

# Radical Equations

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Score: \_\_\_\_\_ / 24

## Q Quick Review

A **radical equation** has the variable inside a root. There's one game plan, two warnings, and one habit that catches every wrong move: *always check*.

**The game plan.** Isolate the radical on one side. Raise both sides to the matching power (square for  $\sqrt{\quad}$ , cube for  $\sqrt[3]{\quad}$ , etc.) to clear the radical. Solve whatever remains. Then check every candidate in the *original* equation.

**Why the check matters — extraneous roots.** Squaring is not reversible. The step “square both sides” creates the *same* new equation from  $\sqrt{A} = B$  and from  $\sqrt{A} = -B$ , so it can introduce solutions that don't satisfy the original. The classic case:  $\sqrt{x+6} = x$  squares to  $x+6 = x^2$ , giving  $x = 3$  or  $x = -2$ . Plug back:  $\sqrt{9} = 3$  (yes), but  $\sqrt{4} = 2 \neq -2$  (no). Throw out  $x = -2$ . The *check is the algebra step*, not the verification — it's how you finish the problem.

**Cube roots don't introduce extraneous roots.** Cubing is one-to-one over the real numbers.  $\sqrt[3]{A} = B$  becomes  $A = B^3$  with no sign trickery. (Still worth a quick check for arithmetic errors, but no extraneous-root surprises.)

**Same-base equations.** If both sides are roots of the same index, set the radicands equal:  $\sqrt{x+4} = \sqrt{2x-5} \Rightarrow x+4 = 2x-5 \Rightarrow x = 9$ . (Check:  $\sqrt{13} = \sqrt{13}$ , valid.)

**Reading the graph.** The graph of  $y = \sqrt{ax+b}$  and  $y = mx+c$  together makes the geometry visible. Real solutions are intersection points; extraneous roots from squaring show up as intersections of  $y = mx+c$  with the *reflection* of the radical curve below the  $x$ -axis — points where the parabola  $y^2 = ax+b$  crosses but  $y = \sqrt{ax+b}$  does not.

**Common slips.** Skipping the check. Squaring without isolating the radical first (which produces a mess of cross-terms). Cubing both sides and then “checking” for extraneous roots out of habit (cubing doesn't create them, but checking never hurts).

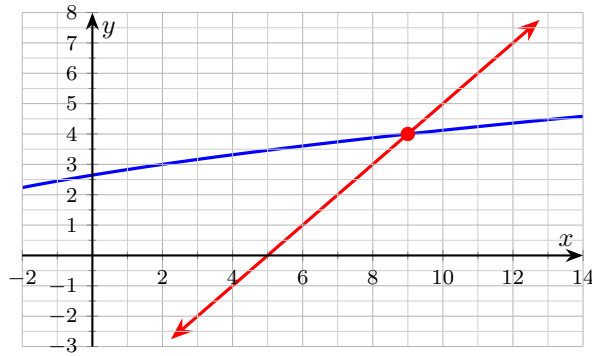
## PRACTICE

Isolate, raise to the matching power, solve, and CHECK every candidate in the original equation.

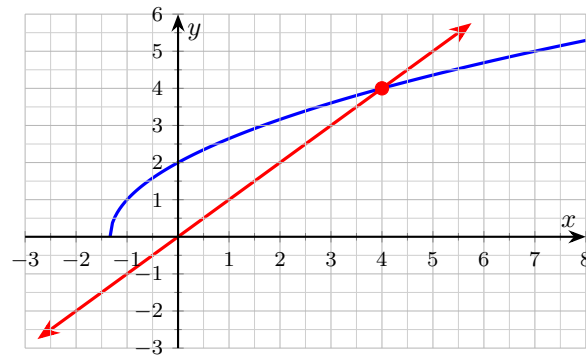
- Solve  $\sqrt{x} = 5$ . \_\_\_\_\_
- Solve  $\sqrt{x} + 3 = 7$ . \_\_\_\_\_
- Solve  $\sqrt{2x-1} = 5$ . \_\_\_\_\_
- Solve  $\sqrt{x+6} = x$ . Identify any extraneous roots. \_\_\_\_\_
- Solve  $\sqrt[3]{x-1} = 2$ . \_\_\_\_\_
- Solve  $\sqrt{x+4} = \sqrt{2x-5}$ . \_\_\_\_\_



7. Solve  $\sqrt{x+7} = x - 5$ . Identify any extraneous roots. \_\_\_\_\_



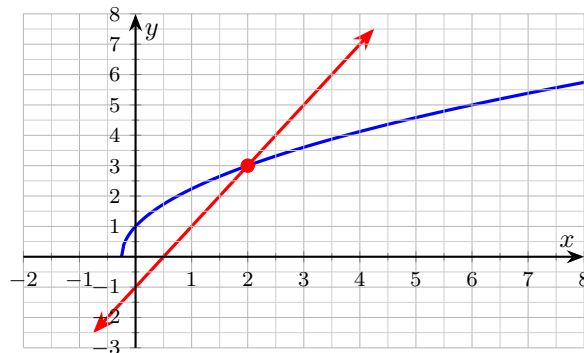
8. Solve  $\sqrt{3x+4} = x$ . Identify any extraneous roots. \_\_\_\_\_



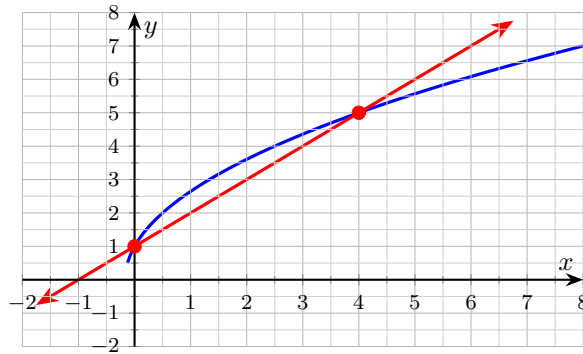
9. Solve  $\sqrt{2x+7} = x + 1$ . Identify any extraneous roots. \_\_\_\_\_

10. Mark TRUE or FALSE: Squaring both sides always preserves the solution set without introducing extraneous roots. \_\_\_\_\_

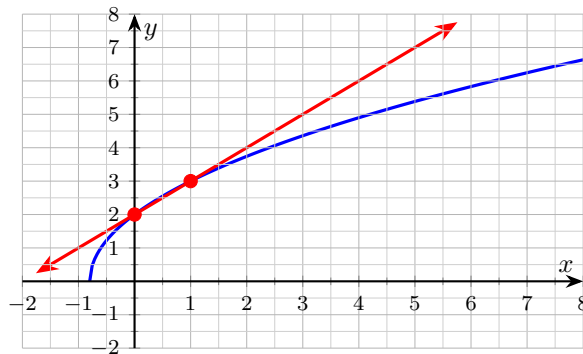
11. Solve  $\sqrt{4x+1} = 2x - 1$ . Identify any extraneous roots. \_\_\_\_\_



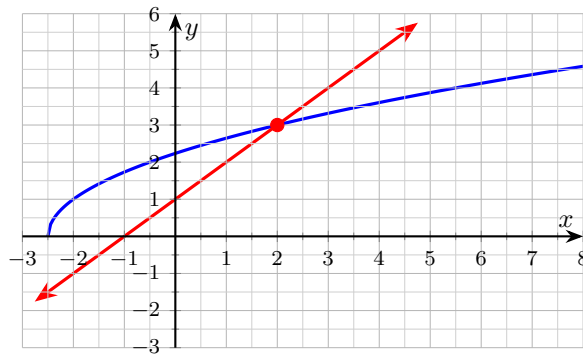
12. Solve  $\sqrt{6x+1} = x+1$ . Identify any extraneous roots. \_\_\_\_\_



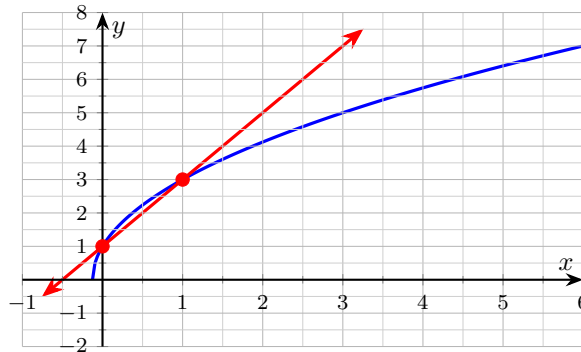
13. Solve  $\sqrt{5x+4} = x+2$ . Identify any extraneous roots. \_\_\_\_\_



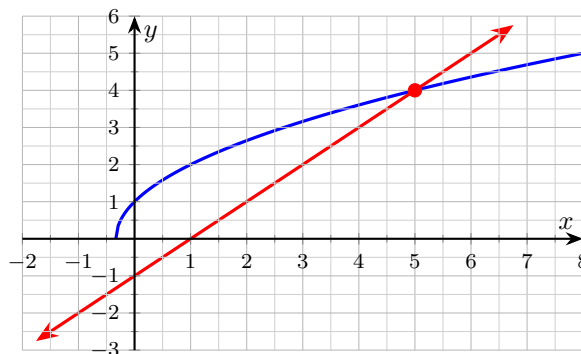
14. Solve  $\sqrt{2x+5} = x+1$ . Identify any extraneous roots. \_\_\_\_\_



15. Solve  $\sqrt{8x + 1} = 2x + 1$ . Identify any extraneous roots. \_\_\_\_\_



16. Solve  $\sqrt{3x + 1} = x - 1$ . Identify any extraneous roots. \_\_\_\_\_



17. Solve  $\sqrt[3]{2x + 3} = 3$ . \_\_\_\_\_

18. Solve  $2\sqrt{x - 1} = 8$ . \_\_\_\_\_

19. Solve  $\sqrt{x + 1} = \sqrt{3x - 7}$ . \_\_\_\_\_

20. Solve  $\sqrt{x} + 2 = x$ . Identify any extraneous roots. \_\_\_\_\_

◆ Word Problems

21. On a flat road, the speed (mph) at which a car will begin to skid in a circular turn of radius  $r$  (feet) is  $v = \sqrt{2.5r}$ . Find the radius of a turn for which the skid speed is 30 mph. Show the check that verifies the candidate radius. \_\_\_\_\_

22. Solve  $\sqrt{2x + 3} + x = 3$  and explain which (if any) candidate solutions are extraneous. \_\_\_\_\_

23. The time (seconds) for a pendulum of length  $L$  (feet) to complete one swing is  $T = \frac{\pi}{4}\sqrt{L}$ . Find the exact length that produces a 2-second swing. Verify by substituting back. \_\_\_\_\_

24. Solve  $\sqrt{x + 11} = \sqrt{x - 1} + 2$ . Carry out the check step explicitly — this is a two-radical equation where extraneous roots are especially common. \_\_\_\_\_



## Answer Keys

1.  $x = 25$   
 2.  $x = 16$   
 3.  $x = 13$   
 4.  $x = 3$   
 5.  $x = 9$   
 6.  $x = 9$   
 7.  $x = 9$   
 8.  $x = 4$   
 9.  $x = \sqrt{6}$   
 10. FALSE  
 11.  $x = 2$   
 12.  $x = 0$  and  $x = 4$   
 13.  $x = 0$  and  $x = 1$   
 14.  $x = 2$   
 15.  $x = 0$  and  $x = 1$   
 16.  $x = 5$   
 17.  $x = 12$   
 18.  $x = 17$   
 19.  $x = 4$   
 20.  $x = 4$   
 21.  $r = 360$  ft  
 22.  $x = 4 - \sqrt{10}$   
 23.  $L = \frac{64}{\pi^2}$  ft  $\approx 6.48$  ft  
 24.  $x = 5$

## Step-by-Step Explanations

1. The radical is already isolated, so square both sides to clear it:  $(\sqrt{x})^2 = 5^2$ , giving  $x = 25$ . **Check:**  $\sqrt{25} = 5 \checkmark$ , so it's valid. Don't write  $\pm 5$  — the radical symbol returns only the principal (non-negative) root.
2. Isolate the radical first by subtracting 3:  $\sqrt{x} = 4$ . Then square both sides:  $x = 16$ . **Check** in the original:  $\sqrt{16} + 3 = 4 + 3 = 7 \checkmark$ . Isolating before squaring keeps the algebra clean.
3. The radical is isolated, so square both sides:  $2x - 1 = 25$ . Solve:  $2x = 26$ , so  $x = 13$ . **Check:**  $\sqrt{2(13)} - 1 = \sqrt{26} = 5 \checkmark$  — it satisfies the original, so it's valid.
4. Square:  $x + 6 = x^2$ , so  $x^2 - x - 6 = 0$  and  $(x - 3)(x + 2) = 0$ , giving  $x = 3$  or  $x = -2$ . **Check**  $x = 3$ :  $\sqrt{9} = 3 \checkmark$ . **Check**  $x = -2$ :  $\sqrt{4} = 2 \neq -2$  (extraneous). Only  $x = 3$  survives.
5. Cube:  $x - 1 = 8$ , so  $x = 9$ . **Check:**  $\sqrt[3]{8} = 2 \checkmark$ . (Cube roots don't introduce extraneous roots, but verifying the arithmetic still pays.)
6. Keep the rule visible: Same index  $\Rightarrow$  set radicands equal:  $x + 4 = 2x - 5$ , so  $x = 9$ . **Check:**  $\sqrt{13} = \sqrt{13} \checkmark$ . That gives a quick check on the answer.
7. Square:  $x + 7 = x^2 - 10x + 25$ , so  $x^2 - 11x + 18 = 0$ , giving  $(x - 9)(x - 2) = 0$  and  $x = 9$  or  $x = 2$ . **Check**  $x = 9$ :  $\sqrt{16} = 4$  and  $9 - 5 = 4 \checkmark$ . **Check**  $x = 2$ :  $\sqrt{9} = 3$  but  $2 - 5 = -3 \neq 3$ . The line sits below the  $x$ -axis at  $x = 2$ , where the radical curve cannot reach. Only  $x = 9$  is valid.
8. Square:  $3x + 4 = x^2$ , so  $x^2 - 3x - 4 = 0$  and  $(x - 4)(x + 1) = 0$ , giving  $x = 4$  or  $x = -1$ . **Check**  $x = 4$ :  $\sqrt{16} = 4 \checkmark$ . **Check**  $x = -1$ :  $\sqrt{1} = 1 \neq -1 \checkmark$ . The line passes through  $(-1, -1)$ , below the radical's reach. Only  $x = 4$  survives.
9. Square:  $2x + 7 = x^2 + 2x + 1$ , so  $x^2 = 6$  and  $x = \pm\sqrt{6}$ . The original requires  $x + 1 \geq 0$ , i.e.,  $x \geq -1$ . **Check**  $x = \sqrt{6} \approx 2.45$ : LHS =  $\sqrt{2(2.45)} + 7 \approx \sqrt{11.9} \approx 3.45$ ; RHS =  $2.45 + 1 = 3.45 \checkmark$ . **Check**  $x = -\sqrt{6} \approx -2.45$ : fails the  $x \geq -1$  constraint — extraneous. Only  $x = \sqrt{6}$  is valid.
10. Squaring is not reversible — it can create extraneous roots that satisfy the squared equation but not the original. That's why the check is mandatory.
11. Square:  $4x + 1 = (2x - 1)^2 = 4x^2 - 4x + 1$ , so  $8x = 4x^2$ , giving  $4x(x - 2) = 0$  and  $x = 0$  or  $x = 2$ . **Check**  $x = 0$ :  $\sqrt{1} = 1$  but  $2(0) - 1 = -1 \neq 1$ . **Check**  $x = 2$ :  $\sqrt{9} = 3$  and  $2(2) - 1 = 3 \checkmark$ . Only  $x = 2$  survives. The graph shows the line  $y = 2x - 1$  dipping below the  $x$ -axis near  $x = 0$ , where the radical can't follow.
12. Square:  $6x + 1 = x^2 + 2x + 1$ , so  $x^2 - 4x = 0$  and  $x(x - 4) = 0$ , giving  $x = 0$  or  $x = 4$ . **Check**  $x = 0$ :  $\sqrt{1} = 1$  and  $0 + 1 = 1 \checkmark$ . **Check**  $x = 4$ :  $\sqrt{25} = 5$  and  $4 + 1 = 5 \checkmark$ . Both check out — the line  $y = x + 1$  meets the radical curve at two valid intersections. (Not every radical equation has an extraneous root; sometimes the algebra is honest.)
13. Square:  $5x + 4 = x^2 + 4x + 4$ , so  $x^2 - x = 0$  and  $x(x - 1) = 0$ , giving  $x = 0$  or  $x = 1$ . **Check**  $x = 0$ :  $\sqrt{4} = 2$  and  $0 + 2 = 2 \checkmark$ . **Check**  $x = 1$ :  $\sqrt{9} = 3$  and  $1 + 2 = 3 \checkmark$ . Both valid.
14. Square:  $2x + 5 = x^2 + 2x + 1$ , so  $x^2 = 4$  and  $x = \pm 2$ . **Check**  $x = 2$ :  $\sqrt{9} = 3$  and  $2 + 1 = 3 \checkmark$ . **Check**  $x = -2$ :  $\sqrt{1} = 1$  but  $-2 + 1 = -1 \neq 1$ . Only  $x = 2$  valid.

15. Square:  $8x + 1 = (2x + 1)^2 = 4x^2 + 4x + 1$ , so  $4x = 4x^2$ , giving  $4x(x - 1) = 0$  and  $x = 0$  or  $x = 1$ . **Check**  $x = 0$ :  $\sqrt{1} = 1$  and  $2(0) + 1 = 1 \checkmark$ . **Check**  $x = 1$ :  $\sqrt{9} = 3$  and  $2(1) + 1 = 3 \checkmark$ . Both valid.
16. Square:  $3x + 1 = x^2 - 2x + 1$ , so  $x^2 - 5x = 0$  and  $x(x - 5) = 0$ , giving  $x = 0$  or  $x = 5$ . **Check**  $x = 5$ :  $\sqrt{16} = 4$  and  $5 - 1 = 4 \checkmark$ . **Check**  $x = 0$ :  $\sqrt{1} = 1$  but  $0 - 1 = -1 \neq 1$ . Only  $x = 5$  survives.
17. Cube both sides to clear the cube root:  $2x + 3 = 3^3 = 27$ . Solve:  $2x = 24$ , so  $x = 12$ . **Check:**  $\sqrt[3]{2(12)} + 3 = \sqrt[3]{24} + 3 = 3 \checkmark$ . Cubing is one-to-one, so it never introduces extraneous roots.
18. Isolate the radical by dividing both sides by 2:  $\sqrt{x - 1} = 4$ . Square:  $x - 1 = 16$ , so  $x = 17$ . **Check:**  $2\sqrt{17 - 1} = 2\sqrt{16} = 2(4) = 8 \checkmark$ . Clear the coefficient before squaring or you'll square a mess.
19. Both sides are square roots of the same index, so square once to set the radicands equal:  $x + 1 = 3x - 7$ . Solve:  $2x = 8$ , so  $x = 4$ . **Check:** both sides give  $\sqrt{5} \checkmark$ . (Still confirm the radicands stay non-negative at the answer — here  $5 \geq 0$ , so we're fine.)
20. Isolate:  $\sqrt{x} = x - 2$ . Square:  $x = (x - 2)^2 = x^2 - 4x + 4$ , so  $x^2 - 5x + 4 = 0$  and  $(x - 1)(x - 4) = 0$ , giving  $x = 1$  or  $x = 4$ . **Check**  $x = 1$ :  $\sqrt{1} + 2 = 3 \neq 1$ . **Check**  $x = 4$ :  $\sqrt{4} + 2 = 4 \checkmark$ . Only  $x = 4$  valid.
21. Set  $\sqrt{2.5r} = 30$ . Square:  $2.5r = 900$ , so  $r = 360$  ft. **Check:**  $\sqrt{2.5(360)} = \sqrt{900} = 30 \checkmark$ . Valid. (A 30-mph corner needs a 360-ft radius — a moderately long curve, which matches highway-design tables.)
22. Isolate the radical first:  $\sqrt{2x + 3} = 3 - x$ . The radical is non-negative, so the right side must be too:  $3 - x \geq 0$ , giving  $x \leq 3$ . Domain of the radicand:  $2x + 3 \geq 0$ , so  $x \geq -\frac{3}{2}$ . **Square both sides:**  $2x + 3 = (3 - x)^2 = 9 - 6x + x^2$ , which rearranges to  $x^2 - 8x + 6 = 0$ . Quadratic formula:  $x = \frac{8 \pm \sqrt{64 - 24}}{2} = \frac{8 \pm \sqrt{40}}{2} = 4 \pm \sqrt{10}$ . **Check**  $x = 4 - \sqrt{10} \approx 0.84$ : satisfies  $-\frac{3}{2} \leq x \leq 3$ . Plug back:  $\sqrt{2(0.84)} + 3 \approx \sqrt{4.68} \approx 2.16$ , and  $3 - 0.84 = 2.16$ . Match  $\checkmark$ . **Check**  $x = 4 + \sqrt{10} \approx 7.16$ : fails  $x \leq 3$ . Extraneous — squaring created this root. Only  $x = 4 - \sqrt{10}$  is valid.
23. Set  $\frac{\pi}{4}\sqrt{L} = 2$ , so  $\sqrt{L} = \frac{8}{\pi}$ . Square:  $L = \frac{64}{\pi^2} \approx 6.48$  ft. **Check:**  $T = \frac{\pi}{4}\sqrt{\frac{64}{\pi^2}} = \frac{\pi}{4} \cdot \frac{8}{\pi} = 2 \checkmark$ . (The constant  $\pi/4$  encodes the physics: longer pendulums swing more slowly, but the relationship is square-root, not linear. Quadrupling the length only doubles the period — consistent with the length ratio  $64/\pi^2$  producing a clean  $T = 2$  s.)
24. Square both sides:  $x + 11 = (x - 1) + 4\sqrt{x - 1} + 4$ , so  $x + 11 = x + 3 + 4\sqrt{x - 1}$ . Subtract:  $8 = 4\sqrt{x - 1}$ , giving  $\sqrt{x - 1} = 2$  and  $x - 1 = 4$ , so  $x = 5$ . **Check:**  $\sqrt{16} = 4$  on the left;  $\sqrt{4} + 2 = 2 + 2 = 4$  on the right. Match  $\checkmark$ . (When two radicals appear, squaring once leaves one radical; isolating and squaring a second time finishes the job. Each squaring step is a chance for extraneous roots, so the final check is non-negotiable.)



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