Reference Angles and the Calculator

Name: _____	Date: _____	Score: _____ / 29

Quick Review

For any angle θ in standard position, the **reference angle** is the positive acute angle the terminal side makes with the x-axis. Reference angles let you reduce every trig calculation to a Q1 lookup followed by a quadrant sign.

Procedure for any trig function at any θ.

1) Reduce θ into $[0, 360^\circ)$ (or $[0, 2\pi)$) by adding or subtracting full rotations.

2) Identify the quadrant of θ.

3) Find the reference angle (acute, positive).

4) Compute the trig value at the reference angle (with calculator or special-angle table).

5) Apply the quadrant sign (ASTC: *A*ll in Q1, *S*ine in Q2, *T*angent in Q3, *C*osine in Q4).

Example. $\sin 200^\circ$: Q3, reference 20°. Sine is negative in Q3. $\sin 200^\circ = -\sin 20^\circ \approx -0.342$.

Why bother with reference angles when the calculator can do it directly? Two reasons. First, the procedure works for exact special-angle values too (no calculator needed for $\sin 210^\circ$). Second, it builds the geometric intuition that makes the unit-circle picture stick. Calculators do give the same answer either way – but "$-\sin 20^\circ$" tells you something the decimal doesn't.

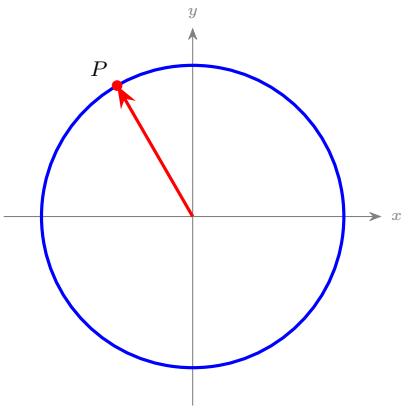
Common slips. Forgetting to negate when the quadrant calls for it (e.g., reporting $+\sin 20^\circ$ for $\sin 200^\circ$). Using the reference angle's value for *both* the magnitude and the quadrant sign (the calculator already gives the signed value if you input the full angle). Reducing into $[0, 360^\circ)$ but then forgetting which quadrant the reduced angle landed in.

PRACTICE

Combine reference angles with a calculator. Give exact values when possible and round to 4 decimals otherwise.

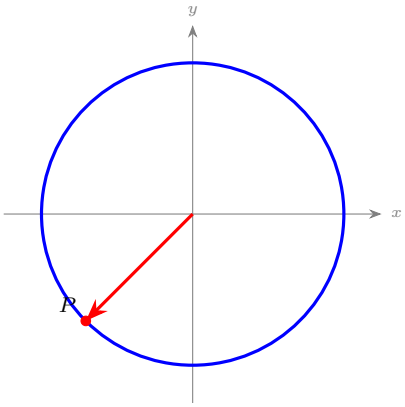
1. Reference angle for 120°. _____

2. $\sin 120^\circ$ (exact) for the angle drawn below. _____

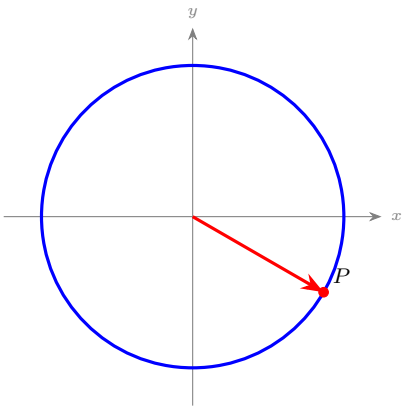


Scan Me

3. $\cos 225°$ (exact) for the angle drawn below. _____



4. $\tan 330°$ (exact) for the angle drawn below. _____



5. $\sin 200°$ (four decimals). _____

6. $\cos 110°$ (four decimals). _____

7. $\sin 240°$ (exact). _____

8. $\cos 315°$ (exact). _____

9. $\tan 150°$ (exact). _____

10. $\tan 235°$ (four decimals). _____

11. $\cos 248°$ (three decimals). _____

12. $\sin 155°$ (four decimals). _____

13. $\cos 300°$ (exact). _____

14. $\sin\left(\frac{7\pi}{6}\right)$ (exact). _____

15. $\cos\left(\frac{5\pi}{3}\right)$ (exact). _____

16. $\tan 130°$ (four decimals). _____

17. Reference angle for $\frac{4\pi}{3}$. _____

18. $\sin 260°$ (four decimals). _____

19. $\cos 165°$ (four decimals). _____

20. $\tan 290°$ (four decimals). _____

Scan Me

Word Problems

21. A pendulum swings to angle $\theta = 140^\circ$ measured counterclockwise from the positive x-axis. Find $\sin 140^\circ$ to four decimals, and state both the quadrant and reference angle used. ____

22. Compute $\cos 215^\circ + \sin 215^\circ$ to four decimals. ____

23. An angle $\theta = 355^\circ$. Find $\cos\theta$ exactly to a reference-angle expression, then approximate to four decimals. ____

24. Find every angle θ in $[0^\circ, 360^\circ)$ with $\sin\theta = -\dfrac{1}{2}$ exactly. ____

Additional Practice

25. Find $\sin\theta$ if opposite $= 5$, hypotenuse $= 13$. _____

26. Find $\cos\theta$ if adjacent $= 12$, hypotenuse $= 13$. _____

27. Find $\tan\theta$ if opposite $= 7$, adjacent $= 4$. _____

28. Find $\sin 30^\circ$. _____

29. Find $\cos 60^\circ$. _____

Scan Me

Answer Keys

1. $\boxed{60^\circ}$
2. $\boxed{\frac{\sqrt{3}}{2}}$
3. $\boxed{-\frac{\sqrt{2}}{2}}$
4. $\boxed{-\frac{\sqrt{3}}{3}}$
5. $\boxed{\approx -0.3420}$
6. $\boxed{\approx -0.3420}$
7. $\boxed{-\frac{\sqrt{3}}{2}}$
8. $\boxed{\frac{\sqrt{2}}{2}}$
9. $\boxed{-\frac{\sqrt{3}}{3}}$
10. $\boxed{\approx 1.4281}$
11. $\boxed{\approx -0.375}$
12. $\boxed{\approx 0.4226}$
13. $\boxed{\frac{1}{2}}$
14. $\boxed{-\frac{1}{2}}$
15. $\boxed{\frac{1}{2}}$
16. $\boxed{\approx -1.1918}$
17. $\boxed{\frac{\pi}{3}}$
18. $\boxed{\approx -0.9848}$
19. $\boxed{\approx -0.9659}$
20. $\boxed{\approx -2.7475}$
21. $\boxed{\approx 0.6428;\ \text{Q2, reference } 40^\circ}$
22. $\boxed{\approx -1.3927}$
23. $\boxed{\cos 5^\circ \approx 0.9962}$
24. $\boxed{\theta = 210^\circ \text{ or } 330^\circ}$

Additional Practice Answers

25. $\boxed{\frac{5}{13}}$
26. $\boxed{\frac{12}{13}}$
27. $\boxed{\frac{7}{4}}$
28. $\boxed{\frac{1}{2}}$
29. $\boxed{\frac{1}{2}}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Q2. $180^\circ - 120^\circ = 60^\circ$. This is the part to check before moving on, because it keeps the answer tied to the original question.

2. Keep the rule visible: Q2, reference 60°, sine positive. $\sin 60^\circ = \frac{\sqrt{3}}{2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

3. One steady path is: Q3, reference 45°, cosine negative. $-\cos 45^\circ = -\frac{\sqrt{2}}{2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

4. Start with the key idea: Q4, reference 30°, tangent negative. $-\tan 30^\circ = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$. That gives a quick check on the answer.

5. A careful way to see it: Q3, reference 20°, sine negative. $-\sin 20^\circ \approx -0.3420$. This is the part to check before moving on, because it keeps the answer tied to the original question.

6. Keep the rule visible: Q2, reference 70°, cosine negative. $-\cos 70^\circ \approx -0.3420$. This is the part to check before moving on, because it keeps the answer tied to the original question.

7. One steady path is: Q3, reference 60°, sine negative. $-\sin 60^\circ = -\frac{\sqrt{3}}{2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

8. Start with the key idea: Q4, reference 45°, cosine positive. $\cos 45^\circ = \frac{\sqrt{2}}{2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

9. A careful way to see it: Q2, reference 30°, tangent negative. $-\tan 30^\circ = -\frac{\sqrt{3}}{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

10. Keep the rule visible: Q3, reference 55°, tangent positive. $\tan 55^\circ \approx 1.4281$. This is the part to check before moving on, because it keeps the answer tied to the original question.

11. One steady path is: Q3, reference 68°, cosine negative. $-\cos 68^\circ \approx -0.375$. This is the part to check before moving on, because it keeps the answer tied to the original question.

12. Start with the key idea: Q2, reference 25°, sine positive. $\sin 25^\circ \approx 0.4226$. This is the part to check before moving on, because it keeps the answer tied to the original question.

13. A careful way to see it: Q4, reference 60°, cosine positive. $\cos 60^\circ = \frac{1}{2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

14. Keep the rule visible: $\frac{7\pi}{6} = 210^\circ$, Q3, reference 30°, sine negative. This is the part to check before moving on, because it keeps the answer tied to the original question.

15. One steady path is: $\frac{5\pi}{3} = 300^\circ$, Q4, reference $\frac{\pi}{3}$, cosine positive. This is the part to check before moving on, because it keeps the answer tied to the original question.

16. Start with the key idea: Q2, reference 50°, tangent negative. $-\tan 50^\circ \approx -1.1918$. This is the part to check before moving on, because it keeps the answer tied to the original question.

17. A careful way to see it: Q3. $\frac{4\pi}{3} - \pi = \frac{\pi}{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

18. Keep the rule visible: Q3, reference 80°, sine negative. $-\sin 80^\circ \approx -0.9848$. This is the part to check before moving on, because it keeps the answer tied to the original question.

19. One steady path is: Q2, reference 15°, cosine negative. $-\cos 15^\circ \approx -0.9659$. This is the part to check before moving on, because it keeps the answer tied to the original question.

20. Start with the key idea: Q4, reference 70°, tangent negative. $-\tan 70^\circ \approx -2.7475$. This is the part to check before moving on, because it keeps the answer tied to the original question.

21. A careful way to see it: 140° is in Q2 (between 90° and 180°). Reference angle $= 180^\circ - 140^\circ = 40^\circ$. Sine is positive in Q2, so $\sin 140^\circ = +\sin 40^\circ \approx 0.6428$. That gives a quick check on the answer.

22. Keep the rule visible: 215° is in Q3 with reference 35°. Both sine and cosine are negative in Q3, so $\sin 215^\circ \approx -\sin 35^\circ \approx -0.5736$ and $\cos 215^\circ \approx$

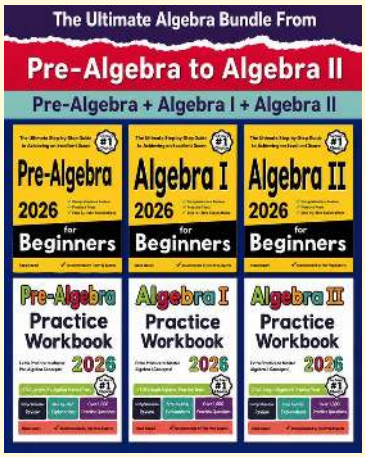
Scan Me

$-\cos 35^\circ \approx -0.8192$. Add them: $-0.8192 + (-0.5736) \approx -1.3928$. Carry full precision and the sum rounds to -1.3927 (the slight difference is just round-then-add versus add-then-round). That gives a quick check on the answer.

23. One steady path is: 355° is in Q4 with reference 5°. Cosine is positive in Q4, so $\cos 355^\circ = \cos 5^\circ \approx 0.9962$. (Reality check: 5° is very close to 0° where cosine is 1, so a value just under 1 is expected.) That gives a quick check on the answer.

24. Start with the key idea: $\sin\theta$ is negative, so θ is in Q3 or Q4. The reference angle for $|\sin\theta| = \frac{1}{2}$ is 30°. Q3 angle: $180^\circ + 30^\circ = 210^\circ$. Q4 angle: $360^\circ - 30^\circ = 330^\circ$. Both check: $\sin 210^\circ = \sin 330^\circ = -\frac{1}{2}$. That gives a quick check on the answer.

Build Algebra Confidence From Pre-Algebra Through Algebra II



The Complete Algebra Success Bundle

Pre-Algebra, Algebra I, and Algebra II in one clear path

Friendly lessons, focused practice, and full-review support for every stage.

Scan Me

Scan for the Bundle

6 Books
3 Courses
1 Path

Bundle Value: Six coordinated books help students review missing skills, learn new algebra topics, and practice until the steps feel natural.

Complete Course Path

- ✓ Starts with Pre-Algebra foundations
- ✓ Moves smoothly into Algebra I skills
- ✓ Extends learning through Algebra II topics
- ✓ Great for review, tutoring, and summer study

One bundle, one steady path.

Step-by-Step Lessons

- ✓ Plain-English explanations students can follow
- ✓ Worked examples that show every important step
- ✓ Common mistakes called out before they stick
- ✓ Skill-building practice after each lesson
- ✓ Helpful for independent study or class support

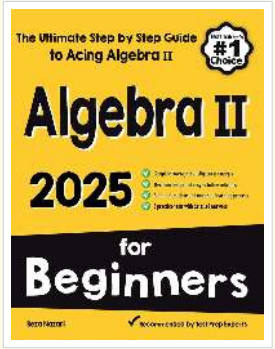
Less guessing. More understanding.

Practice That Sticks

- ✓ Matching practice workbooks for extra repetition
- ✓ Review sets to keep older skills fresh
- ✓ Answer explanations for checking thinking
- ✓ Strong support before tests and final exams
- ✓ Designed to build fluency and confidence

Practice today. Remember tomorrow.

STUDENT FAVORITE • Master Algebra II From the Ground Up



✓ 100% Guaranteed
✓ Lifetime Support
✓ Trusted by Teachers

Start Your Algebra Journey Today! →

Algebra II for Beginners

Written by a top math teacher & aligned with national and state Algebra II courses. From polynomial functions to logarithms, trigonometry, and rational expressions — explained the easy way.

- ✓ **Complete coverage** of every Algebra II concept — perfect companion to these worksheets
- ✓ **Step-by-step explanations** with worked examples on every topic
- ✓ **QR codes in every chapter** for free video lessons & bonus practice
- ✓ **2 full-length practice tests** with detailed answer keys

★ STUDENT'S #1 CHOICE ★

Teacher-recommended • 12,000+ Happy Students

↓ PDF EDITION

Scan Me

Instant download • any device

PAPERBACK

Scan Me

Paperback on Amazon

Hold it in your hands

Pair these free worksheets with *Algebra II for Beginners* and you have a complete self-paced course — concept lessons, daily practice, and full exam-style reviews, all in one path. → **EffortlessMath.com/product/algebra-ii-for-beginners**