Rational and Radical Forms of Expressions

Name: ____________	Date: ____________	Score: ______ / 24

Quick Review

Radicals and rational exponents are two ways of writing the same thing. The key conversion is $\sqrt[n]{x^m} = x^{m/n}$ — the **root index** (n) becomes the **denominator** of the exponent, and the **power inside** (m) becomes the **numerator**. A square root has index 2, so $\sqrt{x} = x^{1/2}$. A cube root of x^2 is $x^{2/3}$. The *denominator* is the root; the *numerator* is the power. **Negative exponents** mean reciprocal: $x^{-n} = \frac{1}{x^n}$, so $\frac{1}{\sqrt[3]{x^4}} = x^{-4/3}$. All the usual exponent rules still apply — product, quotient, power-of-a-power — they just use fractional exponents instead of integers. **Computing** numeric rational powers: think of $a^{m/n}$ as $(\sqrt[n]{a})^m$. Take the root first (smaller numbers, easier), then apply the integer power. For example, $8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$.

PRACTICE

Simplify each expression. Use positive exponents (or simplest radical form) in the final answer.

1. $\sqrt{x}$ $(x \geq 0)$ ________
2. The table shows radicals rewritten with rational exponents. Continue the pattern for $\sqrt[3]{x^2}$ (take $x \geq 0$). ________

radical	rational exponent
$\sqrt{x}$	$x^{1/2}$
$\sqrt[3]{x}$	$x^{1/3}$
$\sqrt[3]{x^2}$	$?$

3. $x^{3/4} \cdot x^{1/4}$ ________
4. $\sqrt[5]{x^{10}}$ $(x \geq 0)$ ________
5. The table splits the rational power $4^{3/2}$ into a root step and a power step. Use it to finish the computation. ________

step	result
$\sqrt{4}$ (the $\frac{1}{2}$)	2
then cube it (the 3)	$?$

6. $\dfrac{1}{\sqrt[3]{x^4}}$ $(x > 0)$ ________
7. $\sqrt{32x^6} \cdot \sqrt[4]{x^4}$ $(x \geq 0)$ ________
8. $\sqrt[3]{x^9}$ $(x > 0)$ ________
9. $s = V^{1/3}$ with $V = 125x^6$ $(x \geq 0)$ ________
10. $(81y^8)^{3/4}$ $(y \geq 0)$ ________
11. The table breaks $27^{2/3}$ into its two steps. Use it to finish. ________

step	result
$\sqrt[3]{27}$ (the $\frac{1}{3}$)	3
then square it (the 2)	$?$

12. $16^{3/4}$ ________
13. $x^{2/3} \cdot x^{1/3}$ ________
14. $\dfrac{x^{5/2}}{x^{1/2}}$ $(x > 0)$ ________
15. $(x^{1/2})^4$ $(x \geq 0)$ ________

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16. $\sqrt{50}$ ________

17. $\sqrt{12x^4}\ (x \geq 0)$ ________

18. $\sqrt[3]{-8x^3}$ ________

19. $x^{-1/2}\ (x > 0)$ ________

20. $(8x^3)^{2/3}\ (x \geq 0)$ ________

♦ Word Problems

21. The side length of a cube with volume V is given by $s = V^{1/3}$. If $V = 125x^6$ and $x \geq 0$, find a simplified expression for the side length. ____

22. The time for a pendulum's swing is $T = 2\pi\sqrt{L/9.8}$ seconds, where L is the length in meters. Find T when $L = 2.45$ m. Use $\pi \approx 3.14$. ____

23. A satellite dish has an area $A = \pi r^2$ where the radius is $r = \sqrt{A/\pi}$. If $A = 12.56$ square meters, find r. Use $\pi \approx 3.14$. ____

24. In physics, the period of orbit at radius r for a satellite around a planet is $T = k \cdot r^{3/2}$ for some constant k (Kepler's Third Law). If a satellite at radius $r = 4$ has period $T = 80$, what is k? ____

Answer Keys

1. $\boxed{x^{1/2}}$
2. $\boxed{x^{2/3}}$
3. $\boxed{x}$
4. $\boxed{x^2}$
5. $\boxed{8}$
6. $\boxed{x^{-4/3}}$
7. $\boxed{4x^4\sqrt{2}}$
8. $\boxed{x^3}$
9. $\boxed{5x^2}$
10. $\boxed{27y^6}$
11. $\boxed{9}$
12. $\boxed{8}$
13. $\boxed{x}$
14. $\boxed{x^2}$
15. $\boxed{x^2}$
16. $\boxed{5\sqrt{2}}$
17. $\boxed{2x^2\sqrt{3}}$
18. $\boxed{-2x}$
19. $\boxed{\dfrac{1}{\sqrt{x}}}$
20. $\boxed{4x^2}$
21. $\boxed{5x^2}$
22. $\boxed{T \approx 3.14 \text{ sec}}$
23. $\boxed{r = 2 \text{ m}}$
24. $\boxed{k = 10}$

Step-by-Step Explanations

1. A careful way to see it: The index 2 becomes the denominator: $\sqrt{x} = x^{1/2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

2. The root index goes on the bottom of the exponent and the inside power on top. Index 3, power 2 gives $x^{2/3}$.

3. One steady path is: Same base, so add: $\frac{3}{4} + \frac{1}{4} = 1$. The product is $x^1 = x$. That gives a quick check on the answer.

4. Start with the key idea: Convert: $x^{10/5} = x^2$. The fifth root of a tenth power is a second power. That gives a quick check on the answer.

5. Read $4^{3/2}$ as $(\sqrt{4})^3$. The table does the root first: $\sqrt{4} = 2$. Now apply the power: $2^3 = 8$. (Squaring first instead gives $\sqrt{64} = 8$ too — either order works.)

6. Convert the radical: $\sqrt[3]{x^4} = x^{4/3}$. The whole expression is the reciprocal: $\frac{1}{x^{4/3}} = x^{-4/3}$.

7. First radical: $\sqrt{32x^6} = \sqrt{16 \cdot 2} \cdot \sqrt{x^6} = 4x^3\sqrt{2}$. Second: $\sqrt[4]{x^4} = x$. Multiply: $4x^3\sqrt{2} \cdot x = 4x^4\sqrt{2}$.

8. Start with the key idea: $x^{9/3} = x^3$. Quick check: $x^3 \cdot x^3 \cdot x^3 = x^9$. ✓This is the part to check before moving on, because it keeps the answer tied to the original question.

9. A careful way to see it: $(125x^6)^{1/3}$: distribute the exponent. $125^{1/3} = 5$ (since $5^3 = 125$). $(x^6)^{1/3} = x^2$. So $s = 5x^2$. That gives a quick check on the answer.

10. Keep the rule visible: $81^{3/4} = (3^4)^{3/4} = 3^3 = 27$. $(y^8)^{3/4} = y^6$. Product: $27y^6$. That gives a quick check on the answer.

11. Think of $27^{2/3}$ as $(\sqrt[3]{27})^2$. The cube root gives 3, and squaring gives $3^2 = 9$.

12. Start with the key idea: Fourth root first: $\sqrt[4]{16} = 2$ (since $2^4 = 16$). Cube it: $2^3 = 8$. That gives a quick check on the answer.

13. A careful way to see it: $\frac{2}{3} + \frac{1}{3} = 1$, so the product is x. This is the part to check before moving on, because it keeps the answer tied to the original question.

14. Keep the rule visible: Subtract: $\frac{5}{2} - \frac{1}{2} = 2$. So x^2. This is the part to check before moving on, because it keeps the answer tied to the original question.

15. One steady path is: Multiply: $\frac{1}{2} \cdot 4 = 2$, so x^2. (Makes sense: $(\sqrt{x})^4 = x^2$.) That gives a quick check on the answer.

16. Start with the key idea: Pull out perfect-square factors: $50 = 25 \cdot 2$. So $\sqrt{50} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$. That gives a quick check on the answer.

17. A careful way to see it: $12 = 4 \cdot 3$. $\sqrt{12x^4} = \sqrt{4} \cdot \sqrt{3} \cdot \sqrt{x^4} = 2\sqrt{3} \cdot x^2 = 2x^2\sqrt{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

18. Cube roots take negatives. $\sqrt[3]{-8} = -2$ (since $(-2)^3 = -8$). $\sqrt[3]{x^3} = x$. Product: $-2x$.

19. One steady path is: Negative exponent flips, fractional becomes a root: $x^{-1/2} = \frac{1}{x^{1/2}} = \frac{1}{\sqrt{x}}$. That gives a quick check on the answer.

20. Distribute the exponent. $8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$. $(x^3)^{2/3} = x^2$. Product: $4x^2$.

21. Substitute and distribute the exponent. $(125x^6)^{1/3} = 125^{1/3} \cdot (x^6)^{1/3} = 5 \cdot x^2 = 5x^2$. (Reality check: cubing $5x^2$ gives $125x^6$. ✓)

22. Inside the radical first: $\frac{2.45}{9.8} = 0.25$. Then $\sqrt{0.25} = 0.5$. Multiply by 2π: $T = 2\pi(0.5) = \pi \approx 3.14$ seconds. (A clean designed-for-the-classroom value.)

23. Inside the radical: $\frac{12.56}{3.14} = 4$. Then $\sqrt{4} = 2$. So the radius is 2 meters. (Check: $\pi(2)^2 = 4\pi \approx 12.56$. ✓)

24. Solve $80 = k \cdot 4^{3/2}$. Compute $4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$. So $80 = 8k$, giving $k = 10$. (Rational exponents do real work in physics — this is the same law Kepler used in the 1600s.)

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