# Pythagorean Identities

| Name: ______________ | Date: ______________ | Score: ______ / 34 |
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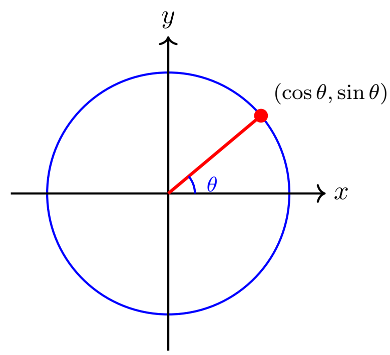
## Quick Review

The unit circle gives us our most useful trig identity for free. Any point on the unit circle has $x^2 + y^2 = 1$, and the coordinates are $x = \cos\theta$, $y = \sin\theta$. Substitute and you get the **fundamental Pythagorean identity**: $\sin^2\theta + \cos^2\theta = 1$ for every $\theta$.




**Two more identities for free.** Divide everything by $\cos^2\theta$ (where $\cos\theta \neq 0$):

$$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta} \;\Rightarrow\; \tan^2\theta + 1 = \sec^2\theta.$$

Divide everything by $\sin^2\theta$ (where $\sin\theta \neq 0$):

$$1 + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} \;\Rightarrow\; 1 + \cot^2\theta = \csc^2\theta.$$

**The three Pythagorean identities (memorize).**

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

**Finding one trig value from another.** If you know one value and a quadrant, you can find the other five. Plug into the Pythagorean identity, solve for the missing function squared, and choose the sign from the quadrant.

**Useful simplifications.** $1 - \sin^2\theta = \cos^2\theta$; $1 - \cos^2\theta = \sin^2\theta$; $\sec^2\theta - \tan^2\theta = 1$; $\csc^2\theta - \cot^2\theta = 1$. These let you simplify expressions that look messy at first glance.

**Common slips.** Forgetting the $\pm$ when square-rooting (you need the quadrant to fix the sign). Writing $\sin\theta + \cos\theta = 1$ (it's the *squares* that sum to $1$, never the values themselves). Using $\tan^2\theta - 1 = \sec^2\theta$ (wrong sign: it's $\tan^2\theta + 1 = \sec^2\theta$).

## PRACTICE

*Use the three Pythagorean identities. Take signs from the given quadrant.*

**1.** State the fundamental Pythagorean identity. ________

**2.** If $\sin\theta = \frac{3}{5}$ and $\theta$ is acute, find $\cos\theta$. ________

**3.** Simplify $1 + \tan^2\theta$. ________

**4.** Simplify $1 + \cot^2\theta$. ________

**5.** If $\cos\theta = \frac{12}{13}$ and $\theta$ is acute, find $\sec\theta$ and $\sin\theta$. ________

**6.** Simplify $\dfrac{\sin^2\theta}{1 - \sin^2\theta}$. ________

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7. Evaluate $\sin^2\theta(1+\cot^2\theta)$ at $\theta = \frac{\pi}{4}$. ________

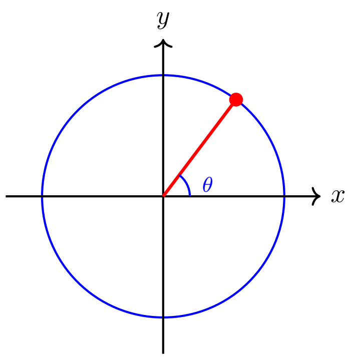
8. If $\sec\theta = \frac{5}{3}$ and $\theta$ is in Q4, find $\tan\theta$. ________

9. Simplify $(1+\tan^2\theta)\cos^2\theta$. ________

10. True or False: $\sec^2\theta - \tan^2\theta = 1$. ________

11. If $\sin\theta = -\frac{5}{13}$ and $\theta$ is in Q3, find $\cos\theta$. ________

12. The unit-circle diagram shows $\cos\theta = \frac{3}{5}$ for $\theta$ in Q1. Use the Pythagorean identity to find $\sin\theta$. ________

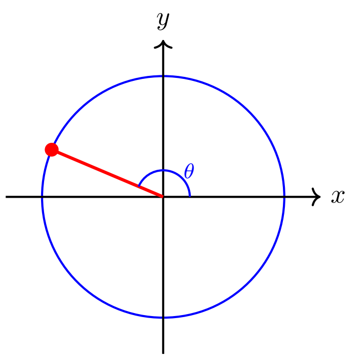



13. Simplify $\frac{\cos^2\theta}{1-\sin\theta}$ (assume $\sin\theta \neq 1$). ________

14. If $\tan\theta = 2$ and $\theta$ is acute, find $\sec\theta$. ________

15. If $\csc\theta = \frac{5}{3}$ for an acute $\theta$, find $\cot\theta$. ________

16. The unit circle shows angle $\theta$ in Q2 with $\sin\theta = \frac{5}{13}$. Find $\cos\theta$. ________




17. Simplify $\sin\theta \cdot \sec\theta$. ________

18. Simplify $(\sec\theta - 1)(\sec\theta + 1)$. ________

19. If $\cos\theta = -\frac{2}{3}$ and $\theta$ is in Q3, find $\sin\theta$. ________

20. Simplify $\frac{1-\cos^2\theta}{\sin\theta}$ (assume $\sin\theta \neq 0$). ________

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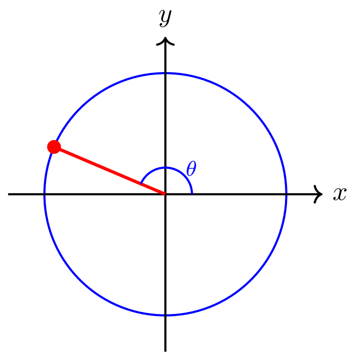
## Word Problems

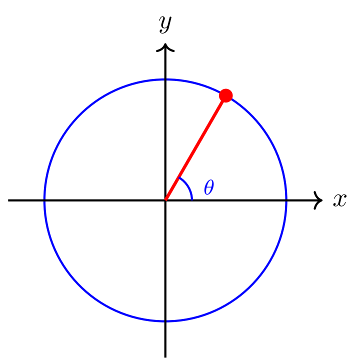
**21.** An angle $\theta$ in Q2 satisfies $\sin\theta = \frac{5}{13}$. Use a Pythagorean identity to find $\cos\theta$ exactly, and then compute $\tan\theta$. ____




**22.** Show that $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$ for every $\theta$. ____

**23.** If $\theta$ is acute and $\tan\theta = \frac{3}{4}$, find $\sec\theta$, $\sin\theta$, and $\cos\theta$ exactly. ____

**24.** Simplify the expression $\frac{\sec\theta}{\tan\theta + \cot\theta}$. (Assume all functions are defined.) ____




## Additional Practice

**25.** Find $\sin\theta$ if opposite $= 5$, hypotenuse $= 13$. ________

**26.** Find $\cos\theta$ if adjacent $= 12$, hypotenuse $= 13$. ________

**27.** Find $\tan\theta$ if opposite $= 7$, adjacent $= 4$. ________

**28.** Find $\sin 30°$. ________

**29.** Find $\cos 60°$. ________

**30.** Find $\tan 45°$. ________

**31.** Convert $180°$ to radians. ________

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**32.** Convert $\frac{\pi}{3}$ radians to degrees. ________

**33.** Find a coterminal angle with $70^\circ$. ________

**34.** Reference angle for $150^\circ$. ________

# Answer Keys

1. $\boxed{\sin^2\theta + \cos^2\theta = 1}$
2. $\boxed{\frac{4}{5}}$
3. $\boxed{\sec^2\theta}$
4. $\boxed{\csc^2\theta}$
5. $\boxed{\sec\theta = \frac{13}{12},\ \sin\theta = \frac{5}{13}}$
6. $\boxed{\tan^2\theta}$
7. $\boxed{1}$
8. $\boxed{-\frac{4}{3}}$
9. $\boxed{1}$
10. True
11. $\boxed{-\frac{12}{13}}$
12. $\boxed{\frac{4}{5}}$
13. $\boxed{1+\sin\theta}$
14. $\boxed{\sqrt{5}}$
15. $\boxed{\frac{4}{3}}$
16. $\boxed{-\frac{12}{13}}$
17. $\boxed{\tan\theta}$
18. $\boxed{\tan^2\theta}$
19. $\boxed{-\frac{\sqrt{5}}{3}}$
20. $\boxed{\sin\theta}$
21. $\boxed{\cos\theta = -\frac{12}{13},\ \tan\theta = -\frac{5}{12}}$
22. see explanation
23. $\boxed{\sec\theta = \frac{5}{4},\ \sin\theta = \frac{3}{5},\ \cos\theta = \frac{4}{5}}$
24. $\boxed{\sin\theta}$

### Additional Practice Answers

25. $\boxed{\frac{5}{13}}$
26. $\boxed{\frac{12}{13}}$
27. $\boxed{\frac{7}{4}}$
28. $\boxed{\frac{1}{2}}$
29. $\boxed{\frac{1}{2}}$
30. $\boxed{1}$
31. $\boxed{\pi}$
32. $\boxed{60^\circ}$
33. $\boxed{430^\circ}$
34. $\boxed{30^\circ}$

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

**1.** A careful way to see it: Direct from the unit-circle equation $x^2+y^2=1$ with $x=\cos\theta$ and $y=\sin\theta$. That gives a quick check on the answer.

**2.** Keep the rule visible: $\cos^2\theta = 1-\frac{9}{25}=\frac{16}{25}$. Acute angle means $\cos\theta>0$, so $\cos\theta=\frac{4}{5}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**3.** One steady path is: This is the second Pythagorean identity: $1+\tan^2\theta = \sec^2\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**4.** Start from the main Pythagorean identity $\sin^2\theta+\cos^2\theta=1$ and divide every term by $\sin^2\theta$: that turns $\cos^2\theta/\sin^2\theta$ into $\cot^2\theta$ and $1/\sin^2\theta$ into $\csc^2\theta$, giving $1+\cot^2\theta=\csc^2\theta$. (It's the cousin of $1+\tan^2\theta=\sec^2\theta$ — same move, divided by $\cos^2\theta$ instead.)

**5.** A careful way to see it: sec is the reciprocal of cos. For sine: $\sin^2\theta = 1-\frac{144}{169}=\frac{25}{169}$, so $\sin\theta=\frac{5}{13}$ (positive, acute). That gives a quick check on the answer.

**6.** Keep the rule visible: $1-\sin^2\theta=\cos^2\theta$, so the expression becomes $\frac{\sin^2\theta}{\cos^2\theta}=\tan^2\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**7.** One steady path is: $1+\cot^2\theta=\csc^2\theta=\frac{1}{\sin^2\theta}$, so $\sin^2\theta\cdot\csc^2\theta = \sin^2\theta\cdot\frac{1}{\sin^2\theta}=1$ (whenever $\sin\theta\neq 0$). The value $\theta=\frac{\pi}{4}$ is just one of many that work. That gives a quick check on the answer.

**8.** Start with the key idea: $\tan^2\theta=\sec^2\theta-1=\frac{25}{9}-1=\frac{16}{9}$, so $\tan\theta=\pm\frac{4}{3}$. Q4 has tangent negative, so $\tan\theta=-\frac{4}{3}$. That gives a quick check on the answer.

**9.** A careful way to see it: $(1+\tan^2\theta)\cos^2\theta = \sec^2\theta\cdot\cos^2\theta = \frac{1}{\cos^2\theta}\cdot\cos^2\theta=1$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**10.** Keep the rule visible: Rearrange $1+\tan^2\theta=\sec^2\theta$ to get $\sec^2\theta-\tan^2\theta=1$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**11.** One steady path is: $\cos^2\theta=1-\frac{25}{169}=\frac{144}{169}$, so $\cos\theta=\pm\frac{12}{13}$. Q3 has cosine negative, so $\cos\theta=-\frac{12}{13}$. That gives a quick check on the answer.

**12.** Start with the key idea: $\sin^2\theta = 1-\left(\frac{3}{5}\right)^2 = 1-\frac{9}{25}=\frac{16}{25}$. Q1 has sine positive, so $\sin\theta=\frac{4}{5}$. That gives a quick check on the answer.

**13.** A careful way to see it: $\cos^2\theta=1-\sin^2\theta=(1-\sin\theta)(1+\sin\theta)$. Cancel $(1-\sin\theta)$: $1+\sin\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**14.** Keep the rule visible: $\sec^2\theta=1+\tan^2\theta=1+4=5$. Acute angle has $\sec\theta>0$, so $\sec\theta=\sqrt{5}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**15.** One steady path is: $\cot^2\theta=\csc^2\theta-1=\frac{25}{9}-1=\frac{16}{9}$, so $\cot\theta=\pm\frac{4}{3}$. Acute angle has cotangent positive: $\cot\theta=\frac{4}{3}$. That gives a quick check on the answer.

**16.** Start with the key idea: $\cos^2\theta=1-\frac{25}{169}=\frac{144}{169}$, so $\cos\theta=\pm\frac{12}{13}$. Q2 has cosine negative, so $\cos\theta=-\frac{12}{13}$. That gives a quick check on the answer.

**17.** A careful way to see it: $\sin\theta\cdot\frac{1}{\cos\theta}=\frac{\sin\theta}{\cos\theta}=\tan\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**18.** Keep the rule visible: Difference of squares: $\sec^2\theta-1=\tan^2\theta$ (rearranged second Pythagorean identity). That gives a quick check on the answer.

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**19.** One steady path is: $\sin^2\theta = 1 - \frac{4}{9} = \frac{5}{9}$, so $\sin\theta = \pm\frac{\sqrt{5}}{3}$. Q3 has sine negative, so $\sin\theta = -\frac{\sqrt{5}}{3}$. That gives a quick check on the answer.

**20.** Start with the key idea: $1-\cos^2\theta = \sin^2\theta$. So the quotient is $\frac{\sin^2\theta}{\sin\theta} = \sin\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

**21.** From $\sin^2\theta + \cos^2\theta = 1$: $\cos^2\theta = 1 - \frac{25}{169} = \frac{144}{169}$. In Q2 cosine is negative, so $\cos\theta = -\frac{12}{13}$. Then $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5/13}{-12/13} = -\frac{5}{12}$.

**22.** Expand the left side: $(\sin\theta + \cos\theta)^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$. Group: $(\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta = 1 + 2\sin\theta\cos\theta$ by the Pythagorean identity. So the identity holds for every $\theta$. (Bonus: this is one step of the derivation of $\sin(2\theta) = 2\sin\theta\cos\theta$, which you'll meet in a future chapter.)

**23.** One steady path is: $\sec^2\theta = 1 + \tan^2\theta = 1 + \frac{9}{16} = \frac{25}{16}$. Acute angle has secant positive: $\sec\theta = \frac{5}{4}$, so $\cos\theta = \frac{4}{5}$. Then $\sin\theta = \tan\theta\cdot\cos\theta = \frac{3}{4}\cdot\frac{4}{5} = \frac{3}{5}$. (Quick check: $(\frac{3}{5})^2 + (\frac{4}{5})^2 = \frac{9+16}{25} = 1$ ✓.) That gives a quick check on the answer.

**24.** Write in terms of sin and cos: $\frac{1/\cos\theta}{\sin\theta/\cos\theta + \cos\theta/\sin\theta}$. Combine the denominator: $\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$. So the expression is $\frac{1/\cos\theta}{1/(\sin\theta\cos\theta)} = \frac{\sin\theta\cos\theta}{\cos\theta} = \sin\theta$.

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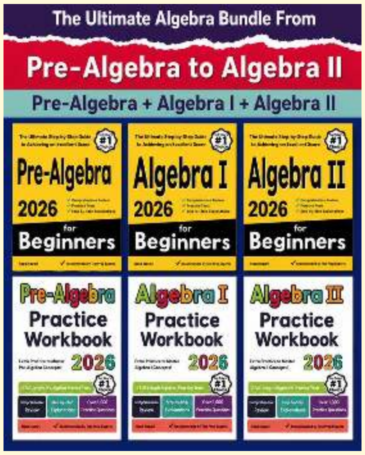


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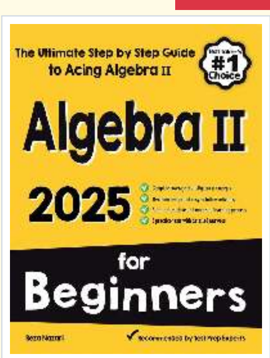


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