Polynomial Division (Synthetic Division)

Name: ____________	Date: ____________	Score: ______ / 34

Quick Review

Synthetic division is a faster, cleaner version of long division — but it only works when the divisor is **linear and of the form** $(x - c)$. For divisors like $(2x + 1)$ or $(x^2 - 1)$, you have to use ordinary long division.

The algorithm: write the value c (note the sign flip for $(x + a)$, where $c = -a$). Then list all the coefficients of the dividend in descending order, with 0 placeholders for any missing degrees.

1. Bring down the leading coefficient unchanged.
2. Multiply by c and write under the next coefficient.
3. Add. Write the sum below.
4. Repeat: multiply by c, write under the next coefficient, add.

The bottom row gives the quotient's coefficients (starting one degree lower than the dividend) followed by the remainder as the very last entry.

Quick check: divide $x^3 - 4x^2 + 5x - 2$ by $(x - 1)$. With $c = 1$ and coefficients $1, -4, 5, -2$, the synthetic-division box looks like:

$$\begin{array}{c|cccc} 1 & 1 & -4 & 5 & -2 \\ & & 1 & -3 & 2 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

Reading the bottom row, the quotient is $x^2 - 3x + 2$ and the remainder is 0.

Connection to the Remainder Theorem: the final bottom-row entry equals $f(c)$. If it's 0, $(x - c)$ is a factor.

Common traps: forgetting the sign flip for $(x + a)$; omitting zero placeholders for missing degrees (the columns slip out of alignment); confusing the quotient's degree (it's always one less than the dividend's).

PRACTICE

Use synthetic division (when applicable) to find the quotient and remainder.

1. Which divisor allows synthetic division: $(2x + 1),\ (x + 5),\ (x^2 - 1)$? ________

2. Find c for synthetic division by (x+5) ________

3. Quotient when $(x^2 + 5x + 6) is$ divided by (x+2) ________

4. Quotient and remainder when $x^3 - 4x^2 + 5x - 2$ is divided by $(x - 1)$ ________

5. The synthetic division setup below shows division of $x^3 + 2x^2 - 5x + 1$ by $(x - 2)$. Find the remainder. ________

$$\begin{array}{c|cccc} 2 & 1 & 2 & -5 & 1 \\ & & 2 & 8 & 6 \\ \hline & 1 & 4 & 3 & 7 \end{array}$$

6. Quotient when $x^4 - 16 is$ divided by (x-2) ________

7. Quotient when $2x^3 - 3x^2 + 4x - 5 is$ divided by (x+1) ________

8. Quotient when $x^4 - 3x^2 + 2x - 4 is$ divided by (x+2) ________

9. If $f(x) = x^3 + kx^2 - 2x + 5$ divided by $(x - 3)$ has remainder 8, find k ________

Scan Me

10. Use the synthetic-division box below to find the quotient of $(2x^3 + 3x^2 - 11x - 6) \div (x - 2)$: ________

2	2	3	-11	-6
		4	14	6
	2	7	3	0

11. Quotient when $x^3 - 1 is$ divided by (x-1) ________

12. Quotient when $x^3 + 1 is$ divided by (x+1) ________

13. Find f(-3) for $f(x) = x^3 + 4x^2 - 2x + 1$ via synthetic division ________

14. Quotient when $3x^3 + x^2 - 7x + 2 is$ divided by (x+2) ________

15. Should missing-degree terms be omitted in synthetic division? ________

16. Quotient when $x^2 - 25 is$ divided by (x-5) ________

17. The last bottom-row entry in synthetic division represents ________

18. Quotient when $4x^3 - 12x^2 + 9x - 1 is$ divided by (x-1) ________

19. Quotient when $x^4 + 2x^3 - 3 is$ divided by (x+1) ________

20. If synthetic division of $p(x)$ by $(x - c)$ yields a remainder of 0, what does that mean? ________

♦ Word Problems

21. Use synthetic division to divide $p(x) = x^3 + 2x^2 - 5x - 6$ by $(x - 2)$, then write $p(x)$ in the form (divisor)(quotient) + remainder. ____

22. A polynomial $f(x) = 2x^4 - 3x^3 - 11x^2 + 5x + 6$ is tested for the factor $(x - 3)$ using synthetic division. Decide whether $(x - 3)$ is a factor and explain. ____

23. Use synthetic division to find $f(-2)$ when $f(x) = x^4 + 3x^3 - x^2 + 2x - 5$. ____

24. A polynomial $p(x) = x^3 + 5x^2 + 3x - 9$ has remainder 0 when divided by $(x + 3)$. Use synthetic division to find the quotient, then factor $p(x)$ completely. ____

Additional Practice

25. Write $3x - 5 + x^3$ in standard form. ________

26. Find the degree of $7x^4 - 2x^2 + 9$. ________

27. Add $(2x^2 + 3x - 1) + (x^2 - 5x + 4)$. ________

28. Subtract $(5x^2 - x + 6) - (2x^2 + 3x - 1)$. ________

29. Multiply $(x + 4)(x - 3)$. ________

30. Factor $x^2 + 9x + 20$. ________

31. Factor $6x^2 + 9x$. ________

32. Find the GCF of $12x^3$ and $18x^2$. ________

33. Divide $(x^2 + 5x + 6)$ by $(x + 2)$. ________

34. Find the remainder when $x^2 - 1$ is divided by $x - 3$. ________

Scan Me

Answer Keys

1. $(x+5)$
2. $c=-5$
3. $x+3$
4. quotient x^2-3x+2, remainder 0
5. 7
6. x^3+2x^2+4x+8
7. $2x^2-5x+9,\ remainder-14$
8. $x^3-2x^2+x,\ remainder-4$
9. $k=-2$
10. $2x^2+7x+3$
11. x^2+x+1
12. x^2-x+1
13. $f(-3)=16$
14. $3x^2-5x+3,\ remainder-4$
15. $no---use0placeholders$
16. $x+5$
17. $theremainder$
18. $4x^2-8x+1$
19. $x^3+x^2-x+1,\ remainder-4$
20. $(x-c)isafactorofp(x)$
21. $p(x)=(x-2)(x^2+4x+3)+0$
22. no; the synthetic remainder is $3\neq 0$
23. $f(-2)=-21$
24. $p(x)=(x+3)(x-1)(x+3)=(x+3)^2(x-1)$

Additional Practice Answers

25. x^3+3x-5
26. 4
27. $3x^2-2x+3$
28. $3x^2-4x+7$
29. x^2+x-12
30. $(x+4)(x+5)$
31. $3x(2x+3)$
32. $6x^2$
33. $x+3$
34. 8

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Only divisors of the form $(x-c)$ qualify. $(x+5)=(x-(-5))$ is linear with leading coefficient 1. The others fail the form requirement.

2. Keep the rule visible: Write $(x+5)=(x-(-5))$, so $c=-5$. Sign flip is the most common slip. That gives a quick check on the answer.

3. Use $c=-2$ with coefficients $1,5,6$. Bring down 1. Multiply $1\cdot(-2)=-2$, add to 5: 3. Multiply $3\cdot(-2)=-6$, add to 6: 0. Bottom row $1,3,0\Rightarrow$ quotient $x+3$, remainder 0.

4. Start with the key idea: $c=1$ with $1,-4,5,-2$. Bring down 1; $1\cdot1=1$, add to -4: -3; $-3\cdot1=-3$, add to 5: 2; $2\cdot1=2$, add to -2: 0. Bottom row $1,-3,2,0$. That gives a quick check on the answer.

5. A careful way to see it: $c=2$ with $1,2,-5,1$. Bring down 1; $1\cdot2=2$, sum $2+2=4$; $4\cdot2=8$, sum $-5+8=3$; $3\cdot2=6$, sum $1+6=7$. Bottom row $1,4,3,7\Rightarrow$ remainder 7. This matches $f(2)=8+8-10+1=7$ by the Remainder Theorem. That gives a quick check on the answer.

6. Keep the rule visible: $c=2$ with placeholders $1,0,0,0,-16$. Bring down 1; $1\cdot2=2$, sum $0+2=2$; $2\cdot2=4$, sum $0+4=4$; $4\cdot2=8$, sum $0+8=8$; $8\cdot2=16$, sum $-16+16=0$. Bottom row $1,2,4,8,0$. That gives a quick check on the answer.

7. One steady path is: $c=-1$ with $2,-3,4,-5$. Bring down 2; $2\cdot(-1)=-2$, sum $-3+(-2)=-5$; $-5\cdot(-1)=5$, sum $4+5=9$; $9\cdot(-1)=-9$, sum $-5+(-9)=-14$. Bottom row $2,-5,9,-14$. That gives a quick check on the answer.

8. Start with the key idea: $c=-2$ with $1,0,-3,2,-4$. Bring down 1; $1\cdot(-2)=-2$, sum $0+(-2)=-2$; $-2\cdot(-2)=4$, sum $-3+4=1$; $1\cdot(-2)=-2$, sum $2+(-2)=0$; $0\cdot(-2)=0$, sum $-4+0=-4$. Bottom row $1,-2,1,0,-4$. That gives a quick check on the answer.

9. A careful way to see it: $f(3)=27+9k-6+5=26+9k$. Set $26+9k=8$: $9k=-18$, so $k=-2$. That gives a quick check on the answer.

10. Keep the rule visible: $c=2$ with $2,3,-11,-6$. Bring down 2; $2\cdot2=4$, sum $3+4=7$; $7\cdot2=14$, sum $-11+14=3$; $3\cdot2=6$, sum $-6+6=0$. Bottom row $2,7,3,0\Rightarrow$ quotient $2x^2+7x+3$, remainder 0. That gives a quick check on the answer.

11. Use $c=1$ with coefficients $1,0,0,-1$ (placeholders for the missing x^2 and x). Bring down 1; $1\cdot1=1$, sum $0+1=1$; $1\cdot1=1$, sum $0+1=1$; $1\cdot1=1$, sum $-1+1=0$. Bottom row $1,1,1,0$, so quotient x^2+x+1, remainder 0.

12. Use $c=-1$ with coefficients $1,0,0,1$. Bring down 1; $1\cdot(-1)=-1$, sum $0-1=-1$; $-1\cdot(-1)=1$, sum $0+1=1$; $1\cdot(-1)=-1$, sum $1-1=0$. Bottom row $1,-1,1,0$, so quotient x^2-x+1 with remainder 0.

13. A careful way to see it: $c=-3$ with $1,4,-2,1$. Bring down 1; sum $4+(-3)=1$; sum $-2+(-3)=-5$; sum $1+15=16$. Last entry is the remainder, which by the Remainder Theorem equals $f(-3)=16$. That gives a quick check on the answer.

14. Keep the rule visible: $c=-2$ with $3,1,-7,2$. Bring down 3; $3\cdot(-2)=-6$, sum $1-6=-5$; $-5\cdot(-2)=10$, sum $-7+10=3$; $3\cdot(-2)=-6$, sum $2-6=-4$. Bottom row $3,-5,3,-4$. That gives a quick check on the answer.

15. Missing terms must be written with 0 coefficient, otherwise the columns shift out of alignment and every step is wrong.

16. Use $c=5$ with coefficients $1,0,-25$ (zero placeholder for the missing x). Bring down 1; $1\cdot5=5$, sum $0+5=5$; $5\cdot5=25$, sum $-25+25=0$. Bottom row $1,5,0$, so quotient $x+5$, remainder 0.

17. By construction, the final entry is what's left after the algorithm completes. By the Remainder Theorem, it also equals $f(c)$.

18. Use $c=1$ with coefficients $4,-12,9,-1$. Bring down 4; $4\cdot1=4$, sum $-12+4=-8$; $-8\cdot1=-8$, sum $9-8=1$; $1\cdot1=1$, sum $-1+1=0$. Bottom row $4,-8,1,0$, so quotient $4x^2-8x+1$, remainder 0.

19. One steady path is: $c=-1$ with $1,2,0,0,-3$. Bring down 1; sum $2-1=1$; sum $0-1=-1$; sum $0+1=1$; sum $-3-1=-4$. Bottom row $1,1,-1,1,-4$. That gives a quick check on the answer.

20. Start with the key idea: Zero remainder $\Leftrightarrow p(c)=0\Leftrightarrow(x-c)$ is a factor (Factor Theorem). That gives a quick check on the answer.

21. A careful way to see it: $c=2$ with $1,2,-5,-6$. Bring down 1; $1\cdot2=2$, sum $2+2=4$; $4\cdot2=8$, sum $-5+8=3$; $3\cdot2=6$, sum $-6+6=0$. Bottom row $1,4,3,0$. Quotient x^2+4x+3, remainder 0. So $p(x)=(x-2)(x^2+4x+3)$. That gives a quick check on the answer.

22. Keep the rule visible: $c=3$ with $2,-3,-11,5,6$. Bring down 2; $2\cdot3=6$, sum $-3+6=3$; $3\cdot3=9$, sum $-11+9=-2$; $-2\cdot3=-6$, sum $5-6=-1$; $-1\cdot3=-3$, sum $6-3=3$. Bottom row $2,3,-2,-1,3$. The final entry 3 is the remainder. Since the remainder isn't 0, $(x-3)$ is *not* a factor of f. Cross-check by direct evaluation: $f(3)=2(81)-3(27)-11(9)+5(3)+6=162-81-99+15+6=3$ ✓. That gives a quick check on the answer.

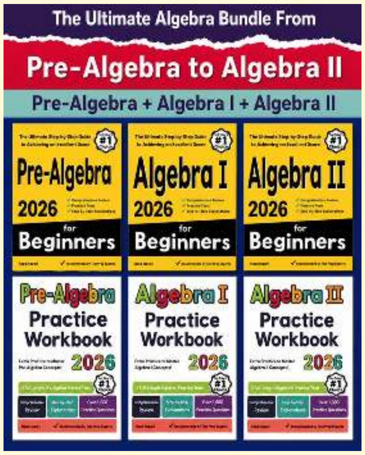
23. One steady path is: $c=-2$ with $1,3,-1,2,-5$. Bring down 1; $1\cdot(-2)=-2$, sum $3-2=1$; $1\cdot(-2)=-2$, sum $-1-2=-3$; $-3\cdot(-2)=6$, sum $2+6=8$; $8\cdot(-2)=-16$, sum $-5-16=-21$. The last entry is the remainder, which by the Remainder Theorem equals $f(-2)=-21$. Direct check: $f(-2)=16-24-4-4-5=-21$ ✓. That gives a quick check on the answer.

24. Start with the key idea: $c=-3$ with $1,5,3,-9$. Bring down 1; $1\cdot(-3)=-3$, sum $5-3=2$; $2\cdot(-3)=-6$, sum $3-6=-3$; $-3\cdot(-3)=9$, sum $-9+9=0$.

Scan Me

Quotient $x^2 + 2x - 3 = (x + 3)(x - 1)$. So $p(x) = (x + 3)(x^2 + 2x - 3) = (x + 3)(x + 3)(x - 1) = (x + 3)^2(x - 1)$. The double factor means $x = -3$ is a zero of multiplicity 2. That gives a quick check on the answer.

Build Algebra Confidence From Pre-Algebra Through Algebra II



The Complete Algebra Success Bundle

Pre-Algebra, Algebra I, and Algebra II in one clear path

Friendly lessons, focused practice, and full-review support for every stage.

Scan Me

Scan for the Bundle

6 Books
3 Courses
1 Path

Bundle Value: Six coordinated books help students review missing skills, learn new algebra topics, and practice until the steps feel natural.

Complete Course Path

- ✓ Starts with Pre-Algebra foundations
- ✓ Moves smoothly into Algebra I skills
- ✓ Extends learning through Algebra II topics
- ✓ Great for review, tutoring, and summer study

One bundle, one steady path.

Step-by-Step Lessons

- ✓ Plain-English explanations students can follow
- ✓ Worked examples that show every important step
- ✓ Common mistakes called out before they stick
- ✓ Skill-building practice after each lesson
- ✓ Helpful for independent study or class support

Less guessing. More understanding.

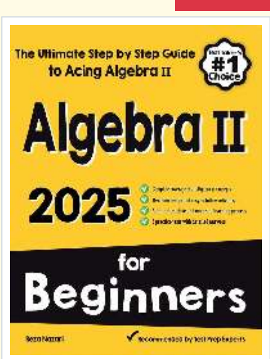
Practice That Sticks

- ✓ Matching practice workbooks for extra repetition
- ✓ Review sets to keep older skills fresh
- ✓ Answer explanations for checking thinking
- ✓ Strong support before tests and final exams
- ✓ Designed to build fluency and confidence

Practice today. Remember tomorrow.

STUDENT FAVORITE • Master Algebra II From the Ground Up

Algebra II for Beginners



✓ 100% Guaranteed
✓ Lifetime Support
✓ Trusted by Teachers

Start Your Algebra Journey Today! →

Written by a top math teacher & aligned with national and state Algebra II courses. From polynomial functions to logarithms, trigonometry, and rational expressions — explained the easy way.

- ✓ **Complete coverage** of every Algebra II concept — perfect companion to these worksheets
- ✓ **Step-by-step explanations** with worked examples on every topic
- ✓ **QR codes in every chapter** for free video lessons & bonus practice
- ✓ **2 full-length practice tests** with detailed answer keys

⋆ STUDENT'S #1 CHOICE ⋆

Teacher-recommended • 12,000+ Happy Students

↓ PDF EDITION

Scan Me

Instant download • any device

PAPERBACK

Scan Me

Paperback on Amazon

Hold it in your hands

Pair these free worksheets with *Algebra II for Beginners* and you have a complete self-paced course — concept lessons, daily practice, and full exam-style reviews, all in one path. → EffortlessMath.com/product/algebra-ii-for-beginners