Rational and Irrational Numbers

Name: ______________________	Date: ______________	Score: _____ / 34

Quick Review

A **rational number** is anything you can write as a ratio $\frac{a}{b}$ of two integers with $b \neq 0$. Everything else is **irrational**. The whole game lives in the decimal expansion.

Decimal test. If the expansion *terminates* (like 0.75) or *eventually repeats* (like $0.\overline{142857}$), the number is rational. If it goes on forever without ever locking into a repeat, it's irrational. π, e, $\sqrt{2}$, and oddballs like $0.10110011100011110000\ldots$ live in the second camp.

Square roots. $\sqrt{n}$ is rational only when n is a perfect square. $\sqrt{16} = 4$, rational. $\sqrt{5}$, irrational. $\sqrt{20} = 2\sqrt{5}$ – still irrational, because the $\sqrt{5}$ inside refuses to cooperate.

Closure shortcuts. Rationals are closed under $+$, $-$, $\times$, and $\div$ by a nonzero rational. So if you build a number out of $\frac{3}{7}, 0.\overline{2}, -5$ using those operations, the result stays rational.

Rational plus irrational. The sum is always irrational. Why? If $r + i$ were rational, you could subtract r and get a rational answer for i – contradiction. The same kind of argument says a nonzero rational times an irrational is irrational.

Irrational plus irrational? Use caution. Sometimes irrational, sometimes not. $\sqrt{2} + \sqrt{3}$ is irrational, but $\sqrt{2} + (-\sqrt{2}) = 0$ is as rational as numbers come. The product $\sqrt{3} \cdot \sqrt{3} = 3$ is rational too. "Always" claims about two irrationals deserve a counter-example check.

Simplify before classifying. $\sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$. The original two radicals look irrational; the simplified form makes it obvious.

PRACTICE

Classify each number as rational or irrational. Simplify first when a radical or fraction is hiding the answer.

1. Classify $\frac{3}{7}$. ________
2. Classify $\sqrt{5}$. ________
3. Classify $0.\overline{6}$. ________
4. Classify π. ________
5. Classify $\sqrt{16}$. The reference table reminds you which square roots land on integers. ________

$\sqrt{n}$	Value	Type
$\sqrt{1}$	1	rational
$\sqrt{4}$	2	rational
$\sqrt{9}$	3	rational
$\sqrt{25}$	5	rational

6. Classify $\sqrt{20}$. The table shows a few non-perfect-square radicands for contrast. ________

$\sqrt{n}$	Perfect square?	Type
$\sqrt{2}$	no	irrational
$\sqrt{5}$	no	irrational
$\sqrt{12}$	no	irrational

7. Classify $\frac{22}{7}$. ________

8. Classify $0.123456789101112\ldots$ (concatenating positive integers). The table sorts a few decimals by how their expansion behaves. ________

Decimal	Expansion	Type
0.75	terminates	rational
$0.\overline{3}$	repeats	rational
$0.1010010001\ldots$	no repeat	irrational

9. Compute and classify $(\sqrt{2}+5)-\sqrt{2}$. ________

10. Compute and classify $\sqrt{3}\cdot\sqrt{3}$. ________

11. Classify $\sqrt{18}-\sqrt{8}$. ________

12. Mark TRUE or FALSE: The sum of two rational numbers is always rational. ________

13. Mark TRUE or FALSE: A rational number plus an irrational number is always irrational. ________

14. Mark TRUE or FALSE: The product of a nonzero rational and an irrational is always irrational. ________

15. Mark TRUE or FALSE: The sum of two irrational numbers is always irrational. ________

16. Classify -7. ________

17. Classify $\sqrt[3]{8}$. ________

18. Classify $\sqrt[3]{2}$. ________

19. Compute and classify $\dfrac{\sqrt{50}}{\sqrt{2}}$. ________

20. Classify $0.101001000100001\ldots$ (the gaps between 1's grow by one zero each time). ________

♦ Word Problems

21. A student claims that $\sqrt{2}+\sqrt{8}$ is rational because the radicands 2 and 8 are both even. Show that the claim is wrong: simplify the expression and classify the result correctly. ____

22. Two students argue: Alex says the product of any two irrational numbers is irrational. Brie says it can be rational. Give one example that settles the argument, and explain who is right. ____

23. A geometry problem asks for the exact diagonal of a 1×1 square. The student writes 1.41 and calls the diagonal rational. Find the exact diagonal length, classify it, and explain why 1.41 is misleading. ____

24. Decide whether the following number is rational or irrational, and justify briefly: $\dfrac{\sqrt{12}+\sqrt{27}}{\sqrt{3}}$. ____

Additional Practice

25. Simplify $\sqrt{72}$. ________

26. Simplify $\sqrt{45}$. ________

27. Simplify $\sqrt[3]{64}$. ________

28. Solve $\sqrt{x+5}=9$. ________

29. Solve $\sqrt{x}-3=4$. ________

30. Domain of $y=\sqrt{x-6}$. ________

31. Add $3\sqrt{5} + 2\sqrt{5}$. ________

32. Multiply $\sqrt{3} \cdot \sqrt{12}$. ________

33. Rationalize $\frac{4}{\sqrt{2}}$. ________

34. Write $x^{3/2}$ using radicals. ________

Answer Keys

1. rational	**13.** TRUE
2. irrational	**14.** TRUE
3. rational	**15.** FALSE
4. irrational	**16.** rational
5. rational	**17.** rational
6. irrational	**18.** irrational
7. rational	**19.** $5,$ rational
8. irrational	**20.** irrational
9. $5,$ rational	**21.** $3\sqrt{2},$ irrational
10. $3,$ rational	**22.** Brie. Quick check: $\sqrt{2} \cdot \sqrt{8} = 4.$
11. irrational	**23.** $\sqrt{2},$ irrational; 1.41 is an approximation
12. TRUE	**24.** $5,$ rational

Additional Practice Answers

25. $6\sqrt{2}$	**30.** $x \geq 6$
26. $3\sqrt{5}$	**31.** $5\sqrt{5}$
27. 4	**32.** 6
28. $x = 76$	**33.** $2\sqrt{2}$
29. $x = 49$	**34.** $\sqrt{x^3}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A ratio of two integers with a nonzero denominator. That's the definition of rational.

2. Keep the rule visible: 5 isn't a perfect square, so $\sqrt{5}$ can't be written as a ratio of two integers. (Proof of this is a classic contradiction argument – worth seeing once.) That gives a quick check on the answer.

3. The bar means 6 repeats forever: $0.\overline{6} = \frac{2}{3}$. Repeating decimals are always rational.

4. Start with the key idea: $\pi = 3.14159265\ldots$ never terminates and never repeats. The decimal 3.14 is a rational *approximation*, not the actual number. That gives a quick check on the answer.

5. A careful way to see it: $\sqrt{16} = 4$. Every integer is rational – write 4 as $\frac{4}{1}$. (16 is a perfect square, so it joins the list in the table.) That gives a quick check on the answer.

6. Keep the rule visible: $\sqrt{20} = 2\sqrt{5}$. The $\sqrt{5}$ factor is irrational, and multiplying by a nonzero rational (2) keeps it that way. (20 is not a perfect square – it belongs with the table's entries.) That gives a quick check on the answer.

7. A ratio of two integers. (Common confusion: $\frac{22}{7}$ is a famous approximation of π, but it's a different, rational number.)

8. The decimal doesn't terminate, and the blocks of digits keep growing in length, so no fixed repeating pattern can ever form. It belongs in the bottom row of the table – irrational.

9. The two $\sqrt{2}$ terms cancel, leaving 5. Two irrationals can subtract to a rational – watch for it.

10. Keep the rule visible: $\sqrt{3} \cdot \sqrt{3} = 3$. Two irrationals can multiply to a rational. (The product of two irrationals is not always irrational – this counter-example proves it.) That gives a quick check on the answer.

11. Simplify each: $\sqrt{18} = 3\sqrt{2}$, $\sqrt{8} = 2\sqrt{2}$. Subtract: $3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$, which is irrational. (Always simplify before classifying.)

12. Rationals are closed under addition: $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$, a ratio of integers (with $bd \neq 0$).

13. If the sum were rational, subtracting the rational piece would force the irrational piece to be rational too. Contradiction.

14. Same kind of contradiction argument: divide by the rational to recover the irrational. The "nonzero" tag matters – 0 times anything is 0.

15. One steady path is: $\sqrt{2} + (-\sqrt{2}) = 0$, which is rational. Whenever someone says "always" about irrational sums, hunt for this kind of counter-example. That gives a quick check on the answer.

16. Start with the key idea: Every integer is rational. $-7 = \frac{-7}{1}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

17. A careful way to see it: $\sqrt[3]{8} = 2$, an integer. (Cube roots can land on rationals even when the radicand isn't a perfect square – check whether it's a perfect cube.) That gives a quick check on the answer.

18. Keep the rule visible: 2 isn't a perfect cube, so $\sqrt[3]{2}$ can't be written as a ratio of integers. Irrational. That gives a quick check on the answer.

19. Combine under the same radical: $\sqrt{\frac{50}{2}} = \sqrt{25} = 5$. The quotient collapses to an integer.

20. The decimal doesn't terminate and never repeats – the gap pattern keeps stretching, so no fixed block can lock in. Irrational.

21. Simplify each radical first. $\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$. So $\sqrt{2}+\sqrt{8} = \sqrt{2}+2\sqrt{2} = 3\sqrt{2}$. Since $\sqrt{2}$ is irrational and 3 is a nonzero rational, the product $3\sqrt{2}$ is irrational. (The student's reasoning – that even radicands give rational results – has no basis. Whether a square root is rational depends on whether the radicand is a perfect square, not on parity.)

22. Take $\sqrt{2}$ (irrational) and $\sqrt{8}$ (also irrational). Their product is $\sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$, which is rational. So the product of two irrationals *can* be rational – Brie is right. (Other valid examples: $\sqrt{3} \cdot \sqrt{3} = 3$, or $\sqrt{2} \cdot \sqrt{2} = 2$. One counter-example is all you need.)

23. By the Pythagorean theorem the diagonal is $\sqrt{1^2 + 1^2} = \sqrt{2}$, which is irrational. The decimal 1.41 terminates – so it *is* rational, but it's not equal to $\sqrt{2}$. It's a rounded approximation (the actual decimal is $1.41421356\ldots$ without ever locking into a repeat). The student confused the exact value with its calculator display.

24. Simplify the top: $\sqrt{12} = 2\sqrt{3}$ and $\sqrt{27} = 3\sqrt{3}$, so the top is $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$. Dividing by $\sqrt{3}$ gives $\frac{5\sqrt{3}}{\sqrt{3}} = 5$, an integer and hence rational. (The expression *looks* thoroughly irrational at first glance – always simplify before classifying.)

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