

Natural Logarithms

Name: _____ Date: _____ Score: _____ / 34

Q Quick Review

The **natural logarithm**, written $\ln(x)$, is just $\log_e(x)$ — a logarithm with base $e \approx 2.71828$. Everything you know about logarithms still applies; the base just happens to be Euler’s number, which shows up in continuous-growth models because of its calculus-friendly properties.

The three key values. $\ln(1) = 0$ (since $e^0 = 1$), $\ln(e) = 1$ (since $e^1 = e$), and $\ln(e^n) = n$ for every real n . The natural log peels exponents off powers of e the same way base-2 log peels them off powers of 2.

Inverse with e^x . The natural log undoes the natural exponential: $e^{\ln x} = x$ for $x > 0$, and $\ln(e^x) = x$ for every real x . This is the magic step that solves continuous-growth equations: when the unknown is in the exponent, take \ln of both sides to bring it down.

Why e ? Continuous compounding gives $A = Pe^{rt}$, where the rate r is the *instantaneous* growth fraction. Natural logs are the inverse of that machine. Anytime you see e^{kt} in a model, \ln is the tool that solves for t or k .

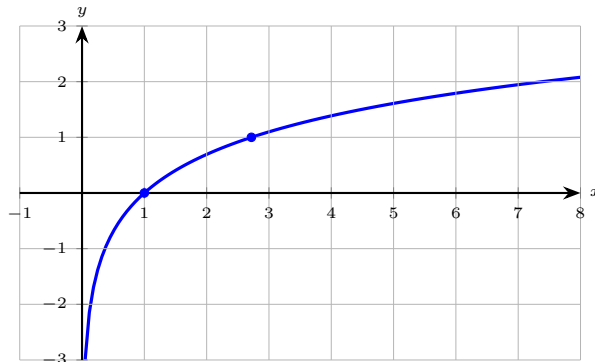
Domain reminder. $\ln(x)$ requires $x > 0$. So $\ln(x - 3)$ requires $x > 3$; $\ln(x^2 + 1)$ is fine for every real x because $x^2 + 1 \geq 1 > 0$.

Common slips. Writing $\ln(M + N)$ as $\ln M + \ln N$ (wrong — same trap as the common log). Using \log (base 10) instead of \ln on the calculator when the problem involves e . Forgetting to back-substitute and check that the log argument stayed positive.

PRACTICE

Evaluate natural logs; isolate exponentials with \ln ; check the domain after solving.

1. Definition: $\ln(x) =$ _____
2. Evaluate $\ln(1)$. The graph of $y = \ln(x)$ is shown to help you find where the curve crosses the x -axis. _____



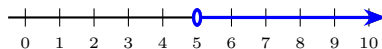
3. A short table of values for $g(x) = \ln(x)$ is given below. Use it to estimate $\ln(3)$ to two decimals. _____

x	1	2	3	4	5
$\ln(x)$	0	0.69	1.10	1.39	1.61

4. Evaluate $\ln(e^5)$. _____
5. Simplify $e^{\ln(7)}$. _____
6. Solve $\ln(x) = 4$ for x . _____
7. Simplify $\ln(e^3) - \ln(e^2) + 2\ln(e)$. _____
8. Solve $3e^x - 5 = 16$ to four decimals. _____
9. Mark TRUE or FALSE: $\ln(0) = 0$. _____
10. If $\ln(2x - 1) = \ln(11)$, find x . _____
11. Solve $A(t) = 12e^{0.18t} = 30$ for t . Round to two decimals. _____



- 12. Solve $\ln(x) + \ln(5) = \ln(15)$. _____
- 13. Solve $\ln(x - 3) = 2$. _____
- 14. Solve $2\ln(x) = \ln(36)$ with $x > 0$. _____
- 15. Solve $e^{2x} = 15$ for x to two decimals. _____
- 16. Solve $\ln(x) - \ln(2) = \ln(7)$. _____
- 17. Find the domain of $f(x) = \ln(x - 5)$. The number line below marks the boundary value of the argument; use it to write the domain as an inequality. _____



- 18. Find the domain of $f(x) = \ln(x^2 + 1)$. _____
- 19. Solve $e^{x+1} = e^{2x-3}$ for x . _____
- 20. Solve $5e^{0.04t} = 20$ for t to two decimals. _____

◆ Word Problems

- 21. A radioactive sample decays as $A(t) = 200e^{-0.04t}$ grams, where t is in years. How long until 50 grams remain? Round to two decimals. _____
- 22. The number of bacteria in a culture is $N(t) = 500e^{0.12t}$, with t in hours. Find when the population reaches 2000. Round to two decimals. _____
- 23. Solve $\ln(x + 3) - \ln(x - 1) = \ln 2$ for x . Check the domain of every log in the original equation. _____
- 24. A car's value depreciates continuously, modeled by $V(t) = 25000e^{-0.15t}$ dollars after t years. After how many years does the car's value drop to \$10,000? Round to two decimals. _____

Additional Practice

- 25. Evaluate $\log_2 32$. _____
- 26. Evaluate $\log_5 125$. _____
- 27. Rewrite $\log_3 81 = 4$ exponentially. _____
- 28. Solve $\log_4 x = 3$. _____
- 29. Domain of $y = \log(x - 7)$. _____
- 30. Expand $\log(ab)$. _____
- 31. Expand $\log(x^3)$. _____
- 32. Condense $\log x + \log 5$. _____
- 33. Condense $2\log x$. _____
- 34. Solve $\ln x = 0$. _____



Answer Keys

<p>1. $\log_e(x)$</p> <p>2. 0</p> <p>3. ≈ 1.10</p> <p>4. 5</p> <p>5. 7</p> <p>6. $x = e^4$</p> <p>7. 3</p> <p>8. $\ln(7) \approx 1.9459$</p> <p>9. FALSE</p> <p>10. $x = 6$</p> <p>11. $t \approx 5.09$</p> <p>12. $x = 3$</p>	<p>13. $x = e^2 + 3 \approx 10.39$</p> <p>14. $x = 6$</p> <p>15. $x \approx 1.35$</p> <p>16. $x = 14$</p> <p>17. $x > 5$</p> <p>18. all real numbers</p> <p>19. $x = 4$</p> <p>20. $t \approx 34.66$</p> <p>21. $t \approx 34.66$ years</p> <p>22. $t \approx 11.55$ hours</p> <p>23. $x = 5$</p> <p>24. $t \approx 6.11$ years</p>
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Additional Practice Answers

<p>25. 5</p> <p>26. 3</p> <p>27. $3^4 = 81$</p> <p>28. $x = 64$</p> <p>29. $x > 7$</p>	<p>30. $\log a + \log b$</p> <p>31. $3 \log x$</p> <p>32. $\log(5x)$</p> <p>33. $\log(x^2)$</p> <p>34. $x = 1$</p>
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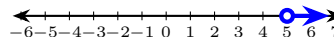
Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. The natural log is just a logarithm whose base is Euler's number $e \approx 2.71828$, so $\ln(x) = \log_e(x)$. Every log rule you know still applies; only the base is fixed.
2. Keep the rule visible: $e^0 = 1$, so $\ln(1) = 0$. (Same identity as every other log: $\log_b(1) = 0$.) The curve crosses the x -axis right at $x = 1$, which is exactly what the marked point $(1, 0)$ shows. That gives a quick check on the answer.
3. Read straight down the $x = 3$ column: $\ln(3) \approx 1.10$. (Check the shape — the values climb but more and more slowly, the signature of a logarithm.)
4. Ask: $e^? = e^5$. The exponents must match, so the answer is 5. Since $\ln = \log_e$, the natural log peels the exponent straight off a power of e : $\ln(e^n) = n$.
5. Here e^x and \ln undo each other in the other order: $\ln(7)$ is the exponent you put on e to get 7, so raising e to it lands right back on 7. The identity is $e^{\ln x} = x$ for $x > 0$.
6. Keep the rule visible: Flip to exponential form: $x = e^4 \approx 54.6$. (Domain check: $e^4 > 0$, so valid.) That gives a quick check on the answer.
7. Use $\ln(e^k) = k$ on each term: $\ln(e^3) = 3$, $\ln(e^2) = 2$, and $\ln(e) = 1$. Substitute and combine: $3 - 2 + 2(1) = 3$.
8. Isolate the exponential first: add 5 to get $3e^x = 21$, then divide by 3 to get $e^x = 7$. Take \ln of both sides so the inverse brings the exponent down: $x = \ln(7) \approx 1.9459$.
9. A careful way to see it: $\ln(0)$ is undefined — no real power of e equals 0. The natural log shoots down to $-\infty$ as the argument approaches 0 from above. That gives a quick check on the answer.
10. Same base, same argument: $2x - 1 = 11$, so $x = 6$. Domain check: $2(6) - 1 = 11 > 0$, valid.
11. Set $12e^{0.18t} = 30$ and divide by 12 to isolate the exponential: $e^{0.18t} = 2.5$. Take \ln so the inverse frees the exponent: $0.18t = \ln(2.5)$. Divide by 0.18: $t = \frac{\ln 2.5}{0.18} \approx 5.09$.
12. Product rule (in reverse): $\ln(5x) = \ln(15)$, so $5x = 15$ and $x = 3$. Domain check: $x = 3 > 0$, valid.
13. Exponential form: $x - 3 = e^2$, so $x = e^2 + 3 \approx 7.389 + 3 = 10.39$. Domain: $x - 3 = e^2 > 0$, valid.
14. Power rule first: $\ln(x^2) = \ln(36)$, so $x^2 = 36$. Since the original $\ln(x)$ requires $x > 0$, only $x = 6$ survives; $x = -6$ is extraneous.
15. The exponential is already isolated, so take \ln of both sides; the inverse pulls the exponent down: $2x = \ln(15)$. Divide by 2: $x = \frac{\ln 15}{2} \approx \frac{2.708}{2} \approx 1.35$.
16. Quotient property: $\ln(x/2) = \ln(7)$, so $x/2 = 7$ and $x = 14$. Domain check: $x = 14 > 0$, valid.

17. The argument $x - 5$ must be positive: $x - 5 > 0$, so $x > 5$. The open circle at 5 on the number line shows that $x = 5$ itself is excluded (it would make the argument zero).

Answer graph



18. Keep the rule visible: $x^2 + 1 \geq 1 > 0$ for every real x , so the log is defined everywhere. That gives a quick check on the answer.
19. Both sides are powers of the same base e , and e^x is one-to-one, so the exponents must be equal: $x + 1 = 2x - 3$. Solve: subtract x and add 3 to get $x = 4$.
20. Divide by 5 to isolate the exponential: $e^{0.04t} = 4$. Take \ln so the inverse brings the exponent down: $0.04t = \ln(4)$. Divide by 0.04: $t = \frac{\ln 4}{0.04} \approx \frac{1.3863}{0.04} \approx 34.66$.
21. Set $200e^{-0.04t} = 50$. Divide by 200: $e^{-0.04t} = 0.25$. Take \ln of both sides: $-0.04t = \ln(0.25) \approx -1.3863$. Solve: $t = \frac{1.3863}{0.04} \approx 34.66$ years. (Sanity check by plugging back: $200e^{-0.04(34.66)} = 200e^{-1.386} \approx 200(0.25) = 50$. Match.)
22. Set $500e^{0.12t} = 2000$. Divide: $e^{0.12t} = 4$. Take \ln : $0.12t = \ln 4 \approx 1.3863$. Solve: $t \approx 11.55$ hours. (Quick check: from 500, two doublings get us to 2000, and one doubling takes $\ln 2 / 0.12 \approx 5.78$ hours, so two doublings ≈ 11.55 — agrees with the direct calculation.)
23. Quotient property: $\ln\left(\frac{x+3}{x-1}\right) = \ln 2$. Same-base logs equal means the arguments are equal: $\frac{x+3}{x-1} = 2$, so $x+3 = 2(x-1) = 2x-2$, giving $x = 5$. Domain check: $x+3 = 8 > 0$ and $x-1 = 4 > 0$, both valid. (Always check both original arguments stay positive — one of them going non-positive would have killed the solution.)
24. Set $25000e^{-0.15t} = 10000$. Divide by 25000: $e^{-0.15t} = 0.4$. Take \ln : $-0.15t = \ln(0.4) \approx -0.9163$. Solve: $t = \frac{0.9163}{0.15} \approx 6.11$ years. (Continuous decay is faster than the same nominal rate compounded annually — the $e^{-0.15} \approx 0.861$ per year is a 13.9% effective annual loss.)



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