Reciprocal Trigonometric Functions

Name: ____________	Date: ____________	Score: _____ / 24

Quick Review

The three **reciprocal trig functions** are the partners of sine, cosine, and tangent:

$\csc\theta = \dfrac{1}{\sin\theta}$ (where $\sin\theta \neq 0$).

$\sec\theta = \dfrac{1}{\cos\theta}$ (where $\cos\theta \neq 0$).

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$ (where $\sin\theta \neq 0$).

Watch the pairings. Cosine pairs with *secant*, not cosecant. The "co" in cosecant tells you it pairs with sine, the function *without* "co". A quick way to remember: each reciprocal has exactly one "co" between it and its partner.

To compute a reciprocal value. Find the underlying $\sin$, $\cos$, or $\tan$ value, then flip the fraction. The sign is unchanged (reciprocating doesn't flip sign). So if $\sin\theta = -\dfrac{\sqrt{2}}{2}$, then $\csc\theta = -\dfrac{2}{\sqrt{2}} = -\sqrt{2}$.

Where each is undefined. $\csc$ blows up where $\sin\theta = 0$ (at integer multiples of $180°$). $\sec$ blows up where $\cos\theta = 0$ (at $90° + 180°n$). $\cot$ blows up where $\sin\theta = 0$ (same as $\csc$).

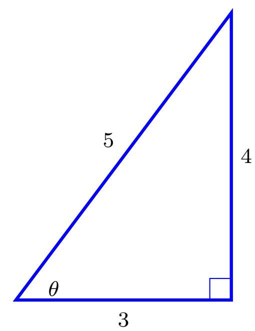
Range facts. For real θ wherever defined, $|\sec\theta| \geq 1$ and $|\csc\theta| \geq 1$. (Because the original sine and cosine are at most 1 in absolute value, their reciprocals are at least 1 in absolute value.) Cotangent, like tangent, covers all real numbers.

Common slips. Pairing secant with sine. Treating $\sin^{-1}\theta$ (inverse sine – gives an angle) as the same thing as $\csc\theta$ (reciprocal of sine – gives a ratio). Computing $\sec 90°$ as if it equals 1 – it's actually undefined because $\cos 90° = 0$.

PRACTICE

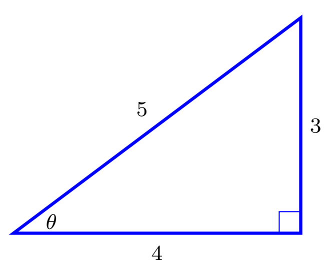
Compute reciprocal trig values. Flip the parent ratio and keep the sign.

1. For the acute angle θ in the triangle below, $\sin\theta = \dfrac{4}{5}$. Find $\csc\theta$. ________



2. $\sec\theta$ if $\cos\theta = -\dfrac{2}{3}$. ________

3. For the acute angle θ below, $\sin\theta = \dfrac{3}{5}$ and $\cos\theta = \dfrac{4}{5}$. Find $\cot\theta$. ________



4. $\sec 60°$. ________

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5. $\csc 30°$. ______

6. Where is $\sec\theta$ undefined? ______

7. $\csc\theta$ if $\sin\theta = -\frac{\sqrt{2}}{2}$. ______

8. $\sec 180°$. ______

9. $\csc 180°$. ______

10. $\cot 45°$. ______

11. $\cot 60°$. ______

12. $\sec 45°$. ______

13. If $\sin\theta = -\frac{7}{25}$ and $\cos\theta = -\frac{24}{25}$ with θ in Q3, find $\cot\theta$. ______

14. $\csc 60°$. ______

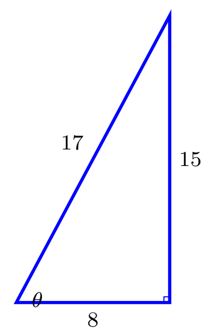
15. $\sec 90°$. ______

16. $\cot 90°$. ______

17. If $\sec\theta = \frac{5}{3}$, find $\cos\theta$. ______

18. $\csc 270°$. ______

19. For the acute angle θ below, $\cos\theta = \frac{8}{17}$. Find $\sec\theta$. ______



20. $\sec 0°$. ______

♦ Word Problems

21. In a right triangle, the side opposite the acute angle θ is 7 and the hypotenuse is 25. Find $\csc\theta$, $\sec\theta$, and $\cot\theta$ exactly. ____

22. An angle θ in Quadrant II satisfies $\sin\theta = \frac{4}{5}$. Find $\sec\theta$ and $\csc\theta$ exactly. ____

23. Why is $\sec\theta$ never strictly between -1 and 1? Answer in one short sentence. ____

24. A surveyor's protractor reads angle θ where $\cos\theta = \frac{5}{13}$ and θ is acute. The calculations call for $\csc\theta$. Find it exactly. ____

Answer Keys

1. $\boxed{\frac{5}{4}}$
2. $\boxed{-\frac{3}{2}}$
3. $\boxed{\frac{4}{3}}$
4. $\boxed{2}$
5. $\boxed{2}$
6. $\boxed{\cos\theta = 0}$
7. $\boxed{-\sqrt{2}}$
8. $\boxed{-1}$
9. $\boxed{\text{undefined}}$
10. $\boxed{1}$
11. $\boxed{\frac{\sqrt{3}}{3}}$
12. $\boxed{\sqrt{2}}$
13. $\boxed{\frac{24}{7}}$
14. $\boxed{\frac{2\sqrt{3}}{3}}$
15. $\boxed{\text{undefined}}$
16. $\boxed{0}$
17. $\boxed{\frac{3}{5}}$
18. $\boxed{-1}$
19. $\boxed{\frac{17}{8}}$
20. $\boxed{1}$
21. $\boxed{\csc\theta = \frac{25}{7},\ \sec\theta = \frac{25}{24},\ \cot\theta = \frac{24}{7}}$
22. $\boxed{\sec\theta = -\frac{5}{3},\ \csc\theta = \frac{5}{4}}$
23. $\boxed{|\sec\theta| \geq 1}$
24. $\boxed{\frac{13}{12}}$

Step-by-Step Explanations

1. In the triangle, opposite $= 4$ and hypotenuse $= 5$, so $\sin\theta = \frac{4}{5}$. Cosecant flips that: $\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$. (For an acute angle every ratio is positive, so no sign to worry about.)

2. Secant is the reciprocal of cosine, so flip the fraction and keep the sign: $\sec\theta = \frac{1}{-2/3} = -\frac{3}{2}$. Taking a reciprocal never flips a negative to a positive.

3. One steady path is: $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{4/5}{3/5} = \frac{4}{3}$, the reciprocal of $\tan\theta = \frac{3}{4}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

4. Start with the key idea: $\cos 60° = \frac{1}{2}$, so $\sec 60° = 2$. This is the part to check before moving on, because it keeps the answer tied to the original question.

5. A careful way to see it: $\sin 30° = \frac{1}{2}$, so $\csc 30° = 2$. This is the part to check before moving on, because it keeps the answer tied to the original question.

6. Because $\sec\theta = \frac{1}{\cos\theta}$, it blows up wherever the denominator is zero – that is, wherever $\cos\theta = 0$, which happens at $90° + 180°n$ for any integer n.

7. One steady path is: Flip: $\frac{-2}{\sqrt{2}} = -\sqrt{2}$ (multiply by $\sqrt{2}/\sqrt{2}$ to rationalize). This is the part to check before moving on, because it keeps the answer tied to the original question.

8. At $180°$ the unit-circle point is $(-1, 0)$, so $\cos 180° = -1$. Then $\sec 180° = \frac{1}{-1} = -1$. (Notice $|\sec| = 1$ here, the smallest it can be.)

9. A careful way to see it: $\sin 180° = 0$, so $\csc 180° = \frac{1}{0}$ – undefined. This is the part to check before moving on, because it keeps the answer tied to the original question.

10. Keep the rule visible: $\tan 45° = 1$, so $\cot 45° = 1$. This is the part to check before moving on, because it keeps the answer tied to the original question.

11. One steady path is: $\tan 60° = \sqrt{3}$, so $\cot 60° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

12. Start with the key idea: $\cos 45° = \frac{\sqrt{2}}{2}$, so $\sec 45° = \frac{2}{\sqrt{2}} = \sqrt{2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

13. A careful way to see it: $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{-24/25}{-7/25} = \frac{24}{7}$. (Two negatives in Q3 -> positive cotangent, consistent with ASTC.) That gives a quick check on the answer.

14. Keep the rule visible: $\sin 60° = \frac{\sqrt{3}}{2}$, so $\csc 60° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ (rationalize). This is the part to check before moving on, because it keeps the answer tied to the original question.

15. One steady path is: $\cos 90° = 0$, so $\sec 90° = \frac{1}{0}$ – undefined. This is the part to check before moving on, because it keeps the answer tied to the original question.

16. Start with the key idea: $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{0}{1} = 0$. (Tangent is undefined at $90°$, but cotangent equals zero.) That gives a quick check on the answer.

17. Cosine and secant are reciprocals of each other, so flipping secant lands you back on cosine: $\cos\theta = \frac{1}{5/3} = \frac{3}{5}$.

18. The point at $270°$ is $(0, -1)$, so $\sin 270° = -1$. Cosecant is its reciprocal: $\csc 270° = \frac{1}{-1} = -1$.

19. With $\cos\theta = \frac{8}{17}$ the adjacent leg is 8 and the hypotenuse is 17. Secant is the reciprocal of cosine: $\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{17}{8}$.

20. Start with the key idea: At $0°$ the unit-circle point is $(1, 0)$, so $\cos 0° = 1$ and $\sec 0° = \frac{1}{1} = 1$. That gives a quick check on the answer.

21. Adjacent $= \sqrt{25^2 - 7^2} = \sqrt{576} = 24$. So $\sin\theta = \frac{7}{25}$, $\cos\theta = \frac{24}{25}$, $\tan\theta = \frac{7}{24}$. Flip each: $\csc = \frac{25}{7}$, $\sec = \frac{25}{24}$, $\cot = \frac{24}{7}$.

22. In Q2, sine is positive (given) and cosine is negative. From $\sin^2 + \cos^2 = 1$: $\cos^2\theta = 1 - \frac{16}{25} = \frac{9}{25}$, so $\cos\theta = -\frac{3}{5}$ (negative in Q2). Then $\sec\theta = -\frac{5}{3}$ and $\csc\theta = \frac{5}{4}$ (positive because sine is).

23. Because $|\cos\theta| \leq 1$ always, its reciprocal $|\sec\theta| = \frac{1}{|\cos\theta|}$ must be at least 1 wherever it's defined. (The reciprocal of a number with absolute value ≤ 1 has absolute value ≥ 1.)

24. Acute angle with $\cos\theta = \frac{5}{13}$ means adjacent $= 5$, hyp $= 13$, opposite $= \sqrt{169 - 25} = 12$. So $\sin\theta = \frac{12}{13}$ and $\csc\theta = \frac{13}{12}$. (A 5-12-13 triangle once again.)

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