Polynomial Division (Long Division)

Name: ______________________	Date: ______________	Score: ______ / 28

Quick Review

Polynomial long division mirrors the long-division algorithm you learned for numbers. The structure: *divide, multiply, subtract, bring down*, repeat until the leftover has lower degree than the divisor.

Step-by-step for $(x^3 - 2x^2 - 5x + 6) \div (x - 1)$**:**

1. Divide leading terms: $x^3 \div x = x^2$. Write x^2 in the quotient.
2. Multiply $x^2(x-1) = x^3 - x^2$. Subtract from the dividend: $(x^3 - 2x^2) - (x^3 - x^2) = -x^2$. Bring down $-5x$ to get $-x^2 - 5x$.
3. Divide: $-x^2 \div x = -x$. Multiply: $-x(x-1) = -x^2 + x$. Subtract: $(-x^2 - 5x) - (-x^2 + x) = -6x$. Bring down $+6$ to get $-6x + 6$.
4. Divide: $-6x \div x = -6$. Multiply: $-6(x-1) = -6x + 6$. Subtract: 0.

Quotient: $x^2 - x - 6$. Remainder: 0.

Division algorithm: dividend = (divisor)(quotient) + remainder, where the remainder is 0 or has degree strictly less than the divisor's. For a linear divisor, the remainder is a constant. For a quadratic divisor, the remainder has degree ≤ 1.

Use placeholder zeros for any missing degrees in the dividend, otherwise the columns slip out of alignment. Quick check: $x^3 - 8$ should be written $x^3 + 0x^2 + 0x - 8$ for the division.

Sanity check: multiply (divisor)(quotient) and add the remainder. If you get the original dividend back, the division is correct.

PRACTICE

Divide using polynomial long division. State the quotient and remainder.

1. $\dfrac{x^2 + 5x + 6}{x + 2}$ ________

2. Divide $\dfrac{x^2 + 4x + 7}{x + 1}$. The table of $f(x) = x^2 + 4x + 7$ lets you read off the remainder by the Remainder Theorem; find the quotient by division. ________

x	-3	-2	0	1
$f(x)$	4	3	7	12

3. $\dfrac{x^3 - 2x^2 - 5x + 6}{x - 1}$ ________

4. $\dfrac{2x^2 + 7x + 5}{2x + 1}$ ________

5. $\dfrac{x^3 - 8}{x - 2}$ ________

6. $\dfrac{x^4 - 3x^2 + 2x + 5}{x^2 - 1}$ ________

7. $\dfrac{2x^3 + 3x^2 - 5x - 6}{x + 2}$ ________

8. Divide $\dfrac{3x^3 - 10x + 8}{x - 2}$. Use the table of $f(x) = 3x^3 - 10x + 8$ to read the remainder; find the quotient by division. ________

x	-1	0	1	3
$f(x)$	15	8	1	59

9. $\dfrac{x^2 - 9}{x - 3}$ ________

Scan Me

10. $\dfrac{x^3 + x^2 - 4x - 4}{x + 1}$ ________

11. $\dfrac{x^3 - 1}{x - 1}$ ________

12. Divide $\dfrac{x^2 + 5x + 9}{x + 2}$. Use the table of $f(x) = x^2 + 5x + 9$ to read the remainder; find the quotient by division. ________

x	-3	-1	0	1
$f(x)$	3	5	9	15

13. $\dfrac{4x^2 - 8x + 3}{2x - 1}$ ________

14. If $x^3 + 2x^2 - 5x + 1 = (x - 2)(x^2 + 4x + 3) + 7$, what is the remainder? ________

15. $\dfrac{x^4 - 1}{x + 1}$ ________

16. $\dfrac{6x^2 + 11x - 7}{2x - 1}$ ________

17. $\dfrac{x^3 + 2x - 3}{x - 1}$ ________

18. Closure check: divide $2x^2 - 3x + 1$ by $x - 2$. State quotient and remainder. ________

19. $\dfrac{x^4 + x^3 - 7x^2 - x + 6}{x^2 - 1}$ ________

20. Multiply your way back: if quotient is $x + 5$ and divisor is $x - 3$ with remainder 2, what was the dividend? ________

♦ Word Problems

21. A rectangular box has volume $V(x) = x^3 + 5x^2 + 6x$ cubic inches and height $(x + 2)$ inches. Use polynomial division to find a polynomial for the base area. ____

22. A polynomial $D(x) = 2x^3 + x^2 - 13x + 6$ is divided by $(x - 2)$. Find the quotient and remainder, and use them to write $D(x)$ in the form (divisor)(quotient) + remainder. ____

23. Use long division to find the quotient and remainder when $f(x) = x^4 - 2x^3 + x^2 + x - 1$ is divided by $g(x) = x^2 - 1$. ____

24. A polynomial expression for a fuel-flow model is $f(x) = 3x^3 - 10x + 8$. Divide by $(x - 2)$ and write the result using the division algorithm form. ____

Additional Practice

25. Write $3x - 5 + x^3$ in standard form. ________

26. Find the degree of $7x^4 - 2x^2 + 9$. ________

27. Add $(2x^2 + 3x - 1) + (x^2 - 5x + 4)$. ________

28. Subtract $(5x^2 - x + 6) - (2x^2 + 3x - 1)$. ________

Scan Me

Answer Keys

1. quotient $x+3$, remainder 0
2. quotient $x+3$, remainder 4
3. quotient x^2-x-6, remainder 0
4. quotient $x+3$, remainder 2
5. quotient x^2+2x+4, remainder 0
6. quotient x^2-2, remainder $2x+3$
7. quotient $2x^2-x-3$, remainder 0
8. quotient $3x^2+6x+2$, remainder 12
9. quotient $x+3$, remainder 0
10. quotient x^2-4, remainder 0
11. quotient x^2+x+1, remainder 0
12. quotient $x+3$, remainder 3
13. quotient $2x-3$, remainder 0
14. 7
15. quotient x^3-x^2+x-1, remainder 0
16. quotient $3x+7$, remainder 0
17. quotient x^2+x+3, remainder 0
18. quotient $2x+1$, remainder 3
19. quotient x^2+x-6, remainder 0
20. $x^2+2x-13$
21. x^2+3x square inches
22. $D(x)=(x-2)(2x^2+5x-3)+0$
23. quotient x^2-2x+2, remainder $-x+1$
24. $f(x)=(x-2)(3x^2+6x+2)+12$

Additional Practice Answers

25. x^3+3x-5
26. 4
27. $3x^2-2x+3$
28. $3x^2-4x+7$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Divide $x^2/x=x$. Multiply $x(x+2)=x^2+2x$; subtract: $5x-2x=3x$, bring down $+6$. Divide $3x/x=3$. Multiply $3(x+2)=3x+6$; subtract: 0. Quotient $x+3$, remainder 0.

2. Dividing by $(x+1)$ means $c=-1$, so the remainder is $f(-1)$. The table omits it, so compute: $f(-1)=1-4+7=4$. The long division gives quotient $x+3$ (and indeed the remainder is 4).

3. One steady path is: See review for the worked-out steps. Final: x^2-x-6 with remainder 0. That gives a quick check on the answer.

4. Start with the key idea: $2x^2/(2x)=x$, $x(2x+1)=2x^2+x$, subtract: $6x+5$. Then $6x/(2x)=3$, $3(2x+1)=6x+3$, subtract: 2. That gives a quick check on the answer.

5. Use placeholders: x^3+0x^2+0x-8. $x^3/x=x^2$, $x^2(x-2)=x^3-2x^2$, subtract: $2x^2+0x$. $2x^2/x=2x$, $2x(x-2)=2x^2-4x$, subtract: $4x-8$. $4x/x=4$, $4(x-2)=4x-8$, subtract: 0. (This matches the sum-of-cubes factorization $x^3-8=(x-2)(x^2+2x+4)$.)

6. Keep the rule visible: $x^4/x^2=x^2$, $x^2(x^2-1)=x^4-x^2$, subtract: $-2x^2+2x+5$. $-2x^2/x^2=-2$, $-2(x^2-1)=-2x^2+2$, subtract: $2x+3$. Remainder has degree $1<$ divisor degree 2, so stop. That gives a quick check on the answer.

7. One steady path is: $2x^3/x=2x^2$, $2x^2(x+2)=2x^3+4x^2$, subtract: $-x^2-5x$. $-x^2/x=-x$, $-x(x+2)=-x^2-2x$, subtract: $-3x-6$. $-3x/x=-3$, $-3(x+2)=-3x-6$, subtract: 0. That gives a quick check on the answer.

8. Here $c=2$, so the remainder is $f(2)$, omitted from the table: $f(2)=24-20+8=12$. Long division (with placeholder $0x^2$) gives quotient $3x^2+6x+2$.

9. A careful way to see it: Difference of squares: $(x-3)(x+3)$, so dividing by $(x-3)$ leaves $x+3$. That gives a quick check on the answer.

10. Notice the grouping: $x^3+x^2=x^2(x+1)$ and $-4x-4=-4(x+1)$. So the numerator factors as $(x+1)(x^2-4)$, and dividing by $(x+1)$ leaves x^2-4 with zero remainder. (Long division gets the same answer in three steps: x^2, then 0 for the middle term, then -4.)

11. Difference of cubes pattern: $x^3-1=(x-1)(x^2+x+1)$. So the quotient is x^2+x+1.

12. Here $c=-2$, so the remainder is $f(-2)$, which the table leaves out: $f(-2)=4-10+9=3$. Long division gives quotient $x+3$, remainder 3.

13. A careful way to see it: $4x^2/(2x)=2x$, $2x(2x-1)=4x^2-2x$, subtract: $-6x+3$. $-6x/(2x)=-3$, $-3(2x-1)=-6x+3$, subtract: 0. That gives a quick check on the answer.

14. The form is dividend = (divisor)(quotient) + remainder. The constant added at the end is the remainder.

15. One steady path is: $x^4-1=(x-1)(x+1)(x^2+1)$, so dividing by $(x+1)$ gives $(x-1)(x^2+1)=x^3-x^2+x-1$. That gives a quick check on the answer.

16. Start with the key idea: $6x^2/(2x)=3x$, $3x(2x-1)=6x^2-3x$, subtract: $14x-7$. $14x/(2x)=7$, $7(2x-1)=14x-7$, subtract: 0. That gives a quick check on the answer.

17. Placeholder: x^3+0x^2+2x-3. $x^3/x=x^2$, $x^2(x-1)=x^3-x^2$, subtract: x^2+2x. $x^2/x=x$, $x(x-1)=x^2-x$, subtract: $3x-3$. $3x/x=3$, $3(x-1)=3x-3$, subtract: 0.

18. Keep the rule visible: $2x^2/x=2x$, $2x(x-2)=2x^2-4x$, subtract: $x+1$. $x/x=1$, $1(x-2)=x-2$, subtract: 3. That gives a quick check on the answer.

19. One steady path is: $x^4/x^2=x^2$; multiply, subtract: x^3-6x^2-x+6. $x^3/x^2=x$; multiply, subtract: $-6x^2+6$. $-6x^2/x^2=-6$; multiply, subtract: 0. Quotient x^2+x-6. That gives a quick check on the answer.

20. Start with the key idea: (divisor)(quotient) + remainder $=(x-3)(x+5)+2=x^2+2x-15+2=x^2+2x-13$. That gives a quick check on the answer.

21. Base area $=V/h=(x^3+5x^2+6x)/(x+2)$. Use long division. $x^3/x=x^2$, $x^2(x+2)=x^3+2x^2$, subtract: $3x^2+6x$. $3x^2/x=3x$, $3x(x+2)=3x^2+6x$, subtract: 0. Quotient: x^2+3x. (Quick factor check: $x^3+5x^2+6x=x(x+2)(x+3)$, so dividing by $(x+2)$ leaves $x(x+3)=x^2+3x$ ✓.)

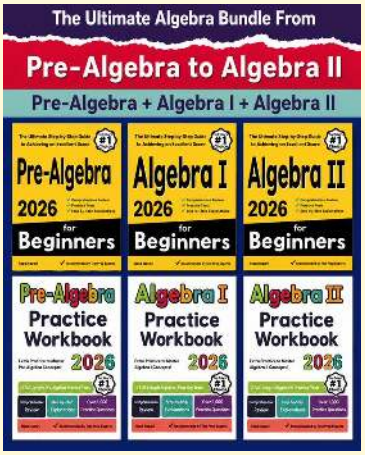
22. Long division: $2x^3/x=2x^2$, $2x^2(x-2)=2x^3-4x^2$, subtract: $5x^2-13x$. $5x^2/x=5x$, $5x(x-2)=5x^2-10x$, subtract: $-3x+6$. $-3x/x=-3$, $-3(x-2)=-3x+6$, subtract: 0. Quotient $2x^2+5x-3$, remainder 0. So $D(x)=(x-2)(2x^2+5x-3)$.

23. One steady path is: $x^4/x^2=x^2$, subtract: $-2x^3+2x^2+x-1$. $-2x^3/x^2=-2x$, $-2x(x^2-1)=-2x^3+2x$, subtract: $2x^2-x-1$. $2x^2/x^2=2$, $2(x^2-1)=2x^2-2$, subtract: $-x+1$. The remainder has degree 1, which is less than divisor degree 2, so stop. Quotient: x^2-2x+2; remainder: $-x+1$. That gives a quick check on the answer.

24. Use placeholders: $3x^3+0x^2-10x+8$. $3x^3/x=3x^2$, $3x^2(x-2)=3x^3-6x^2$, subtract: $6x^2-10x$. $6x^2/x=6x$, $6x(x-2)=6x^2-12x$, subtract: $2x+8$. $2x/x=2$, $2(x-2)=2x-4$, subtract: 12. Quotient $3x^2+6x+2$, remainder 12. So $f(x)=(x-2)(3x^2+6x+2)+12$. (Verify: $(x-2)(3x^2+6x+2)+12=3x^3+6x^2+2x-6x^2-12x-4+12=3x^3-10x+8$ ✓.)

Scan Me

Build Algebra Confidence From Pre-Algebra Through Algebra II



The Complete Algebra Success Bundle

Pre-Algebra, Algebra I, and Algebra II in one clear path

Friendly lessons, focused practice, and full-review support for every stage.

Scan Me

Scan for the Bundle

6 Books
3 Courses
1 Path

Bundle Value: Six coordinated books help students review missing skills, learn new algebra topics, and practice until the steps feel natural.

Complete Course Path

- ✓ Starts with Pre-Algebra foundations
- ✓ Moves smoothly into Algebra I skills
- ✓ Extends learning through Algebra II topics
- ✓ Great for review, tutoring, and summer study

One bundle, one steady path.

Step-by-Step Lessons

- ✓ Plain-English explanations students can follow
- ✓ Worked examples that show every important step
- ✓ Common mistakes called out before they stick
- ✓ Skill-building practice after each lesson
- ✓ Helpful for independent study or class support

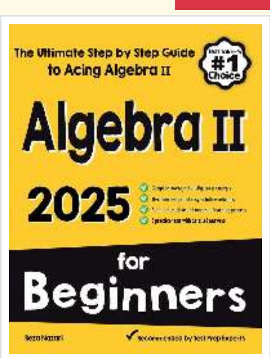
Less guessing. More understanding.

Practice That Sticks

- ✓ Matching practice workbooks for extra repetition
- ✓ Review sets to keep older skills fresh
- ✓ Answer explanations for checking thinking
- ✓ Strong support before tests and final exams
- ✓ Designed to build fluency and confidence

Practice today. Remember tomorrow.

STUDENT FAVORITE • Master Algebra II From the Ground Up



Algebra II for Beginners

Written by a top math teacher & aligned with national and state Algebra II courses. From polynomial functions to logarithms, trigonometry, and rational expressions — explained the easy way.

- ✓ **Complete coverage** of every Algebra II concept — perfect companion to these worksheets
- ✓ **Step-by-step explanations** with worked examples on every topic
- ✓ **QR codes in every chapter** for free video lessons & bonus practice
- ✓ **2 full-length practice tests** with detailed answer keys

✓ 100% Guaranteed
✓ Lifetime Support
✓ Trusted by Teachers

Start Your Algebra Journey Today! →

⋆ STUDENT'S #1 CHOICE ⋆

Teacher-recommended • 12,000+ Happy Students

↓ PDF EDITION

Scan Me

Instant download • any device

PAPERBACK

Scan Me

Paperback on Amazon

Hold it in your hands

Pair these free worksheets with *Algebra II for Beginners* and you have a complete self-paced course — concept lessons, daily practice, and full exam-style reviews, all in one path. → **EffortlessMath.com/product/algebra-ii-for-beginners**