Multiplying and Dividing Complex Numbers

Name: ____________	Date: ____________	Score: ______ / 35

Quick Review

Multiplying complex numbers uses the same FOIL rules as multiplying binomials, with one extra move at the end: replace i^2 with -1. Concretely, $(a+bi)(c+di) = ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)i$. The $i^2 = -1$ step is what turns an imaginary $\times$ imaginary product into a real contribution, often with a sign flip.

Powers of i cycle every four: $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, then the pattern repeats. To simplify i^n for big n, divide n by 4 and use the remainder. For example, $i^{15} = i^{12} \cdot i^3 = 1 \cdot (-i) = -i$.

Dividing a complex number means rationalizing the denominator. To compute $\frac{a+bi}{c+di}$, multiply top and bottom by the *conjugate* of the denominator, $c - di$. That kills the imaginary part of the denominator because $(c+di)(c-di) = c^2 + d^2$, a real number. Then split into $a + bi$ form. **Trap:** when you distribute the minus to set up the conjugate, only the imaginary sign flips — not the real sign. The conjugate of $3 - 2i$ is $3 + 2i$, not $-3 - 2i$.

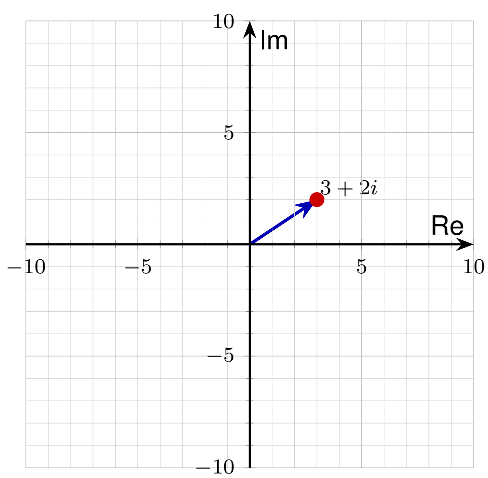
PRACTICE

Simplify each expression. Write the result in standard form $a + bi$.

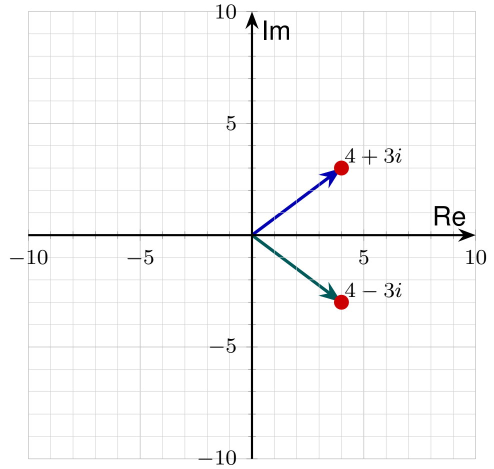
1. $i \cdot (3 + 2i)$ ________

2. $(2 + i)(3 - i)$ ________

3. $(3 + 2i)^2$. The base $3 + 2i$ is plotted below; squaring it doubles its argument and squares its modulus. ________



4. $(4+3i)(4-3i)$. The two factors, shown below, are a conjugate pair — mirror images across the real axis. ________



5. $(2+3i)(1-4i)$ ________
6. i^{15} ________
7. $i^{15} \cdot (2+i)$ ________
8. i^{27} ________
9. i^{100} ________
10. $(1+i)^2$ ________
11. $(5-2i)(3+i)$ ________
12. $\dfrac{2+3i}{1+i}$ ________
13. $\dfrac{4-5i}{2-i}$ ________
14. $\dfrac{7+i}{3-2i}$ ________
15. $\dfrac{1}{2-3i}$ ________
16. $(2-i)(3+4i)+(1+i)$ ________
17. $(3+i)(2-i)-(4-2i)$ ________
18. $(1+i)^4$ ________
19. $\dfrac{3+2i}{1-i}$ ________

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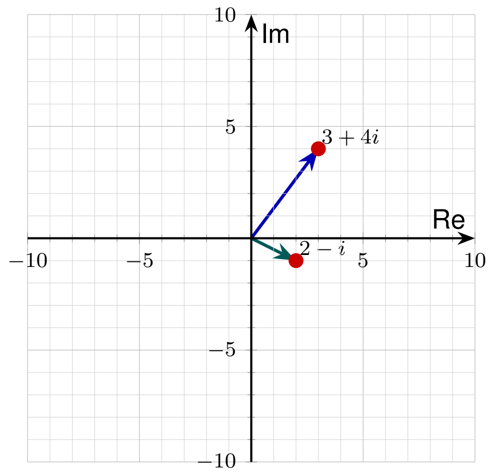
20. $(3+4i)(2-i)$. Both factors are plotted below; the product scales one modulus by the other and adds their arguments. ________



♦ Word Problems

21. Multiply $(3+i)(3-i)$ to find the value of $|3+i|^2$ (the squared modulus). What's the value? ____

22. In the complex plane, multiplying a point by i rotates it $90°$ counterclockwise about the origin. Starting from $3+4i$, apply this rotation twice. Where does the point end up? ____

23. The current in an AC circuit is $I = 2+i$ amps and the impedance is $Z = 3-4i$ ohms. Ohm's law gives the voltage $V = IZ$. Find the voltage. ____

24. The output of a system is $V_{\text{out}} = 12+5i$ volts when the input is $V_{\text{in}} = 3+2i$ volts. The transfer function is $H = \dfrac{V_{\text{out}}}{V_{\text{in}}}$. Find H in standard form. ____

Additional Practice

25. Add $(3+2i)+(5-i)$. ________

26. Subtract $(4-i)-(1+6i)$. ________

27. Multiply $(2+3i)(1-i)$. ________

28. Simplify i^{17}. ________

29. Simplify i^{22}. ________

30. Find the conjugate of $6-5i$. ________

31. Find $|3+4i|$. ________

32. Write $\sqrt{-36}$ in simplest form. ________

33. Divide $\dfrac{8+6i}{2}$. ________

34. Multiply $(5+i)(5-i)$. ________

35. Solve $x^2+25=0$. ________

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Answer Keys

1. $\boxed{-2+3i}$
2. $\boxed{7+i}$
3. $\boxed{5+12i}$
4. $\boxed{25}$
5. $\boxed{14-5i}$
6. $\boxed{-i}$
7. $\boxed{1-2i}$
8. $\boxed{-i}$
9. $\boxed{1}$
10. $\boxed{2i}$
11. $\boxed{17-i}$
12. $\boxed{\frac{5}{2}+\frac{1}{2}i}$
13. $\boxed{\frac{13}{5}-\frac{6}{5}i}$
14. $\boxed{\frac{19}{13}+\frac{17}{13}i}$
15. $\boxed{\frac{2}{13}+\frac{3}{13}i}$
16. $\boxed{11+6i}$
17. $\boxed{3+i}$
18. $\boxed{-4}$
19. $\boxed{\frac{1}{2}+\frac{5}{2}i}$
20. $\boxed{10+5i}$
21. $\boxed{10}$
22. $\boxed{-3-4i}$
23. $\boxed{V=10-5i \text{ volts}}$
24. $\boxed{H=\frac{46}{13}-\frac{9}{13}i}$

Additional Practice Answers

25. $\boxed{8+i}$
26. $\boxed{3-7i}$
27. $\boxed{5+i}$
28. $\boxed{i}$
29. $\boxed{-1}$
30. $\boxed{6+5i}$
31. $\boxed{5}$
32. $\boxed{6i}$
33. $\boxed{4+3i}$
34. $\boxed{26}$
35. $\boxed{x=\pm 5i}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Distribute: $i\cdot 3+i\cdot 2i=3i+2i^2$. Replace i^2 with -1: $3i+2(-1)=-2+3i$. Multiplying by i alone is a 90° rotation in the plane.

2. FOIL the two binomials: $6-2i+3i-i^2$. Combine the middle terms ($-2i+3i=i$) and replace $-i^2$ with $+1$: $6+i+1=7+i$. The $i^2=-1$ step is what bumps the real part from 6 up to 7.

3. Use $(a+b)^2=a^2+2ab+b^2$ with $a=3, b=2i$: $9+12i+4i^2=9+12i-4=5+12i$. Many students forget the $4i^2=-4$ step and write $9+4i$ — watch that.

4. Conjugates! Shortcut: $(a+bi)(a-bi)=a^2+b^2$, so $16+9=25$. The imaginary parts cancel and $-9i^2$ becomes $+9$. The product of a conjugate pair is always the squared modulus, a real number.

5. FOIL: $2-8i+3i-12i^2$. Middle terms: $-8i+3i=-5i$. The $-12i^2$ becomes $+12$, so reals are $2+12=14$. Result: $14-5i$. The imaginary $\times$ imaginary product is what feeds the real part.

6. Powers of i cycle every 4, so divide the exponent by 4 and keep the remainder: $15=4(3)+3$, remainder 3. That means $i^{15}=i^3=-i$.

7. From the previous problem $i^{15}=-i$, so distribute: $(-i)(2+i)=-2i-i^2$. Replace $-i^2$ with $+1$: $-2i+1=1-2i$.

8. Divide the exponent by 4 and use the remainder: $27=4(6)+3$, remainder 3. So $i^{27}=i^3=-i$.

9. A careful way to see it: $100=4(25)$ exactly, remainder 0, so $i^{100}=i^0=1$. You can see it directly: $i^{100}=(i^4)^{25}=1^{25}=1$, since $i^4=1$. That gives a quick check on the answer.

10. Square the binomial: $1+2i+i^2$. Since $i^2=-1$, the $+1$ and -1 cancel, leaving $2i$. A purely imaginary result — the real parts wiped each other out.

11. FOIL: $15+5i-6i-2i^2$. Middle: $5i-6i=-i$. The $-2i^2$ becomes $+2$, so reals are $15+2=17$. Result: $17-i$.

12. Multiply top and bottom by the conjugate $1-i$. Bottom: $(1+i)(1-i)=1-i^2=2$. Top: $(2+3i)(1-i)=2-2i+3i-3i^2=5+i$. So $\frac{5+i}{2}=\frac{5}{2}+\frac{1}{2}i$.

13. Conjugate of $2-i$ is $2+i$. Bottom: $(2-i)(2+i)=4+1=5$. Top: $(4-5i)(2+i)=8+4i-10i-5i^2=8-6i+5=13-6i$. Answer: $\frac{13-6i}{5}=\frac{13}{5}-\frac{6}{5}i$.

14. Conjugate of $3-2i$ is $3+2i$. Bottom: $9+4=13$. Top: $(7+i)(3+2i)=21+14i+3i+2i^2=21+17i-2=19+17i$. Answer: $\frac{19}{13}+\frac{17}{13}i$.

15. Conjugate of $2-3i$ is $2+3i$. Bottom: $4+9=13$. Top: $1\cdot(2+3i)=2+3i$. So $\frac{2+3i}{13}=\frac{2}{13}+\frac{3}{13}i$.

16. Do the product first: $(2-i)(3+4i)=6+8i-3i-4i^2$. The $-4i^2=+4$, so this is $6+4+(8-3)i=10+5i$. Then add $(1+i)$ by columns: $10+1=11$ and $5+1=6$, giving $11+6i$.

17. Product first: $(3+i)(2-i)=6-3i+2i-i^2=6-i+1=7-i$. Then subtract $(4-2i)$: real $7-4=3$, imaginary $-1-(-2)=+1$. Final: $3+i$. (The subtraction flips the second imaginary part — watch that step.)

18. Use $(1+i)^2=2i$ from earlier, then square again: $(2i)^2=4i^2=-4$. Stacking the easy square is faster than expanding $(1+i)^4$ directly.

19. Conjugate of $1-i$ is $1+i$. Bottom: $1+1=2$. Top: $(3+2i)(1+i)=3+3i+2i+2i^2=3+5i-2=1+5i$. Answer: $\frac{1+5i}{2}=\frac{1}{2}+\frac{5}{2}i$.

20. FOIL: $6-3i+8i-4i^2=6+5i+4=10+5i$. (Geometrically, this multiplies the modulus of $3+4i$ by the modulus of $2-i$ and adds the arguments — a scale-and-rotate.)

21. Conjugate product: $(3+i)(3-i)=3^2+1^2=10$. That's exactly $|3+i|^2$, because $|a+bi|^2=a^2+b^2$. (The conjugate trick is why modulus calculations come out clean.)

22. Multiplying by i twice is multiplying by $i^2=-1$, a 180° rotation. So $(3+4i)\cdot i^2=(3+4i)(-1)=-3-4i$. The point lands directly opposite the origin from the start.

Answer graph

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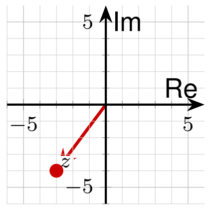

23. One steady path is: $V = IZ = (2+i)(3-4i) = 6-8i+3i-4i^2 = 6-5i+4 = 10-5i$ volts. Engineers use complex multiplication this way every day — $i^2 = -1$ does real work. That gives a quick check on the answer.

24. Start with the key idea: $H = \frac{12+5i}{3+2i}$. Multiply by the conjugate $3-2i$. Bottom: $(3+2i)(3-2i) = 9+4 = 13$. Top: $(12+5i)(3-2i) = 36-24i+15i-10i^2 = 36-9i+10 = 46-9i$. So $H = \frac{46-9i}{13} = \frac{46}{13} - \frac{9}{13}i$. That gives a quick check on the answer.

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