Polynomial Identities

Name: ______	Date: ______	Score: ____ / 36

Quick Review

A **polynomial identity** is an equation between polynomial expressions that holds for *all values* of the variables. Memorize the headline patterns — they appear constantly:

- *Square of a sum:* $(a+b)^2 = a^2 + 2ab + b^2$.
- *Square of a difference:* $(a-b)^2 = a^2 - 2ab + b^2$.
- *Difference of squares:* $(a-b)(a+b) = a^2 - b^2$.
- *Sum of cubes:* $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$.
- *Difference of cubes:* $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.
- *Cube of a sum:* $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.
- *Cube of a difference:* $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.

Common traps to call out by name: $(a+b)^2 \neq a^2 + b^2$ (you must include the $2ab$ middle term). $(a+b)^3 \neq a^3 + b^3$ (the cube has four terms, not two). $a^2 + b^2$ is a *sum* of squares and does *not* factor over the reals — only *differences* of squares factor.

Identities also lend themselves to numerical shortcuts. $99^2 = (100-1)^2 = 100^2 - 200 + 1 = 9801$. $101 \cdot 99 = (100+1)(100-1) = 10000 - 1 = 9999$. These mental-math tricks come from the same identities — pattern-matching can save a lot of arithmetic.

Pythagorean triple identity: for integers $m > n > 0$, the triple $(m^2 - n^2,\ 2mn,\ m^2 + n^2)$ satisfies $(m^2 - n^2)^2 + (2mn)^2 = (m^2 + n^2)^2$. Setting $m = 2, n = 1$ gives $(3, 4, 5)$ — the most famous Pythagorean triple.

PRACTICE

Identify or apply the polynomial identity in each problem.

1. $(a+b)^2$ ________
2. $(a-b)(a+b)$ ________
3. $a^3 + b^3$ ________
4. 99^2 ________
5. $(m^2 - n^2)^2 + (2mn)^2$ ________
6. $(2x+3)^3$ ________
7. $(101)^2$ ________
8. $(x-4)^2 - (x+4)^2$ ________
9. $(a+b)(a^2 - ab + b^2)$ ________
10. Use the box to expand $(x+5)(x-5)$ and confirm the difference-of-squares pattern. ________

$\times$	x	-5
x		
5		

11. $(3x-2)^2$ ________
12. $x^3 - 27$ ________
13. Does $a^2 + b^2$ factor over the reals? ________
14. $(50+1)^3$ ________
15. Which is true: $(a+b)^3 = a^3 + b^3$? ________

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16. The table rewrites $47 \cdot 53$ to fit the difference-of-squares identity $(a-b)(a+b) = a^2 - b^2$. Use it to compute the product. ________

Product	a	b
$47 \cdot 53$	50	3

17. $(x+y)^2 - (x-y)^2$ ________

18. The table gives the Pythagorean-triple formulas. Use it with $m = 3,\ n = 2$ to generate the triple (a, b, c). ________

Leg a	Leg b	Hyp. c
$m^2 - n^2$	$2mn$	$m^2 + n^2$

19. Factor $x^6 - y^6$ using identities ________

20. Verify identity: $(a+b)(a-b) + b^2$ ________

♦ Word Problems

21. Use a polynomial identity to compute 98^2 mentally and show the algebraic step that makes it easy. ____

22. Generate a Pythagorean triple using $m = 4, n = 1$. Verify that the three numbers actually form a right triangle. ____

23. Show that $(x+y)^2 - (x-y)^2 = 4xy$ by expanding both squares and combining like terms. ____

24. Factor $x^6 - 1$ completely over the reals using sum/difference patterns. ____

Additional Practice

25. Write $3x - 5 + x^3$ in standard form. ________

26. Find the degree of $7x^4 - 2x^2 + 9$. ________

27. Add $(2x^2 + 3x - 1) + (x^2 - 5x + 4)$. ________

28. Subtract $(5x^2 - x + 6) - (2x^2 + 3x - 1)$. ________

29. Multiply $(x+4)(x-3)$. ________

30. Factor $x^2 + 9x + 20$. ________

31. Factor $6x^2 + 9x$. ________

32. Find the GCF of $12x^3$ and $18x^2$. ________

33. Divide $(x^2 + 5x + 6)$ by $(x+2)$. ________

34. Find the remainder when $x^2 - 1$ is divided by $x - 3$. ________

35. Zeros of $(x-5)(x+1)$. ________

36. Is $x = 2$ a zero of $x^2 - 4$? ________

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Answer Keys

1. $\boxed{a^2+2ab+b^2}$
2. $\boxed{a^2-b^2}$
3. $\boxed{(a+b)(a^2-ab+b^2)}$
4. $\boxed{9801}$
5. $\boxed{(m^2+n^2)^2}$
6. $\boxed{8x^3+36x^2+54x+27}$
7. $\boxed{10201}$
8. $\boxed{-16x}$
9. $\boxed{a^3+b^3}$
10. $\boxed{x^2-25}$
11. $\boxed{9x^2-12x+4}$
12. $\boxed{(x-3)(x^2+3x+9)}$
13. $\boxed{no}$
14. $\boxed{132651}$
15. $\boxed{false}$
16. $\boxed{2491}$
17. $\boxed{4xy}$
18. $\boxed{(5,12,13)}$
19. $\boxed{(x-y)(x+y)(x^2-xy+y^2)(x^2+xy+y^2)}$
20. $\boxed{a^2}$
21. $\boxed{9604}$
22. $\boxed{(15,8,17)}$
23. $\boxed{4xy}$
24. $\boxed{(x-1)(x+1)(x^2+x+1)(x^2-x+1)}$

Additional Practice Answers

25. $\boxed{x^3+3x-5}$
26. $\boxed{4}$
27. $\boxed{3x^2-2x+3}$
28. $\boxed{3x^2-4x+7}$
29. $\boxed{x^2+x-12}$
30. $\boxed{(x+4)(x+5)}$
31. $\boxed{3x(2x+3)}$
32. $\boxed{6x^2}$
33. $\boxed{x+3}$
34. $\boxed{8}$
35. $\boxed{x=5,\ -1}$
36. $\boxed{\text{yes}}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Square of a sum. Don't forget the $2ab$ middle term. This is the part to check before moving on, because it keeps the answer tied to the original question.

2. Keep the rule visible: Difference of squares. The cross terms $+ab$ and $-ab$ cancel. That gives a quick check on the answer.

3. One steady path is: Sum of cubes. The middle of the second factor is $-ab$, not $+ab$. That gives a quick check on the answer.

4. Start with the key idea: $(100-1)^2 = 10000-200+1 = 9801$. Square of a difference. This is the part to check before moving on, because it keeps the answer tied to the original question.

5. Pythagorean triple identity. Expanding both sides gives $m^4+2m^2n^2+n^4$ either way.

6. Keep the rule visible: Cube of a sum: $(2x)^3+3(2x)^2(3)+3(2x)(3)^2+3^3 = 8x^3+36x^2+54x+27$. That gives a quick check on the answer.

7. One steady path is: $(100+1)^2 = 10000+200+1 = 10201$. This is the part to check before moving on, because it keeps the answer tied to the original question.

8. Use difference of squares with $A = x-4, B = x+4$: $(A-B)(A+B)$. $A-B = -8$, $A+B = 2x$. Product: $-16x$.

9. This matches $(a+b)(a^2-ab+b^2) = a^3+b^3$. Expanding confirms it: the a^2b, ab^2 cross terms all cancel, leaving just a^3+b^3 (the sum of cubes).

10. The cells are $x^2, -5x, 5x, -25$. The two middle cells cancel ($-5x+5x = 0$), leaving x^2-25 — exactly the difference of squares.

11. Square of a difference $(a-b)^2 = a^2-2ab+b^2$ with $a = 3x, b = 2$: $(3x)^2-2(3x)(2)+2^2 = 9x^2-12x+4$. Keep the middle term $-12x$ — dropping it is the classic slip.

12. Difference of cubes with $a = x, b = 3$. The middle of the second factor is $+3x$ for the difference (recall: *sum* has $-ab$, *difference* has $+ab$).

13. Sum of squares is irreducible over the reals. It factors only over the complex numbers as $(a+bi)(a-bi)$.

14. Keep the rule visible: Cube of a sum: $50^3+3(50^2)(1)+3(50)(1)+1 = 125000+7500+150+1 = 132651$. That gives a quick check on the answer.

15. One steady path is: $(a+b)^3$ has four terms; it equals $a^3+3a^2b+3ab^2+b^3$, not just a^3+b^3. That gives a quick check on the answer.

16. Start with the key idea: With $a = 50, b = 3$: $(50-3)(50+3) = 50^2-3^2 = 2500-9 = 2491$. This is the part to check before moving on, because it keeps the answer tied to the original question.

17. A careful way to see it: Expand: $(x^2+2xy+y^2)-(x^2-2xy+y^2) = 4xy$. The x^2 and y^2 terms cancel. That gives a quick check on the answer.

18. Substitute $m = 3, n = 2$ into each formula: $m^2-n^2 = 9-4 = 5$, $2mn = 12$, $m^2+n^2 = 9+4 = 13$. Check: $5^2+12^2 = 169 = 13^2$.

19. View as difference of squares first: $(x^3)^2-(y^3)^2 = (x^3-y^3)(x^3+y^3)$. Then factor each cube: $x^3-y^3 = (x-y)(x^2+xy+y^2)$ and $x^3+y^3 = (x+y)(x^2-xy+y^2)$. Multiply all four factors.

20. Start with the key idea: $(a+b)(a-b) = a^2-b^2$. Add b^2: $a^2-b^2+b^2 = a^2$. This is the part to check before moving on, because it keeps the answer tied to the original question.

21. A careful way to see it: $98^2 = (100-2)^2 = 100^2-2\cdot100\cdot2+2^2 = 10000-400+4 = 9604$. Square of a difference — the identity turns one hard multiplication into three easy ones. That gives a quick check on the answer.

22. Keep the rule visible: $m^2-n^2 = 16-1 = 15$. $2mn = 8$. $m^2+n^2 = 16+1 = 17$. Verify: $15^2+8^2 = 225+64 = 289 = 17^2$ ✓. So $(15,8,17)$ is a Pythagorean triple. That gives a quick check on the answer.

23. One steady path is: $(x+y)^2 = x^2+2xy+y^2$. $(x-y)^2 = x^2-2xy+y^2$. Subtract: $(x^2+2xy+y^2)-(x^2-2xy+y^2)$. The x^2 and y^2 pieces cancel. Left over: $2xy-(-2xy) = 4xy$. This identity is sometimes called *four-times-the-product*: useful for finding xy when you know both squares. That gives a quick check on the answer.

24. Treat as difference of squares: $x^6-1 = (x^3)^2-1^2 = (x^3-1)(x^3+1)$. Now factor each cube. $x^3-1 = (x-1)(x^2+x+1)$ (difference of cubes). $x^3+1 = (x+1)(x^2-x+1)$ (sum of cubes). Both quadratic factors have negative discriminants, so they're irreducible over the reals. Final factorization: $(x-1)(x+1)(x^2+x+1)(x^2-x+1)$.

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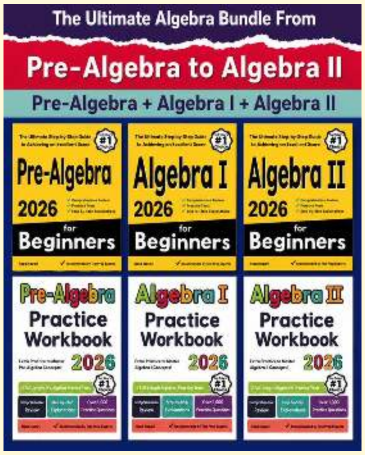

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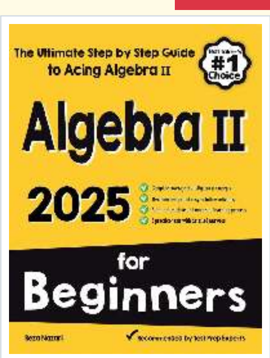

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