f(-3) = 0$? ________
3. Is $(x-1)$ a factor of $f(x) = x^3 - 2x^2 + 5x - 4$? ________
4. Find k if $(x-2)$ is a factor of $f(x) = x^3 + kx^2 - x - 14$ ________
5. Remainder when $f(x) = 2x^3 - x^2 + 4x - 5$ is divided by $(x+1)$ ________
6. If $f(x) = x^3 + ax^2 - 5x + 2$ divided by $(x-1)$ has remainder 4, find a ________
7. If $p(x) = x^3 + kx^2 - 4x - 12$ has $(x+2)$ as a factor, find k ________

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8. Quotient when $f(x) = x^3 - 4x^2 + x + 6$ is divided by $(x - 3)$ ________

$$\begin{array}{c|cccc} 3 & 1 & -4 & 1 & 6 \\ & & 3 & -3 & -6 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

9. Test whether $(x - 2)$ is a factor of $x^3 - 8$ ________

10. Remainder when $x^3 + 2x^2 - 5x + 1$ is divided by $(x + 3)$ ________

11. If $f(c) = -5$, what is the remainder when $f(x)$ is divided by $(x - c)$? ________

12. Is $(x + 1)$ a factor of $x^4 - 1$? ________

13. The table gives some values of $f(x) = 2x^3 - 5x^2 + 4x - 1$. Use the Remainder Theorem to find the remainder when f is divided by $(x - 2)$. ________

x	-1	0	1	3
$f(x)$	-12	-1	0	20

14. If $p(2) = 0$, which linear factor must divide $p(x)$? ________

15. Remainder when $x^4 + 3x^2 - 2$ is divided by $(x - 1)$ ________

16. Remainder when $x^4 + 3x^2 - 2$ is divided by $(x + 1)$ ________

17. Is $(x - 4)$ a factor of $f(x) = x^3 - 3x^2 - x + 3$? ________

18. Find a if $(x + 2)$ is a factor of $x^3 - ax - 6$ ________

19. Remainder when $f(x) = x^2 + 4x + 7$ divided by $(x + 1)$ ________

20. Statement: if $f(c) \neq 0$, then $(x - c)$ is ________

♦ Word Problems

21. A polynomial $p(x)$ satisfies $p(5) = 0$. State what the Factor Theorem tells you and write a quick test you could use to verify it. ____

22. When a polynomial $f(x)$ is divided by $(x - 3)$, the remainder is 7. What is $f(3)$, and what does this tell you about whether $(x - 3)$ is a factor? ____

23. A cubic polynomial $p(x) = x^3 + kx^2 - x - 6$ has $(x - 2)$ as a factor. Find k, then write the full factored form of $p(x)$. ____

24. A polynomial $p(x) = 2x^3 - 7x^2 + ax - 5$ has remainder 3 when divided by $(x - 1)$. Find the value of a. ____

Additional Practice

25. Write $3x - 5 + x^3$ in standard form. ________

26. Find the degree of $7x^4 - 2x^2 + 9$. ________

27. Add $(2x^2 + 3x - 1) + (x^2 - 5x + 4)$. ________

28. Subtract $(5x^2 - x + 6) - (2x^2 + 3x - 1)$. ________

29. Multiply $(x + 4)(x - 3)$. ________

30. Factor $x^2 + 9x + 20$. ________

31. Factor $6x^2 + 9x$. ________

32. Find the GCF of $12x^3$ and $18x^2$. ________

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Answer Keys

1. 6	**13.** 3
2. yes (Factor Theorem)	**14.** $(x-2)$
3. yes, since $f(1)=0$	**15.** 2
4. $k=2$	**16.** 2
5. -12	**17.** no, since $f(4)=15\neq 0$
6. $a=6$	**18.** $a=7$
7. $k=3$	**19.** 4
8. $q(x)=x^2-x-2$	**20.** not a factor
9. yes, since $f(2)=0$	**21.** $(x-5)$ is a factor of $p(x)$
10. 7	**22.** $f(3)=7;\ (x-3)$ is not a factor
11. -5	**23.** $k=0;\ p(x)=(x-2)(x^2+2x+3)$
12. yes	**24.** $a=13$

Additional Practice Answers

25. x^3+3x-5	**29.** x^2+x-12
26. 4	**30.** $(x+4)(x+5)$
27. $3x^2-2x+3$	**31.** $3x(2x+3)$
28. $3x^2-4x+7$	**32.** $6x^2$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. The Remainder Theorem says the remainder equals $f(2)$. The table skips $x=2$, so compute it: $f(2)=4+6-4=6$.
2. Keep the rule visible: $(x+3)=(x-(-3))$, so test $c=-3$. Since $f(-3)=0$, the Factor Theorem says $(x+3)$ is a factor. That gives a quick check on the answer.
3. One steady path is: Test $c=1$: $f(1)=1-2+5-4=0$. By the Factor Theorem, $(x-1)$ is a factor. That gives a quick check on the answer.
4. By the Factor Theorem, $(x-2)$ being a factor means $f(2)=0$. Substitute $x=2$: $f(2)=8+4k-2-14=4k-8$. Set that equal to 0 and solve: $4k=8$, so $k=2$.
5. Test $c=-1$: $f(-1)=2(-1)^3-(-1)^2+4(-1)-5=-2-1-4-5=-12$. Watch the signs — $(-1)^2=+1$, not -1.
6. By the Remainder Theorem, dividing by $(x-1)$ leaves remainder $f(1)$. Substitute $x=1$: $f(1)=1+a-5+2=a-2$. Set $a-2=4$, so $a=6$.
7. One steady path is: $(x+2)=(x-(-2))$, so a factor means $p(-2)=0$. Substitute $x=-2$: $(-2)^3+k(4)-4(-2)-12=-8+4k+8-12=4k-12$. Set $4k-12=0$, giving $k=3$. That gives a quick check on the answer.
8. First confirm the factor: $f(3)=27-36+3+6=0$ ✓. Synthetic division by $c=3$ with $1,-4,1,6$ gives $1,-1,-2$ with remainder 0. Quotient: x^2-x-2.
9. A careful way to see it: $f(2)=8-8=0$. So $(x-2)$ is a factor. (Bonus: $x^3-8=(x-2)(x^2+2x+4)$.) That gives a quick check on the answer.
10. Dividing by $(x+3)$ means $c=-3$, so the remainder is $f(-3)$. Substitute carefully: $(-3)^3+2(9)-5(-3)+1=-27+18+15+1=7$.
11. One steady path is: Remainder Theorem: remainder $=f(c)=-5$. This is the part to check before moving on, because it keeps the answer tied to the original question.
12. Start with the key idea: Test $c=-1$: $f(-1)=1-1=0$. Yes. (And $x^4-1=(x-1)(x+1)(x^2+1)$.) That gives a quick check on the answer.
13. The remainder is $f(2)$, which the table leaves out. Compute it: $f(2)=16-20+8-1=3$.
14. By the Factor Theorem, $p(c)=0$ corresponds to factor $(x-c)$. Here $c=2$, so $(x-2)$.
15. Dividing by $(x-1)$ gives $c=1$, so the remainder is $f(1)=1+3-2=2$ (the x^4 and x^2 terms are both 1 at $x=1$).
16. Start with the key idea: $f(-1)=1+3-2=2$. Same answer because all exponents in the polynomial are even — the function is even. That gives a quick check on the answer.
17. A careful way to see it: $f(4)=64-48-4+3=15$. Nonzero remainder, so $(x-4)$ is not a factor. That gives a quick check on the answer.
18. Keep the rule visible: $f(-2)=(-2)^3-a(-2)-6=-8+2a-6=2a-14$. Set $2a-14=0$: $a=7$. Check: with $a=7$, $f(x)=x^3-7x-6$ and $f(-2)=-8+14-6=0$ ✓. That gives a quick check on the answer.
19. Dividing by $(x+1)$ means $c=-1$, so the remainder is $f(-1)=1-4+7=4$ (note $(-1)^2=1$, giving $+1$).
20. Start with the key idea: Contrapositive of the Factor Theorem: if $f(c)\neq 0$, then $(x-c)$ is not a factor. That gives a quick check on the answer.
21. By the Factor Theorem, $p(c)=0$ iff $(x-c)$ is a factor. With $c=5$, $(x-5)$ is a factor of $p(x)$. To verify, divide $p(x)$ by $(x-5)$ using long or synthetic division and confirm the remainder is 0.
22. The Remainder Theorem says the remainder when dividing by $(x-c)$ equals $f(c)$. So $f(3)=7$. Since $f(3)\neq 0$, $(x-3)$ is not a factor of $f(x)$.
23. Factor Theorem: $p(2)=0$. Compute: $p(2)=8+4k-2-6=4k$. Set $4k=0$: $k=0$. With $k=0$, $p(x)=x^3-x-6$. Divide by $(x-2)$ (synthetic or long): quotient is x^2+2x+3, remainder 0. So $p(x)=(x-2)(x^2+2x+3)$. The quadratic factor has discriminant $4-12=-8<0$, so it's irreducible over the reals.
24. Remainder Theorem: $p(1)=3$. Compute $p(1)=2-7+a-5=a-10$. Set $a-10=3$: $a=13$. Quick check: $p(x)=2x^3-7x^2+13x-5$, and $p(1)=2-7+13-5=3$ ✓.

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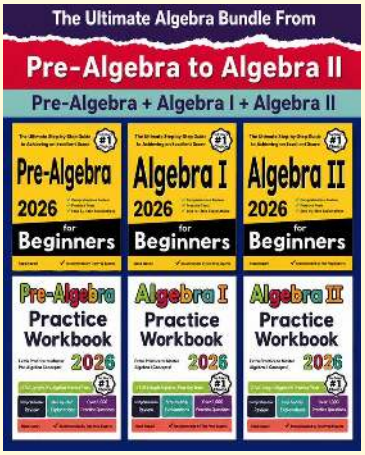

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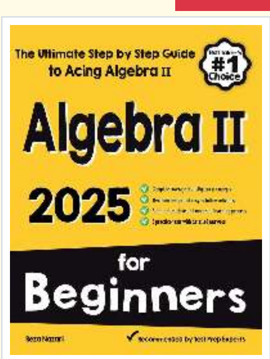

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