Permutations and Combinations

Name: ____________	Date: ____________	Score: ______ / 37

Quick Review

Factorial $n! = n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1$. By convention, $0! = 1$ – a tidy choice that keeps the formulas clean.

Permutation $P(n,r) = \frac{n!}{(n-r)!}$. Use it when *order matters*: arranging books on a shelf, ordering a podium of medalists, building a password from distinct digits.

Combination $C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$. Use it when *order doesn't matter*: picking a committee, choosing a team, selecting a hand of cards. The extra $r!$ in the denominator divides out the redundant orderings of the same chosen set.

The pivot question: *Does swapping two of my selections change the answer?* If yes, it's a permutation. If no, a combination.

Symmetry: $\binom{n}{r} = \binom{n}{n-r}$. Picking which r to include is the same as picking which $n-r$ to leave out.

Arrangements with repeats. For a word like BOOKKEEPER with letters O twice, K twice, E three times, the number of distinct arrangements is $\frac{10!}{2! \cdot 2! \cdot 3!} = 151{,}200$. Divide by the factorial of each repeat group to kill duplicate orderings.

Common slips. Using n^r when no repetition is allowed (that formula allows repeats). Using $P(n,r)$ for a committee (committees don't care about order). Confusing $n!/r!$ with the permutation formula – the correct denominator is $(n-r)!$.

PRACTICE

Decide permutation vs. combination, then compute.

1. $5!$ ________
2. Using the factorial table below, evaluate $7!$. ________

n	1	2	3	4	5	6	7
$n!$	1	2	6	24	120	720	?

3. $P(6,2)$ ________
4. $\binom{8}{3}$ ________
5. $\binom{12}{5}$ ________
6. $P(10,4)$ ________
7. $\binom{10}{4}$ ________
8. 0! ________
9. Using the letter tally below, count the distinct arrangements of all letters in BANANA. ________

Letter	B	A	N
Count	1	3	2

10. Simplify $\binom{n}{n-r}$ using symmetry. ________
11. $\binom{9}{2}$ ________
12. $P(7,3)$ ________
13. How many 3-digit codes from $\{0,\ldots,9\}$ with no repeat ________

Scan Me

14. Using the letter tally below, count the distinct arrangements of MISSISSIPPI. ______

Letter	M	I	S	P
Count	1	4	4	2

15. $\binom{52}{5}$ ______

16. Lock with 4 digits, repeats allowed: total combos ______

17. $\binom{15}{4}$ ______

18. Arrangements of all letters in LEVEL ______

19. $P(9, 3)$ ______

20. $\binom{20}{2}$ ______

♦ Word Problems

21. A bookshelf has 7 different books. In how many different orders can you arrange all 7 books in a row? ____

22. A school is forming a 5-member committee from 12 candidates. The roles are interchangeable – everyone on the committee has the same job. How many different committees are possible? ____

23. A password consists of 4 distinct digits arranged in order, chosen from 0 through 9. How many such passwords are possible? ____

24. Find the number of distinct arrangements of all the letters in the word BOOKKEEPER. ____

Additional Practice

25. Probability of rolling an even number on a fair die. ______

26. Probability of drawing a heart from a standard deck. ______

27. Complement of $P(A) = 0.37$. ______

28. If events are independent, $P(A) = 0.4$, $P(B) = 0.5$, find $P(A \cap B)$. ______

29. Find $P(A \cup B)$ if $P(A) = 0.6$, $P(B) = 0.3$, $P(A \cap B) = 0.1$. ______

30. Choose 3 from 8. ______

31. Arrange 4 distinct books. ______

32. Find $7P2$. ______

33. Find $7C2$. ______

34. Probability of two heads in two coin flips. ______

35. Expected wins in 80 trials with $p = 0.25$. ______

36. Is drawing without replacement independent? ______

37. If 50 of 200 students play chess, find relative frequency. ______

Scan Me

Answer Keys

1. $\boxed{120}$
2. $\boxed{5040}$
3. $\boxed{30}$
4. $\boxed{56}$
5. $\boxed{792}$
6. $\boxed{5040}$
7. $\boxed{210}$
8. $\boxed{1}$
9. $\boxed{60}$
10. $\boxed{\binom{n}{r}}$
11. $\boxed{36}$
12. $\boxed{210}$
13. $\boxed{720}$
14. $\boxed{34{,}650}$
15. $\boxed{2{,}598{,}960}$
16. $\boxed{10{,}000}$
17. $\boxed{1365}$
18. $\boxed{30}$
19. $\boxed{504}$
20. $\boxed{190}$
21. $\boxed{5040}$
22. $\boxed{792}$
23. $\boxed{5040}$
24. $\boxed{151{,}200}$

Additional Practice Answers

25. $\boxed{\frac{1}{2}}$
26. $\boxed{\frac{1}{4}}$
27. $\boxed{0.63}$
28. $\boxed{0.20}$
29. $\boxed{0.8}$
30. $\boxed{56}$
31. $\boxed{24}$
32. $\boxed{42}$
33. $\boxed{21}$
34. $\boxed{\frac{1}{4}}$
35. $\boxed{20}$
36. $\boxed{\text{no}}$
37. $\boxed{0.25}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A factorial multiplies every whole number down to 1: $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$. It counts the orderings of 5 distinct items.

2. Each entry is n times the previous: $7! = 7 \cdot 6! = 7 \cdot 720 = 5040$. A useful number to recognize.

3. Permutation: $P(n, r) = \dfrac{n!}{(n-r)!}$, so $P(6, 2) = \dfrac{6!}{4!} = 6 \cdot 5 = 30$. Order matters, so the first slot has 6 choices and the second has 5.

4. Combination (order doesn't matter): $\binom{8}{3} = \dfrac{8!}{3! \cdot 5!} = \dfrac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = \dfrac{336}{6} = 56$. The $3!$ in the denominator removes the duplicate orderings a permutation would count.

5. A careful way to see it: $\dfrac{12!}{5! \cdot 7!} = \dfrac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{120} = 792$. Useful for committees of 5 from 12 people. That gives a quick check on the answer.

6. Permutation $P(10, 4) = \dfrac{10!}{6!} = 10 \cdot 9 \cdot 8 \cdot 7 = 5040$. Each slot uses one fewer digit since repeats aren't allowed and order matters – exactly 4-digit codes with distinct digits.

7. One steady path is: $\binom{10}{4} = \dfrac{P(10,4)}{4!} = \dfrac{5040}{24} = 210$. Dividing the permutation by $4!$ collapses the 24 orderings of each chosen group into one selection. That gives a quick check on the answer.

8. Start with the key idea: By definition. Keeps formulas like $\binom{n}{0} = \dfrac{n!}{0! \cdot n!} = 1$ working out cleanly. That gives a quick check on the answer.

9. A careful way to see it: $\dfrac{6!}{3! \cdot 2!} = \dfrac{720}{12} = 60$. Three As and two Ns repeat, so divide by $3!$ and $2!$. That gives a quick check on the answer.

10. Symmetry: choosing r to include is the same as choosing $n - r$ to exclude. Both counts match.

11. One steady path is: $\binom{9}{2} = \dfrac{9 \cdot 8}{2 \cdot 1} = \dfrac{72}{2} = 36$. A handshake involves an unordered pair, so combinations fit – 9 people give 36 handshakes. That gives a quick check on the answer.

12. Start with the key idea: $P(7, 3) = \dfrac{7!}{4!} = 7 \cdot 6 \cdot 5 = 210$. Three positions filled in order, each from the dwindling pool $7, 6, 5$. That gives a quick check on the answer.

13. A careful way to see it: $10 \cdot 9 \cdot 8 = P(10, 3) = 720$. Order matters; repeats banned. This is the part to check before moving on, because it keeps the answer tied to the original question.

14. Keep the rule visible: $\dfrac{11!}{4! \cdot 4! \cdot 2!} = \dfrac{39916800}{1152} = 34{,}650$. Four Ss, four Is, two Ps. That gives a quick check on the answer.

15. A hand ignores order, so use a combination: $\binom{52}{5} = \dfrac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!} = 2{,}598{,}960$. That's every possible 5-card poker hand.

16. Start with the key idea: $10^4 = 10{,}000$. Each slot independent with 10 choices, hence the exponent. That gives a quick check on the answer.

17. A careful way to see it: $\dfrac{15 \cdot 14 \cdot 13 \cdot 12}{24} = \dfrac{32760}{24} = 1365$. This is the part to check before moving on, because it keeps the answer tied to the original question.

18. LEVEL has 5 letters with two Ls and two Es repeating. Divide $5!$ by the factorial of each repeat group: $\dfrac{5!}{2! \cdot 2!} = \dfrac{120}{4} = 30$ distinct arrangements.

19. One steady path is: $P(9, 3) = \dfrac{9!}{6!} = 9 \cdot 8 \cdot 7 = 504$. Order matters, so multiply the three descending choices. That gives a quick check on the answer.

20. Start with the key idea: $\binom{20}{2} = \dfrac{20 \cdot 19}{2 \cdot 1} = \dfrac{380}{2} = 190$. Unordered pairs, so it's a combination – 190 handshakes among 20 people. That gives a quick check on the answer.

21. Order matters when arranging a row, so this is a permutation. With all 7 books in 7 positions: $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$ distinct arrangements. (A useful sanity check: with 3 books you'd have 6 orders, with 4 you'd have 24. The factorial grows fast.)

22. Committee membership doesn't depend on order: {Alice, Bob, Carol, Dan, Eve} is the same committee no matter how you list the names. So this is a combination. $\binom{12}{5} = \dfrac{12!}{5! \cdot 7!} = 792$ different committees. (If the roles were

Scan Me

distinct – president, vice-president, etc. – you'd use $P(12,5) = 95{,}040$ instead.)
23. Order matters (a different order is a different password), and digits must be distinct. So this is a permutation of 4 items from 10: $P(10,4) = 10\cdot 9\cdot 8\cdot 7 = 5040$ passwords. (If digits could repeat, you'd have $10^4 = 10{,}000$ – repetition allows more options. If order didn't matter, $\binom{10}{4} = 210$. Three different counts, three different setups.)

24. BOOKKEEPER has 10 letters with these repeats: B(1), O(2), K(2), E(3), P(1), R(1). For arrangements with repeats, divide $n!$ by the factorial of each repeat group's size: $\frac{10!}{2!\cdot 2!\cdot 3!} = \frac{3628800}{24} = 151{,}200$. (Without correcting for repeats you'd get $10! = 3{,}628{,}800$ – but those count BOOKKEEPER and BOOKKEEPER with the two Os swapped as different. Dividing kills that overcount.)