

Rational and Radical Forms of Expressions

Name: _____ Date: _____ Score: _____ / 24

Q Quick Review

Radicals and rational exponents are two ways of writing the same thing. The key conversion is $\sqrt[n]{x^m} = x^{m/n}$ — the **root index** (n) becomes the **denominator** of the exponent, and the **power inside** (m) becomes the **numerator**. A square root has index 2, so $\sqrt{x} = x^{1/2}$. A cube root of x^2 is $x^{2/3}$. The *denominator* is the root; the *numerator* is the power. **Negative exponents** mean reciprocal: $x^{-n} = \frac{1}{x^n}$, so $\frac{1}{\sqrt[3]{x^4}} = x^{-4/3}$. All the usual exponent rules still apply — product, quotient, power-of-a-power — they just use fractional exponents instead of integers. **Computing** numeric rational powers: think of $a^{m/n}$ as $(\sqrt[n]{a})^m$. Take the root first (smaller numbers, easier), then apply the integer power. For example, $8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$.

PRACTICE

Simplify each expression. Use positive exponents (or simplest radical form) in the final answer.

- $\sqrt{x} \ (x \geq 0)$ _____
- The table shows radicals rewritten with rational exponents. Continue the pattern for $\sqrt[3]{x^2}$ (take $x \geq 0$). _____

radical	rational exponent
\sqrt{x}	$x^{1/2}$
$\sqrt[3]{x}$	$x^{1/3}$
$\sqrt[3]{x^2}$?

- $x^{3/4} \cdot x^{1/4}$ _____
- $\sqrt[5]{x^{10}} \ (x \geq 0)$ _____
- The table splits the rational power $4^{3/2}$ into a root step and a power step. Use it to finish the computation. _____

step	result
$\sqrt{4}$ (the $\frac{1}{2}$)	2
then cube it (the 3)	?

- $\frac{1}{\sqrt[3]{x^4}} \ (x > 0)$ _____
- $\sqrt{32x^6} \cdot \sqrt[4]{x^4} \ (x \geq 0)$ _____
- $\sqrt[3]{x^9} \ (x > 0)$ _____
- $s = V^{1/3}$ with $V = 125x^6 \ (x \geq 0)$ _____
- $(81y^8)^{3/4} \ (y \geq 0)$ _____
- The table breaks $27^{2/3}$ into its two steps. Use it to finish. _____

step	result
$\sqrt[3]{27}$ (the $\frac{1}{3}$)	3
then square it (the 2)	?

- $16^{3/4}$ _____
- $x^{2/3} \cdot x^{1/3}$ _____
- $\frac{x^{5/2}}{x^{1/2}} \ (x > 0)$ _____
- $(x^{1/2})^4 \ (x \geq 0)$ _____



16. $\sqrt{50}$ _____

17. $\sqrt{12x^4}$ ($x \geq 0$) _____

18. $\sqrt[3]{-8x^3}$ _____

19. $x^{-1/2}$ ($x > 0$) _____

20. $(8x^3)^{2/3}$ ($x \geq 0$) _____

◆ Word Problems

21. The side length of a cube with volume V is given by $s = V^{1/3}$. If $V = 125x^6$ and $x \geq 0$, find a simplified expression for the side length. _____

22. The time for a pendulum's swing is $T = 2\pi\sqrt{L/9.8}$ seconds, where L is the length in meters. Find T when $L = 2.45$ m. Use $\pi \approx 3.14$. _____

23. A satellite dish has an area $A = \pi r^2$ where the radius is $r = \sqrt{A/\pi}$. If $A = 12.56$ square meters, find r . Use $\pi \approx 3.14$. _____

24. In physics, the period of orbit at radius r for a satellite around a planet is $T = k \cdot r^{3/2}$ for some constant k (Kepler's Third Law). If a satellite at radius $r = 4$ has period $T = 80$, what is k ? _____



Answer Keys

- | | |
|-------------------|----------------------------------|
| 1. $x^{1/2}$ | 13. x |
| 2. $x^{2/3}$ | 14. x^2 |
| 3. x | 15. x^2 |
| 4. x^2 | 16. $5\sqrt{2}$ |
| 5. 8 | 17. $2x^2\sqrt{3}$ |
| 6. $x^{-4/3}$ | 18. $-2x$ |
| 7. $4x^4\sqrt{2}$ | 19. $\frac{1}{\sqrt{x}}$ |
| 8. x^3 | 20. $4x^2$ |
| 9. $5x^2$ | 21. $5x^2$ |
| 10. $27y^6$ | 22. $T \approx 3.14 \text{ sec}$ |
| 11. 9 | 23. $r = 2 \text{ m}$ |
| 12. 8 | 24. $k = 10$ |

Step-by-Step Explanations

- A careful way to see it: The index 2 becomes the denominator: $\sqrt{x} = x^{1/2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- The root index goes on the bottom of the exponent and the inside power on top. Index 3, power 2 gives $x^{2/3}$.
- One steady path is: Same base, so add: $\frac{3}{4} + \frac{1}{4} = 1$. The product is $x^1 = x$. That gives a quick check on the answer.
- Start with the key idea: Convert: $x^{10/5} = x^2$. The fifth root of a tenth power is a second power. That gives a quick check on the answer.
- Read $4^{3/2}$ as $(\sqrt{4})^3$. The table does the root first: $\sqrt{4} = 2$. Now apply the power: $2^3 = 8$. (Squaring first instead gives $\sqrt{64} = 8$ too — either order works.)
- Convert the radical: $\sqrt[3]{x^4} = x^{4/3}$. The whole expression is the reciprocal: $\frac{1}{x^{4/3}} = x^{-4/3}$.
- First radical: $\sqrt{32x^6} = \sqrt{16 \cdot 2} \cdot \sqrt{x^6} = 4x^3\sqrt{2}$. Second: $\sqrt[4]{x^4} = x$. Multiply: $4x^3\sqrt{2} \cdot x = 4x^4\sqrt{2}$.
- Start with the key idea: $x^{9/3} = x^3$. Quick check: $x^3 \cdot x^3 \cdot x^3 = x^9$. ✓ This is the part to check before moving on, because it keeps the answer tied to the original question.
- A careful way to see it: $(125x^6)^{1/3}$: distribute the exponent. $125^{1/3} = 5$ (since $5^3 = 125$), $(x^6)^{1/3} = x^2$. So $s = 5x^2$. That gives a quick check on the answer.
- Keep the rule visible: $81^{3/4} = (3^4)^{3/4} = 3^3 = 27$. $(y^8)^{3/4} = y^6$. Product: $27y^6$. That gives a quick check on the answer.
- Think of $27^{2/3}$ as $(\sqrt[3]{27})^2$. The cube root gives 3, and squaring gives $3^2 = 9$.
- Start with the key idea: Fourth root first: $\sqrt[4]{16} = 2$ (since $2^4 = 16$). Cube it: $2^3 = 8$. That gives a quick check on the answer.
- A careful way to see it: $\frac{2}{3} + \frac{1}{3} = 1$, so the product is x . This is the part to check before moving on, because it keeps the answer tied to the original question.
- Keep the rule visible: Subtract: $\frac{5}{2} - \frac{1}{2} = 2$. So x^2 . This is the part to check before moving on, because it keeps the answer tied to the original question.
- One steady path is: Multiply: $\frac{1}{2} \cdot 4 = 2$, so x^2 . (Makes sense: $(\sqrt{x})^4 = x^2$.) That gives a quick check on the answer.
- Start with the key idea: Pull out perfect-square factors: $50 = 25 \cdot 2$. So $\sqrt{50} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$. That gives a quick check on the answer.
- A careful way to see it: $12 = 4 \cdot 3$. $\sqrt{12x^4} = \sqrt{4} \cdot \sqrt{3} \cdot \sqrt{x^4} = 2\sqrt{3} \cdot x^2 = 2x^2\sqrt{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Cube roots take negatives. $\sqrt[3]{-8} = -2$ (since $(-2)^3 = -8$). $\sqrt[3]{x^3} = x$. Product: $-2x$.
- One steady path is: Negative exponent flips, fractional becomes a root: $x^{-1/2} = \frac{1}{x^{1/2}} = \frac{1}{\sqrt{x}}$. That gives a quick check on the answer.
- Distribute the exponent. $8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$. $(x^3)^{2/3} = x^2$. Product: $4x^2$.
- Substitute and distribute the exponent. $(125x^6)^{1/3} = 125^{1/3} \cdot (x^6)^{1/3} = 5 \cdot x^2 = 5x^2$. (Reality check: cubing $5x^2$ gives $125x^6$. ✓)
- Inside the radical first: $\frac{2 \cdot 45}{9 \cdot 8} = 0.25$. Then $\sqrt{0.25} = 0.5$. Multiply by 2π : $T = 2\pi(0.5) = \pi \approx 3.14$ seconds. (A clean designed-for-the-classroom value.)
- Inside the radical: $\frac{12.56}{3.14} = 4$. Then $\sqrt{4} = 2$. So the radius is 2 meters. (Check: $\pi(2)^2 = 4\pi \approx 12.56$. ✓)
- Solve $80 = k \cdot 4^{3/2}$. Compute $4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$. So $80 = 8k$, giving $k = 10$. (Rational exponents do real work in physics — this is the same law Kepler used in the 1600s.)



Build Algebra Confidence From Pre-Algebra Through Algebra II



The Complete Algebra Success Bundle

Pre-Algebra, Algebra I, and Algebra II in one clear path

Friendly lessons, focused practice, and full-review support for every stage.



Scan for the Bundle

6 Books
3 Courses
1 Path

Bundle Value: Six coordinated books help students review missing skills, learn new algebra topics, and practice until the steps feel natural.

Complete Course Path

- ✓ Starts with Pre-Algebra foundations
- ✓ Moves smoothly into Algebra I skills
- ✓ Extends learning through Algebra II topics
- ✓ Great for review, tutoring, and summer study

One bundle, one steady path.

Step-by-Step Lessons

- ✓ Plain-English explanations students can follow
- ✓ Worked examples that show every important step
- ✓ Common mistakes called out before they stick
- ✓ Skill-building practice after each lesson
- ✓ Helpful for independent study or class support

Less guessing. More understanding.

Practice That Sticks

- ✓ Matching practice workbooks for extra repetition
- ✓ Review sets to keep older skills fresh
- ✓ Answer explanations for checking thinking
- ✓ Strong support before tests and final exams
- ✓ Designed to build fluency and confidence

Practice today. Remember tomorrow.

STUDENT FAVORITE • Master Algebra II From the Ground Up



Algebra II for Beginners

Written by a top math teacher & aligned with national and state Algebra II courses. From polynomial functions to logarithms, trigonometry, and rational expressions — explained the easy way.

- ✓ **Complete coverage** of every Algebra II concept — perfect companion to these worksheets
- ✓ **Step-by-step explanations** with worked examples on every topic
- ✓ **QR codes in every chapter** for free video lessons & bonus practice
- ✓ **2 full-length practice tests** with detailed answer keys

- ✓ 100% Guaranteed
- ✓ Lifetime Support
- ✓ Trusted by Teachers

Start Your Algebra Journey Today! →

★ STUDENT'S #1 CHOICE ★

Teacher-recommended • 12,000+ Happy Students

PDF EDITION



Instant download • any device

PAPERBACK



Paperback on Amazon

Hold it in your hands

Pair these free worksheets with *Algebra II for Beginners* and you have a complete self-paced course — concept lessons, daily practice, and full exam-style reviews, all in one path. → EffortlessMath.com/product/algebra-ii-for-beginners