Pascal's Triangle

Name: ________	Date: ________	Score: ____ / 33

Quick Review

Pascal's triangle is a triangular arrangement of numbers that starts with a single 1 on top (row 0), then builds each new row by starting and ending with 1 and filling the interior with the sum of the two entries above. The first few rows: row 0 is 1; row 1 is $1, 1$; row 2 is $1, 2, 1$; row 3 is $1, 3, 3, 1$; row 4 is $1, 4, 6, 4, 1$; row 5 is $1, 5, 10, 10, 5, 1$.

Binomial coefficient connection. The entry in row n, position k (zero-indexed) equals $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. So row 5 is $\binom{5}{0}, \binom{5}{1}, \binom{5}{2}, \binom{5}{3}, \binom{5}{4}, \binom{5}{5} = 1, 5, 10, 10, 5, 1$. The **addition rule** – each entry is the sum of the two above – is the identity $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.

Three patterns to remember. (1) Row n has $n+1$ entries. (2) Rows are symmetric: $\binom{n}{k} = \binom{n}{n-k}$. (3) The sum of row n is 2^n, because plugging $a = b = 1$ into $(a+b)^n = \sum \binom{n}{k}$ gives 2^n.

Connection to expansions. Row n supplies the coefficients in the expansion of $(a+b)^n$. For $(x+1)^4$, row 4 $(1, 4, 6, 4, 1)$ gives $x^4 + 4x^3 + 6x^2 + 4x + 1$. When one of the binomial pieces is more complex, like $(2x+3)^4$, you still use row 4 for the binomial coefficients, but each term also picks up the appropriate powers of $2x$ and 3.

Common slips. Off-by-one on the row index – "row 4" means $n = 4$, which has 5 entries, not 4. Reading the wrong position (zero-indexed: the first entry is position 0). Forgetting to apply powers of the non-x piece when the binomial isn't just $(x+1)^n$.

PRACTICE

Read entries off Pascal's triangle and use them to find binomial coefficients and expansion terms.

1. Row 0 of Pascal's triangle. ________
2. Use the rows shown to build row 4 of Pascal's triangle. ________

Row	Entries
0	1
1	$1,\ 1$
2	$1,\ 2,\ 1$
3	$1,\ 3,\ 3,\ 1$

3. Find $\binom{5}{2}$. ________
4. The table lists the sum of the entries in each row. Find the sum of the entries in row 7. ________

n	0	1	2	3	4	5	6
row sum	1	2	4	8	16	32	64

5. Coefficient of x^2 in $(x+1)^6$. ________
6. Coefficient of x^4 in $(x+1)^7$. ________
7. Coefficient of x^3 in $(x+2)^5$. ________
8. Coefficient of x^3 in $(2x-1)^5$. ________
9. $\binom{8}{0}$. ________
10. $\binom{8}{8}$. ________
11. In row 6, what's directly below the adjacent pair 10 and 5 in row 5? ________
12. True or False: every row of Pascal's triangle is symmetric. ________
13. $\binom{6}{2}$. ________

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14. The table shows how many entries each row contains. How many entries are in row 10? ______

n	1	2	3	4	5
# entries	2	3	4	5	6

15. Sum of entries in row 5. ______

16. Coefficient of x^4 in $(x+3)^6$. ______

17. Coefficient of y^3 in $(2+y)^4$. ______

18. True or False: the sum of row n of Pascal's triangle is $n!$. ______

19. $\binom{7}{2}+\binom{7}{3}$. ______

20. Coefficient of x^3y^2 in $(x+y)^5$. ______

♦ Word Problems

21. A coin is flipped 6 times. The number of ways to get exactly 3 heads equals an entry in Pascal's triangle. Which entry, and what is its value? ______

22. A pizza shop offers 5 toppings. How many different pizzas can be made if a pizza can have any subset of these toppings (including no toppings or all five)? ______

23. A bag has 4 marbles. How many ways can you pick exactly 2 of them? ______

24. In a triangular arrangement of bowling pins (using row 0 at the front pin and counting backward), the first six rows would have how many total pins? ______

Additional Practice

25. Find the next term: $4, 9, 14, 19, \ldots$. ______

26. Find a_{10} if $a_1 = 3$ and $d = 5$. ______

27. Find the next term: $2, 6, 18, 54, \ldots$. ______

28. Find a_6 if $a_1 = 5$ and $r = 2$. ______

29. Sum $1+2+3+\cdots+20$. ______

30. Find S_5 for $3, 6, 12, 24, 48$. ______

31. Common difference of $12, 7, 2, -3, \ldots$. ______

32. Common ratio of $81, 27, 9, 3, \ldots$. ______

33. Evaluate $\sum_{k=1}^{4} 2k$. ______

Answer Keys

1. $\boxed{1}$
2. $\boxed{1, 4, 6, 4, 1}$
3. $\boxed{10}$
4. $\boxed{128}$
5. $\boxed{15}$
6. $\boxed{35}$
7. $\boxed{40}$
8. $\boxed{80}$
9. $\boxed{1}$
10. $\boxed{1}$
11. $\boxed{15}$
12. True
13. $\boxed{15}$
14. $\boxed{11}$
15. $\boxed{32}$
16. $\boxed{135}$
17. $\boxed{8}$
18. False
19. $\boxed{56}$
20. $\boxed{10}$
21. $\boxed{\binom{6}{3} = 20}$
22. 32 pizzas
23. 6 ways
24. 21 pins

Additional Practice Answers

25. $\boxed{24}$
26. $\boxed{48}$
27. $\boxed{162}$
28. $\boxed{160}$
29. $\boxed{210}$
30. $\boxed{93}$
31. $\boxed{-5}$
32. $\boxed{\frac{1}{3}}$
33. $\boxed{20}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Pascal's triangle starts with a single 1 at the top, in row 0. (Yes, it's a row of length one. Every other row is built up from this seed.)

2. Build up from row $3 = 1, 3, 3, 1$ (the last row in the table). Add adjacent pairs and flank with 1s: $1, 1 + 3 = 4, 3 + 3 = 6, 3 + 1 = 4, 1$. So $1, 4, 6, 4, 1$.

3. One steady path is: $\binom{5}{2} = \frac{5!}{2!\,3!} = \frac{5 \cdot 4}{2} = 10$. (Or read row 5 $(1, 5, 10, 10, 5, 1)$, position 2 zero-indexed: 10.) That gives a quick check on the answer.

4. Reading down the table, each row sum doubles: it's 2^n. So row 7 gives $2^7 = 128$. (Quick verify: row 7 is $1, 7, 21, 35, 35, 21, 7, 1$, which adds to 128 ✓.)

5. Row 6 is $1, 6, 15, 20, 15, 6, 1$. For $(x+1)^6$, the x^2 term needs four 1's and two x's: $\binom{6}{2}x^2(1)^4 = 15x^2$. So coefficient 15.

6. Row 7: $1, 7, 21, 35, 35, 21, 7, 1$. x^4 in $(x + 1)^7$ has coefficient $\binom{7}{4} = 35$. (By symmetry, $\binom{7}{4} = \binom{7}{3} = 35$ – pick whichever index is smaller.)

7. Row 5: $1, 5, 10, 10, 5, 1$. x^3 in $(x + 2)^5$: $\binom{5}{3}x^3(2)^2 = 10 \cdot 4 \cdot x^3 = 40x^3$. So 40. (Don't forget the $(2)^2 = 4$ – the non-x piece contributes.)

8. Row 5. x^3: $\binom{5}{2}(2x)^3(-1)^2 = 10 \cdot 8x^3 \cdot 1 = 80x^3$. ($-1$ to an even power gives $+1$ – coefficient stays positive.)

9. A careful way to see it: Every row of Pascal's triangle starts and ends with 1. $\binom{n}{0} = 1$ always. That gives a quick check on the answer.

10. Keep the rule visible: $\binom{n}{n} = 1$ (the last entry of row n). Or: $\binom{8}{8} = \binom{8}{0} = 1$ by symmetry. That gives a quick check on the answer.

11. Pascal's addition rule: the entry under 10 and 5 is $10 + 5 = 15$. (Row 6 is $1, 6, 15, 20, 15, 6, 1$.)

12. For every n, $\binom{n}{k} = \binom{n}{n-k}$, so reading left-to-right gives the same numbers as right-to-left.

13. A careful way to see it: $\binom{6}{2} = \frac{6 \cdot 5}{2} = 15$. (Or row 6, position 2: 15.) This is the part to check before moving on, because it keeps the answer tied to the original question.

14. Keep the rule visible: From the table, the count is always $n + 1$. So row 10 has $10 + 1 = 11$ entries. That gives a quick check on the answer.

15. One steady path is: Row n sums to 2^n. $2^5 = 32$. (Or add: $1 + 5 + 10 + 10 + 5 + 1 = 32$ ✓.) That gives a quick check on the answer.

16. Start with the key idea: Row 6. x^4: $\binom{6}{4}x^4(3)^2 = \binom{6}{4} \cdot 9 \cdot x^4 = 15 \cdot 9 \cdot x^4 = 135x^4$. So 135. That gives a quick check on the answer.

17. A careful way to see it: Row 4. y^3: $\binom{4}{3}(2)^1(y)^3 = 4 \cdot 2 \cdot y^3 = 8y^3$. So 8. That gives a quick check on the answer.

18. Keep the rule visible: The sum is 2^n, not $n!$. ($2^4 = 16$ but $4! = 24$ – they're different.) That gives a quick check on the answer.

19. Use Pascal's identity: $\binom{7}{2} + \binom{7}{3} = \binom{8}{3} = 56$. (Check: $\binom{7}{2} = 21$, $\binom{7}{3} = 35$, $21 + 35 = 56$ ✓.)

20. Start with the key idea: $(x + y)^5$: the x^3y^2 term has coefficient $\binom{5}{2} = 10$. (No extra constants here – both binomial pieces are just variables.) That gives a quick check on the answer.

21. Choosing which 3 of the 6 flips show heads is exactly $\binom{6}{3}$. From row 6 of Pascal's triangle $(1, 6, 15, 20, 15, 6, 1)$, the position-3 entry is 20. So there are 20 ways. (Sanity check: total outcomes are $2^6 = 64$, and $\binom{6}{0} + \cdots + \binom{6}{6} = 64$ ✓.)

22. For each topping, you either include it or don't – two choices per topping, five toppings, so $2^5 = 32$ pizzas. This is also the sum of row 5 of Pascal's triangle: $\binom{5}{0} + \binom{5}{1} + \cdots + \binom{5}{5} = 1 + 5 + 10 + 10 + 5 + 1 = 32$. Each $\binom{5}{k}$ counts pizzas with exactly k toppings.

23. That's $\binom{4}{2}$. From row 4 of Pascal's triangle $(1, 4, 6, 4, 1)$, position 2 is 6. So 6 ways. (List check: with marbles labeled A, B, C, D, the pairs are AB, AC, AD, BC, BD, CD – six pairs ✓.)

24. The pin count in row n (with row 0 being the single front pin) is $n + 1$. Total for rows 0 through 5 is $1 + 2 + 3 + 4 + 5 + 6 = 21$ pins. (This is the 6th triangular number. Pascal's triangle has these as the diagonal: $\binom{2}{2}, \binom{3}{2}, \binom{4}{2}, \ldots = 1, 3, 6, 10, 15, 21$ – and $\binom{7}{3} = 21$ matches the running total at row 5 via the hockey-stick identity.)

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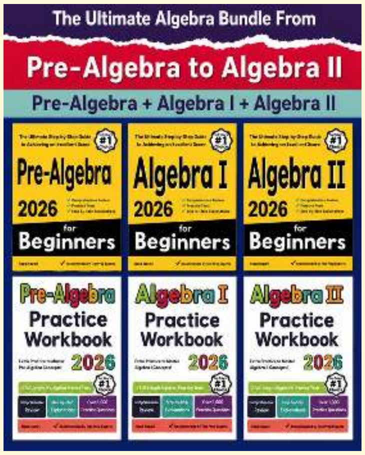

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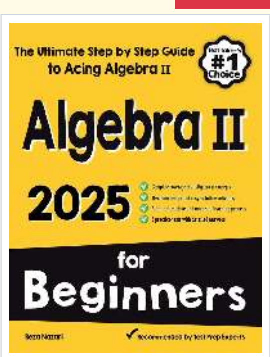

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