

# Rational and Irrational Numbers

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 34

## Quick Review

A **rational number** is anything you can write as a ratio  $\frac{a}{b}$  of two integers with  $b \neq 0$ . Everything else is **irrational**. The whole game lives in the decimal expansion.

**Decimal test.** If the expansion *terminates* (like 0.75) or *eventually repeats* (like  $0.\overline{142857}$ ), the number is rational. If it goes on forever without ever locking into a repeat, it's irrational.  $\pi$ ,  $e$ ,  $\sqrt{2}$ , and oddballs like 0.10110011100011110000... live in the second camp.

**Square roots.**  $\sqrt{n}$  is rational only when  $n$  is a perfect square.  $\sqrt{16} = 4$ , rational.  $\sqrt{5}$ , irrational.  $\sqrt{20} = 2\sqrt{5}$  – still irrational, because the  $\sqrt{5}$  inside refuses to cooperate.

**Closure shortcuts.** Rationals are closed under  $+$ ,  $-$ ,  $\times$ , and  $\div$  by a nonzero rational. So if you build a number out of  $\frac{3}{7}$ ,  $0.\overline{2}$ ,  $-5$  using those operations, the result stays rational.

**Rational plus irrational.** The sum is always irrational. Why? If  $r + i$  were rational, you could subtract  $r$  and get a rational answer for  $i$  – contradiction. The same kind of argument says a nonzero rational times an irrational is irrational.

**Irrational plus irrational? Use caution.** Sometimes irrational, sometimes not.  $\sqrt{2} + \sqrt{3}$  is irrational, but  $\sqrt{2} + (-\sqrt{2}) = 0$  is as rational as numbers come. The product  $\sqrt{3} \cdot \sqrt{3} = 3$  is rational too. "Always" claims about two irrationals deserve a counter-example check.

**Simplify before classifying.**  $\sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$ . The original two radicals look irrational; the simplified form makes it obvious.

## PRACTICE

Classify each number as rational or irrational. Simplify first when a radical or fraction is hiding the answer.

1. Classify  $\frac{3}{7}$ . \_\_\_\_\_
2. Classify  $\sqrt{5}$ . \_\_\_\_\_
3. Classify  $0.\overline{6}$ . \_\_\_\_\_
4. Classify  $\pi$ . \_\_\_\_\_
5. Classify  $\sqrt{16}$ . The reference table reminds you which square roots land on integers. \_\_\_\_\_

$\sqrt{n}$	Value	Type
$\sqrt{1}$	1	rational
$\sqrt{4}$	2	rational
$\sqrt{9}$	3	rational
$\sqrt{25}$	5	rational

6. Classify  $\sqrt{20}$ . The table shows a few non-perfect-square radicands for contrast. \_\_\_\_\_

$\sqrt{n}$	Perfect square?	Type
$\sqrt{2}$	no	irrational
$\sqrt{5}$	no	irrational
$\sqrt{12}$	no	irrational

7. Classify  $\frac{22}{7}$ . \_\_\_\_\_



8. Classify 0.123456789101112... (concatenating positive integers). The table sorts a few decimals by how their expansion behaves.

Decimal	Expansion	Type
0.75	terminates	rational
$0.\overline{3}$	repeats	rational
0.1010010001...	no repeat	irrational

- 9. Compute and classify  $(\sqrt{2} + 5) - \sqrt{2}$ . \_\_\_\_\_
- 10. Compute and classify  $\sqrt{3} \cdot \sqrt{3}$ . \_\_\_\_\_
- 11. Classify  $\sqrt{18} - \sqrt{8}$ . \_\_\_\_\_
- 12. Mark TRUE or FALSE: The sum of two rational numbers is always rational. \_\_\_\_\_
- 13. Mark TRUE or FALSE: A rational number plus an irrational number is always irrational. \_\_\_\_\_
- 14. Mark TRUE or FALSE: The product of a nonzero rational and an irrational is always irrational. \_\_\_\_\_
- 15. Mark TRUE or FALSE: The sum of two irrational numbers is always irrational. \_\_\_\_\_
- 16. Classify  $-7$ . \_\_\_\_\_
- 17. Classify  $\sqrt[3]{8}$ . \_\_\_\_\_
- 18. Classify  $\sqrt[3]{2}$ . \_\_\_\_\_
- 19. Compute and classify  $\frac{\sqrt{50}}{\sqrt{2}}$ . \_\_\_\_\_
- 20. Classify 0.101001000100001... (the gaps between 1's grow by one zero each time). \_\_\_\_\_

◆ Word Problems

- 21. A student claims that  $\sqrt{2} + \sqrt{8}$  is rational because the radicands 2 and 8 are both even. Show that the claim is wrong: simplify the expression and classify the result correctly. \_\_\_\_\_
- 22. Two students argue: Alex says the product of any two irrational numbers is irrational. Brie says it can be rational. Give one example that settles the argument, and explain who is right. \_\_\_\_\_
- 23. A geometry problem asks for the exact diagonal of a  $1 \times 1$  square. The student writes 1.41 and calls the diagonal rational. Find the exact diagonal length, classify it, and explain why 1.41 is misleading. \_\_\_\_\_
- 24. Decide whether the following number is rational or irrational, and justify briefly:  $\frac{\sqrt{12} + \sqrt{27}}{\sqrt{3}}$ . \_\_\_\_\_

Additional Practice

- 25. Simplify  $\sqrt{72}$ . \_\_\_\_\_
- 26. Simplify  $\sqrt{45}$ . \_\_\_\_\_
- 27. Simplify  $\sqrt[3]{64}$ . \_\_\_\_\_
- 28. Solve  $\sqrt{x + 5} = 9$ . \_\_\_\_\_
- 29. Solve  $\sqrt{x} - 3 = 4$ . \_\_\_\_\_
- 30. Domain of  $y = \sqrt{x - 6}$ . \_\_\_\_\_



31. Add  $3\sqrt{5} + 2\sqrt{5}$ . \_\_\_\_\_

32. Multiply  $\sqrt{3} \cdot \sqrt{12}$ . \_\_\_\_\_

33. Rationalize  $\frac{4}{\sqrt{2}}$ . \_\_\_\_\_

34. Write  $x^{3/2}$  using radicals. \_\_\_\_\_



## Answer Keys

- |                 |  |
|-----------------|--|
| 1. rational     | 13. TRUE   |
| 2. irrational   | 14. TRUE   |
| 3. rational     | 15. FALSE  |
| 4. irrational   | 16. rational   |
| 5. rational     | 17. rational   |
| 6. irrational   | 18. irrational   |
| 7. rational     | 19. 5, rational  |
| 8. irrational   | 20. irrational   |
| 9. 5, rational  | 21. $3\sqrt{2}$ , irrational                           |
| 10. 3, rational | 22. Brie. Quick check: $\sqrt{2} \cdot \sqrt{8} = 4$ . |
| 11. irrational  | 23. $\sqrt{2}$ , irrational; 1.41 is an approximation  |
| 12. TRUE        | 24. 5, rational  |

## Additional Practice Answers

- |                 |                  |
|-----------------|------------------|
| 25. $6\sqrt{2}$ | 30. $x \geq 6$   |
| 26. $3\sqrt{5}$ | 31. $5\sqrt{5}$  |
| 27. 4           | 32. 6            |
| 28. $x = 76$    | 33. $2\sqrt{2}$  |
| 29. $x = 49$    | 34. $\sqrt{x^3}$ |

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

- A ratio of two integers with a nonzero denominator. That's the definition of rational.
- Keep the rule visible: 5 isn't a perfect square, so  $\sqrt{5}$  can't be written as a ratio of two integers. (Proof of this is a classic contradiction argument – worth seeing once.) That gives a quick check on the answer.
- The bar means 6 repeats forever:  $0.\overline{6} = \frac{2}{3}$ . Repeating decimals are always rational.
- Start with the key idea:  $\pi = 3.14159265\dots$  never terminates and never repeats. The decimal 3.14 is a rational *approximation*, not the actual number. That gives a quick check on the answer.
- A careful way to see it:  $\sqrt{16} = 4$ . Every integer is rational – write 4 as  $\frac{4}{1}$ . (16 is a perfect square, so it joins the list in the table.) That gives a quick check on the answer.
- Keep the rule visible:  $\sqrt{20} = 2\sqrt{5}$ . The  $\sqrt{5}$  factor is irrational, and multiplying by a nonzero rational (2) keeps it that way. (20 is not a perfect square – it belongs with the table's entries.) That gives a quick check on the answer.
- A ratio of two integers. (Common confusion:  $\frac{22}{7}$  is a famous approximation of  $\pi$ , but it's a different, rational number.)
- The decimal doesn't terminate, and the blocks of digits keep growing in length, so no fixed repeating pattern can ever form. It belongs in the bottom row of the table – irrational.
- The two  $\sqrt{2}$  terms cancel, leaving 5. Two irrationals can subtract to a rational – watch for it.
- Keep the rule visible:  $\sqrt{3} \cdot \sqrt{3} = 3$ . Two irrationals can multiply to a rational. (The product of two irrationals is not always irrational – this counter-example proves it.) That gives a quick check on the answer.
- Simplify each:  $\sqrt{18} = 3\sqrt{2}$ ,  $\sqrt{8} = 2\sqrt{2}$ . Subtract:  $3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$ , which is irrational. (Always simplify before classifying.)
- Rationals are closed under addition:  $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$ , a ratio of integers (with  $bd \neq 0$ ).
- If the sum were rational, subtracting the rational piece would force the irrational piece to be rational too. Contradiction.
- Same kind of contradiction argument: divide by the rational to recover the irrational. The “nonzero” tag matters – 0 times anything is 0.
- One steady path is:  $\sqrt{2} + (-\sqrt{2}) = 0$ , which is rational. Whenever someone says “always” about irrational sums, hunt for this kind of counter-example. That gives a quick check on the answer.
- Start with the key idea: Every integer is rational.  $-7 = \frac{-7}{1}$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
- A careful way to see it:  $\sqrt[3]{8} = 2$ , an integer. (Cube roots can land on rationals even when the radicand isn't a perfect square – check whether it's a perfect cube.) That gives a quick check on the answer.
- Keep the rule visible: 2 isn't a perfect cube, so  $\sqrt[3]{2}$  can't be written as a ratio of integers. Irrational. That gives a quick check on the answer.
- Combine under the same radical:  $\sqrt{\frac{50}{2}} = \sqrt{25} = 5$ . The quotient collapses to an integer.
- The decimal doesn't terminate and never repeats – the gap pattern keeps stretching, so no fixed block can lock in. Irrational.
- Simplify each radical first.  $\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$ . So  $\sqrt{2} + \sqrt{8} = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$ . Since  $\sqrt{2}$  is irrational and 3 is a nonzero rational, the product  $3\sqrt{2}$  is irrational. (The student's reasoning – that even radicands give rational results – has no basis. Whether a square root is rational depends on whether the radicand is a perfect square, not on parity.)
- Take  $\sqrt{2}$  (irrational) and  $\sqrt{8}$  (also irrational). Their product is  $\sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$ , which is rational. So the product of two irrationals *can* be rational – Brie is right. (Other valid examples:  $\sqrt{3} \cdot \sqrt{3} = 3$ , or  $\sqrt{2} \cdot \sqrt{2} = 2$ . One counter-example is all you need.)
- By the Pythagorean theorem the diagonal is  $\sqrt{1^2 + 1^2} = \sqrt{2}$ , which is irrational. The decimal 1.41 terminates – so it *is* rational, but it's not equal to  $\sqrt{2}$ . It's a rounded approximation (the actual decimal is 1.41421356... without ever locking into a repeat). The student confused the exact value with its calculator display.
- Simplify the top:  $\sqrt{12} = 2\sqrt{3}$  and  $\sqrt{27} = 3\sqrt{3}$ , so the top is  $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$ . Dividing by  $\sqrt{3}$  gives  $\frac{5\sqrt{3}}{\sqrt{3}} = 5$ , an integer and hence rational. (The expression *looks* thoroughly irrational at first glance – always simplify before classifying.)



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