Recursive and Explicit Definitions of Sequences

Name: ________________ Date: ____________ Score: ______ / 30

Quick Review

Two different ways to define the same sequence.

Explicit (closed-form). Writes a_n as a formula in n alone: plug in n, get a_n – no need to walk through earlier terms. Examples: $a_n = 4n+1$ (arithmetic), $a_n = 3 \cdot 2^{n-1}$ (geometric), $a_n = n^2 - 3n$ (quadratic). Great for jumping straight to a_{100}.

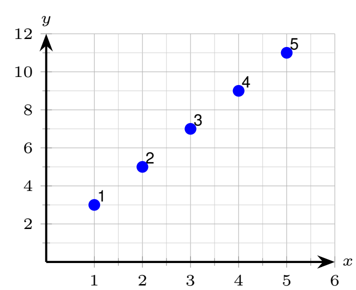
Recursive. Gives one or more starting values, plus a rule that builds each new term from earlier terms. Examples: $a_1 = 3$, $a_n = a_{n-1} + 5$ (arithmetic recursive); $a_1 = 4$, $a_n = 3a_{n-1} - 2$ (linear recurrence); $a_1 = 1$, $a_2 = 1$, $a_n = a_{n-1} + a_{n-2}$ (Fibonacci – needs two starting values). Recursive rules need enough starting terms to "feed" the recursion.

Going from recursive to explicit. If you see the same $+d$ each step, that's arithmetic: $a_n = a_1 + (n-1)d$. If you see the same $\times r$ each step, geometric: $a_n = a_1 r^{n-1}$. For a mixed rule like $a_n = 3a_{n-1} - 2$, look for a fixed point: if the sequence settled at L, then $L = 3L - 2$, so $L = 1$. The offset $a_n - 1$ is geometric with ratio 3, so $a_n = 1 + C \cdot 3^{n-1}$. Use the starting value to find C.

Going from explicit to recursive. For arithmetic $a_n = dn + c$: $a_1 = d + c$, $a_n = a_{n-1} + d$. For geometric $a_n = a_1 r^{n-1}$: a_1 is the constant out front, $a_n = r \cdot a_{n-1}$.

When to use which. Explicit wins for far-away terms ("find a_{1000}"). Recursive wins for short walks and for problems where the rule itself is the interesting part (compound interest, population models).

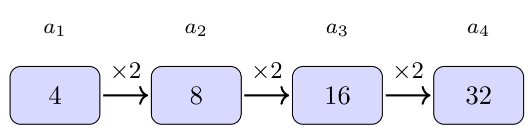
Common slips. Forgetting that a recurrence like $a_n = a_{n-1} + a_{n-2}$ needs *two* starting values to launch. Reading the explicit formula as a recursive one because the $n-1$ shows up in both. Using the geometric recursion symbols on an arithmetic problem (addition, not multiplication).**Sequence plot.** A recursive rule makes one term at a time; plotting the first few terms beside the rule helps students see the same pattern in explicit form.



PRACTICE

Identify recursive vs explicit definitions, convert between them, and compute terms.

1. Is $a_1 = 3$, $a_n = a_{n-1} + 5$ for $n \geq 2$ a recursive or explicit definition? ________
2. If $a_1 = 4$ and $a_n = 2a_{n-1}$, find a_4. ________



3. Which recursive definition matches the explicit formula $a_n = 4n + 1$? ________
4. Convert $a_n = 5 \cdot 3^{n-1}$ to a recursive form. ________
5. If $a_1 = 1$, $a_2 = 2$, $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$, find a_6. ________
6. Which of these is an *explicit* formula? $A : a_n = n^2 - 3n$ $B : a_n = a_{n-1} + 2$ $C : a_n = 3a_{n-1}$ ________
7. Given $a_n = 7 - 3n$, find the common difference. ________
8. Given $a_1 = -2$ and $a_n = a_{n-1} + 6$, find the explicit formula. ________
9. Given $g_1 = 9$ and $g_n = \frac{1}{3} g_{n-1}$, find g_5. ________
10. True or False: a recursive rule like $a_n = a_{n-1} + a_{n-2}$ can be launched with just a_1. ________

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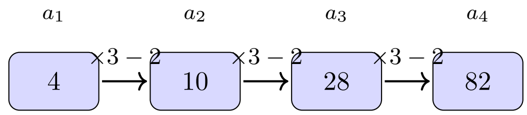
11. Given $a_1 = 4$ and $a_n = 3a_{n-1} - 2$, find the first four terms. ________



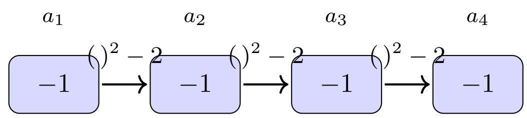
12. For the recurrence $a_1 = 4$, $a_n = 3a_{n-1} - 2$ shown in the previous problem, find the explicit formula. ________

13. Given $a_1 = 2$, $a_n = 2a_{n-1} + 1$, find a_5. ________

14. Convert $a_n = 2n^2 + 1$ to "recursive" form. (Hint: just give a_1 and a rule for $a_n - a_{n-1}$.) ________

15. Match: explicit $a_n = 2^n$ becomes which recursive rule? ________

16. Given $a_1 = -1$ and $a_n = a_{n-1}^2 - 2$, find a_4. ________



17. True or False: an explicit formula always lets you compute a_{1000} faster than the recursive form does. ________

18. Given $a_1 = 5$, $a_n = a_{n-1} - 3$, find a_{20}. ________

19. A sequence is defined by $a_1 = 2$, $a_n = a_{n-1} + n$. Find a_5. ________

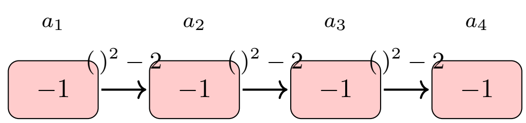
20. Given $a_n = \dfrac{n}{n+1}$, write the recursive form (just give a_1 and the relation between a_n and a_{n-1}). ________

♦ Word Problems

21. A bacteria colony doubles every hour. At hour 0, there are 500 bacteria. Write both a recursive rule and an explicit formula for the count a_n at the start of hour n (so $a_0 = 500$, a_1 is after one hour). ____

22. A loan starts at \$$1{,}000$ and is reduced by a fixed \$$75$ each month. Write the explicit formula for the remaining balance B_n at the end of month n (with $B_0 = 1000$). Then find B_{12}. ____

23. A sequence is defined recursively by $a_1 = -1$ and $a_n = a_{n-1}^2 - 2$ for $n \geq 2$. Identify which statement is true: (A) periodic with period 2, (B) $a_{20} = 1$, (C) arithmetic, (D) constant. ____



24. Sequence P: $a_1 = 3$, $a_n = 2a_{n-1} + 4$. Sequence Q: $b_1 = 3$, $b_n = b_{n-1} + 5$. Compute $a_5 - b_5$. ____

Additional Practice

25. Find the next term: $4, 9, 14, 19, \ldots$. ________

26. Find a_{10} if $a_1 = 3$ and $d = 5$. ________

27. Find the next term: $2, 6, 18, 54, \ldots$. ________

28. Find a_6 if $a_1 = 5$ and $r = 2$. ________

29. Sum $1 + 2 + 3 + \cdots + 20$. ________

30. Find S_5 for $3, 6, 12, 24, 48$. ________

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Answer Keys

1. Recursive
2. $a_4 = 32$
3. $a_1 = 5,\ a_n = a_{n-1} + 4$
4. $a_1 = 5,\ a_n = 3a_{n-1}$
5. $a_6 = 13$
6. A
7. $d = -3$
8. $a_n = 6n - 8$
9. $g_5 = \dfrac{1}{9}$
10. False
11. $4, 10, 28, 82$
12. $a_n = 3^n + 1$
13. $a_5 = 47$
14. $a_1 = 3,\ a_n = a_{n-1} + (4n-2)$
15. $a_1 = 2,\ a_n = 2a_{n-1}$
16. $a_4 = -1$
17. True
18. $a_{20} = -52$
19. $a_5 = 16$
20. $a_1 = \frac{1}{2},\ a_n = \dfrac{n}{n+1}$
21. Recursive: $a_0 = 500,\ a_n = 2a_{n-1}$; Explicit: $a_n = 500 \cdot 2^n$
22. $B_n = 1000 - 75n,\ B_{12} = \$100$
23. (D) constant
24. 85

Additional Practice Answers

25. 24
26. 48
27. 162
28. 160
29. 210
30. 93

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Each new term is built from the previous one. That's the defining trait of a recursive rule – you need a_{n-1} to find a_n.

2. Walk the chain: $a_1 = 4, a_2 = 2(4) = 8, a_3 = 2(8) = 16, a_4 = 2(16) = 32$. (The diagram shows the same path.)

3. Compute the first few terms from $a_n = 4n + 1$: $a_1 = 5$, $a_2 = 9$, $a_3 = 13$. Difference $d = 4$. So $a_1 = 5, a_n = a_{n-1} + 4$.

4. Geometric with $a_1 = 5$ and $r = 3$. So $a_1 = 5$, $a_n = 3a_{n-1}$. Each step multiplies by the ratio.

5. Fibonacci-style: $a_1 = 1, a_2 = 2, a_3 = 3, a_4 = 5, a_5 = 8, a_6 = 13$. (Two starting values needed – one isn't enough to launch this recurrence.)

6. Keep the rule visible: A writes a_n as a function of n alone – plug in and go. B and C both need a previous term, so they're recursive. That gives a quick check on the answer.

7. Arithmetic with d = coefficient of n, so $d = -3$. (Verify: $a_1 = 4, a_2 = 1$, and $1 - 4 = -3$ ✓.)

8. Arithmetic with $a_1 = -2$ and $d = 6$. $a_n = -2 + (n-1)(6) = 6n - 8$. Check: $a_1 = 6(1) - 8 = -2$ ✓.

9. Geometric with $r = \frac{1}{3}$. Walk: $9, 3, 1, \frac{1}{3}, \frac{1}{9}$. (Or explicit: $g_n = 9(\frac{1}{3})^{n-1}$, $g_5 = 9 \cdot (\frac{1}{3})^4 = 9/81 = \frac{1}{9}$.)

10. That rule needs both a_{n-1} and a_{n-2} to fire. So you need both a_1 and a_2 before you can compute a_3.

11. One steady path is: $a_1 = 4$; $a_2 = 3(4) - 2 = 10$; $a_3 = 3(10) - 2 = 28$; $a_4 = 3(28) - 2 = 82$. (Matches the chain diagram.) That gives a quick check on the answer.

12. Find the fixed point: $L = 3L - 2 \Rightarrow L = 1$. So $a_n - 1$ is geometric with ratio 3. Since $a_1 - 1 = 3$, we have $a_n - 1 = 3 \cdot 3^{n-1} = 3^n$, so $a_n = 3^n + 1$. Verify: $a_1 = 3^1 + 1 = 4$, $a_2 = 3^2 + 1 = 10$, $a_3 = 3^3 + 1 = 28$ ✓.

13. This recurrence isn't pure geometric (the $+1$ breaks the constant-ratio pattern), so walk it term by term: $a_1 = 2$, $a_2 = 2(2) + 1 = 5$, $a_3 = 2(5) + 1 = 11$, $a_4 = 2(11) + 1 = 23$, $a_5 = 2(23) + 1 = 47$. Apply the rule once per step – don't try to jump straight to a_5.

14. Keep the rule visible: $a_n - a_{n-1} = 2n^2 + 1 - [2(n-1)^2 + 1] = 2[n^2 - (n-1)^2] = 2(2n-1) = 4n - 2$. So $a_1 = 3$ and $a_n = a_{n-1} + (4n-2)$. (The differences aren't constant – this isn't a standard arithmetic-style recursion.) That gives a quick check on the answer.

15. One steady path is: $a_n = 2^n$: at $n = 1$, $a_1 = 2$. Ratio $a_n/a_{n-1} = 2$. So $a_1 = 2, a_n = 2a_{n-1}$. That gives a quick check on the answer.

16. Start with the key idea: $a_1 = -1$. $a_2 = (-1)^2 - 2 = -1$. So every later term is also -1 – the sequence is constant at -1. (Once you hit the fixed point L where $L = L^2 - 2$, you stay there. $L^2 - L - 2 = 0$ gives $L = 2$ or $L = -1$; we started at -1.) That gives a quick check on the answer.

17. Explicit plugs in $n = 1000$ and computes directly. Recursive needs to walk through all 999 earlier terms. (That's why explicit is so useful for far-out terms.)

18. Arithmetic with $a_1 = 5$, $d = -3$. Use the explicit form: $a_n = 5 + (n-1)(-3) = 8 - 3n$. So $a_{20} = 8 - 60 = -52$.

19. Walk: $a_1 = 2$, $a_2 = 2 + 2 = 4$, $a_3 = 4 + 3 = 7$, $a_4 = 7 + 4 = 11$, $a_5 = 11 + 5 = 16$. (The differences aren't constant here – each step adds the current index.)

20. This sequence is naturally explicit. The cleanest recursive form doesn't reduce to a simple ratio or difference – just keep the explicit. (Sometimes explicit is the only nice description.) Values: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$

21. Doubling each hour is geometric with ratio 2. Recursive: $a_0 = 500$, $a_n = 2a_{n-1}$. Explicit (indexed from 0): $a_n = 500 \cdot 2^n$. (Verify: $a_0 = 500$, $a_1 = 1000$, $a_2 = 2000$ – matches doubling.)

22. Each month subtracts $\$75$, so this is arithmetic with $d = -75$. Explicit: $B_n = 1000 - 75n$. At month 12: $B_{12} = 1000 - 900 = 100$, so \$100 left. (Reality check: positive balance after 12 months – the loan isn't paid off yet; it would clear around month $1000/75 \approx 13.3$.)

23. One steady path is: $a_1 = -1$. $a_2 = (-1)^2 - 2 = 1 - 2 = -1$. So every later term is also -1 – the recursion lands on the fixed point $L = -1$ (which satisfies $L = L^2 - 2$) and stays. The sequence is constant at -1, not periodic-2, not equal to 1 at $n = 20$, and not arithmetic (well, technically a constant is arithmetic with $d = 0$, but the best description here is "constant"). That gives a quick check on the answer.

24. P: $a_1 = 3$, $a_2 = 10$, $a_3 = 24$, $a_4 = 52$, $a_5 = 108$. Q: $b_1 = 3$, $b_2 = 8$, $b_3 = 13$, $b_4 = 18$, $b_5 = 23$. Difference: $108 - 23 = 85$. (P grows much faster because it multiplies; Q grows linearly. Geometric beats arithmetic in the long run.)

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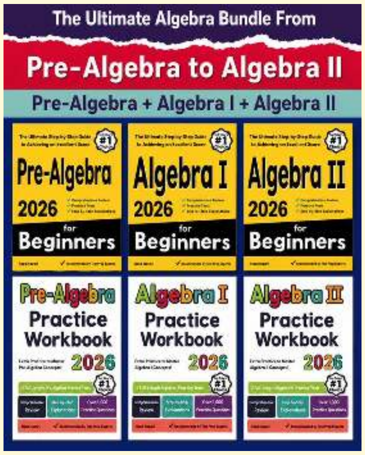

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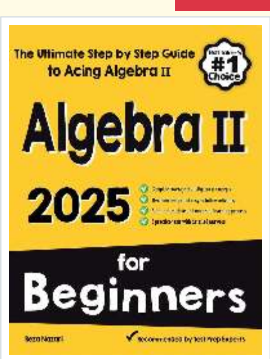

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