

Multiplying and Dividing Complex Numbers

Name: _____

Date: _____

Score: _____ / 35

Q Quick Review

Multiplying complex numbers uses the same FOIL rules as multiplying binomials, with one extra move at the end: replace i^2 with -1 . Concretely, $(a + bi)(c + di) = ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)i$. The $i^2 = -1$ step is what turns an imaginary \times imaginary product into a real contribution, often with a sign flip.

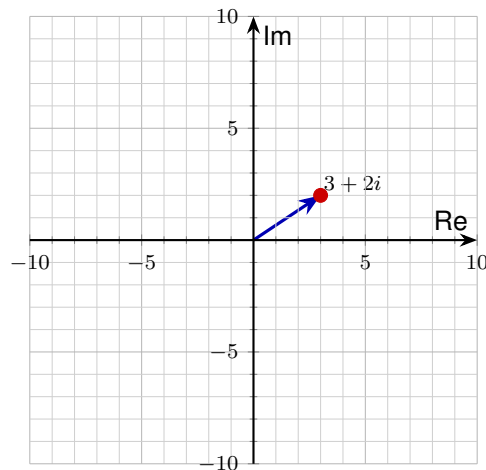
Powers of i cycle every four: $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, then the pattern repeats. To simplify i^n for big n , divide n by 4 and use the remainder. For example, $i^{15} = i^{12} \cdot i^3 = 1 \cdot (-i) = -i$.

Dividing a complex number means rationalizing the denominator. To compute $\frac{a + bi}{c + di}$, multiply top and bottom by the *conjugate* of the denominator, $c - di$. That kills the imaginary part of the denominator because $(c + di)(c - di) = c^2 + d^2$, a real number. Then split into $a + bi$ form. **Trap:** when you distribute the minus to set up the conjugate, only the imaginary sign flips — not the real sign. The conjugate of $3 - 2i$ is $3 + 2i$, not $-3 - 2i$.

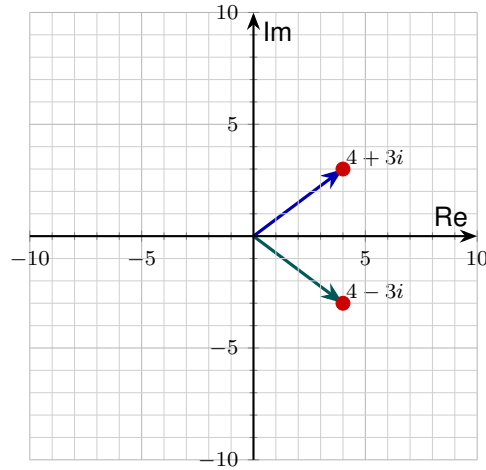
PRACTICE

Simplify each expression. Write the result in standard form $a + bi$.

- $i \cdot (3 + 2i)$ _____
- $(2 + i)(3 - i)$ _____
- $(3 + 2i)^2$. The base $3 + 2i$ is plotted below; squaring it doubles its argument and squares its modulus. _____



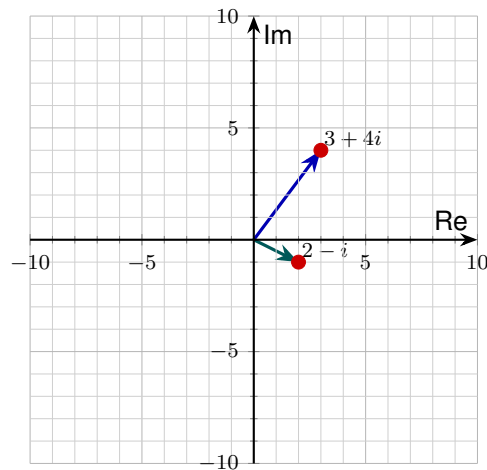
4. $(4 + 3i)(4 - 3i)$. The two factors, shown below, are a conjugate pair — mirror images across the real axis. _____



- 5. $(2 + 3i)(1 - 4i)$ _____
- 6. i^{15} _____
- 7. $i^{15} \cdot (2 + i)$ _____
- 8. i^{27} _____
- 9. i^{100} _____
- 10. $(1 + i)^2$ _____
- 11. $(5 - 2i)(3 + i)$ _____
- 12. $\frac{2 + 3i}{1 + i}$ _____
- 13. $\frac{4 - 5i}{2 - i}$ _____
- 14. $\frac{7 + i}{3 - 2i}$ _____
- 15. $\frac{1}{2 - 3i}$ _____
- 16. $(2 - i)(3 + 4i) + (1 + i)$ _____
- 17. $(3 + i)(2 - i) - (4 - 2i)$ _____
- 18. $(1 + i)^4$ _____
- 19. $\frac{3 + 2i}{1 - i}$ _____



20. $(3 + 4i)(2 - i)$. Both factors are plotted below; the product scales one modulus by the other and adds their arguments. _____



◆ Word Problems

- 21. Multiply $(3 + i)(3 - i)$ to find the value of $|3 + i|^2$ (the squared modulus). What's the value? _____
- 22. In the complex plane, multiplying a point by i rotates it 90° counterclockwise about the origin. Starting from $3 + 4i$, apply this rotation twice. Where does the point end up? _____
- 23. The current in an AC circuit is $I = 2 + i$ amps and the impedance is $Z = 3 - 4i$ ohms. Ohm's law gives the voltage $V = IZ$. Find the voltage. _____
- 24. The output of a system is $V_{out} = 12 + 5i$ volts when the input is $V_{in} = 3 + 2i$ volts. The transfer function is $H = \frac{V_{out}}{V_{in}}$. Find H in standard form. _____

Additional Practice

- 25. Add $(3 + 2i) + (5 - i)$. _____
- 26. Subtract $(4 - i) - (1 + 6i)$. _____
- 27. Multiply $(2 + 3i)(1 - i)$. _____
- 28. Simplify i^{17} . _____
- 29. Simplify i^{22} . _____
- 30. Find the conjugate of $6 - 5i$. _____
- 31. Find $|3 + 4i|$. _____
- 32. Write $\sqrt{-36}$ in simplest form. _____
- 33. Divide $\frac{8 + 6i}{2}$. _____
- 34. Multiply $(5 + i)(5 - i)$. _____
- 35. Solve $x^2 + 25 = 0$. _____



Answer Keys

1. $-2 + 3i$

2. $7 + i$

3. $5 + 12i$

4. 25

5. $14 - 5i$

6. $-i$

7. $1 - 2i$

8. $-i$

9. 1

10. $2i$

11. $17 - i$

12. $\frac{5}{2} + \frac{1}{2}i$

13. $\frac{13}{5} - \frac{6}{5}i$

Additional Practice Answers

25. $8 + i$

26. $3 - 7i$

27. $5 + i$

28. i

29. -1

30. $6 + 5i$

14. $\frac{19}{13} + \frac{17}{13}i$

15. $\frac{2}{13} + \frac{3}{13}i$

16. $11 + 6i$

17. $3 + i$

18. -4

19. $\frac{1}{2} + \frac{5}{2}i$

20. $10 + 5i$

21. 10

22. $-3 - 4i$

23. $V = 10 - 5i$ volts

24. $H = \frac{46}{13} - \frac{9}{13}i$

31. 5

32. $6i$

33. $4 + 3i$

34. 26

35. $x = \pm 5i$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Distribute: $i \cdot 3 + i \cdot 2i = 3i + 2i^2$. Replace i^2 with -1 : $3i + 2(-1) = -2 + 3i$. Multiplying by i alone is a 90° rotation in the plane.

2. FOIL the two binomials: $6 - 2i + 3i - i^2$. Combine the middle terms ($-2i + 3i = i$) and replace $-i^2$ with $+1$: $6 + i + 1 = 7 + i$. The $i^2 = -1$ step is what bumps the real part from 6 up to 7.

3. Use $(a + b)^2 = a^2 + 2ab + b^2$ with $a = 3, b = 2i$: $9 + 12i + 4i^2 = 9 + 12i - 4 = 5 + 12i$. Many students forget the $4i^2 = -4$ step and write $9 + 4i$ — watch that.

4. Conjugates! Shortcut: $(a + bi)(a - bi) = a^2 + b^2$, so $16 + 9 = 25$. The imaginary parts cancel and $-9i^2$ becomes $+9$. The product of a conjugate pair is always the squared modulus, a real number.

5. FOIL: $2 - 8i + 3i - 12i^2$. Middle terms: $-8i + 3i = -5i$. The $-12i^2$ becomes $+12$, so reals are $2 + 12 = 14$. Result: $14 - 5i$. The imaginary \times imaginary product is what feeds the real part.

6. Powers of i cycle every 4, so divide the exponent by 4 and keep the remainder: $15 = 4(3) + 3$, remainder 3. That means $i^{15} = i^3 = -i$.

7. From the previous problem $i^{15} = -i$, so distribute: $(-i)(2 + i) = -2i - i^2$. Replace $-i^2$ with $+1$: $-2i + 1 = 1 - 2i$.

8. Divide the exponent by 4 and use the remainder: $27 = 4(6) + 3$, remainder 3. So $i^{27} = i^3 = -i$.

9. A careful way to see it: $100 = 4(25)$ exactly, remainder 0, so $i^{100} = i^0 = 1$. You can see it directly: $i^{100} = (i^4)^{25} = 1^{25} = 1$, since $i^4 = 1$. That gives a quick check on the answer.

10. Square the binomial: $1 + 2i + i^2$. Since $i^2 = -1$, the $+1$ and -1 cancel, leaving $2i$. A purely imaginary result — the real parts wiped each other out.

11. FOIL: $15 + 5i - 6i - 2i^2$. Middle: $5i - 6i = -i$. The $-2i^2$ becomes $+2$, so reals are $15 + 2 = 17$. Result: $17 - i$.

12. Multiply top and bottom by the conjugate $1 - i$. Bottom: $(1 + i)(1 - i) = 1 - i^2 = 2$. Top: $(2 + 3i)(1 - i) = 2 - 2i + 3i - 3i^2 = 5 + i$. So $\frac{5 + i}{2} = \frac{5}{2} + \frac{1}{2}i$.

13. Conjugate of $2 - i$ is $2 + i$. Bottom: $(2 - i)(2 + i) = 4 + 1 = 5$. Top: $(4 - 5i)(2 + i) = 8 + 4i - 10i - 5i^2 = 8 - 6i + 5 = 13 - 6i$. Answer:

$$\frac{13 - 6i}{5} = \frac{13}{5} - \frac{6}{5}i.$$

14. Conjugate of $3 - 2i$ is $3 + 2i$. Bottom: $9 + 4 = 13$. Top: $(7 + i)(3 + 2i) = 21 + 14i + 3i + 2i^2 = 21 + 17i - 2 = 19 + 17i$. Answer: $\frac{19}{13} + \frac{17}{13}i$.

15. Conjugate of $2 - 3i$ is $2 + 3i$. Bottom: $4 + 9 = 13$. Top: $1 \cdot (2 + 3i) = 2 + 3i$. So $\frac{2 + 3i}{13} = \frac{2}{13} + \frac{3}{13}i$.

16. Do the product first: $(2 - i)(3 + 4i) = 6 + 8i - 3i - 4i^2$. The $-4i^2 = +4$, so this is $6 + 4 + (8 - 3)i = 10 + 5i$. Then add $(1 + i)$ by columns: $10 + 1 = 11$ and $5 + 1 = 6$, giving $11 + 6i$.

17. Product first: $(3 + i)(2 - i) = 6 - 3i + 2i - i^2 = 6 - i + 1 = 7 - i$. Then subtract $(4 - 2i)$: real $7 - 4 = 3$, imaginary $-1 - (-2) = +1$. Final: $3 + i$. (The subtraction flips the second imaginary part — watch that step.)

18. Use $(1 + i)^2 = 2i$ from earlier, then square again: $(2i)^2 = 4i^2 = -4$. Stacking the easy square is faster than expanding $(1 + i)^4$ directly.

19. Conjugate of $1 - i$ is $1 + i$. Bottom: $1 + 1 = 2$. Top: $(3 + 2i)(1 + i) = 3 + 3i + 2i + 2i^2 = 3 + 5i - 2 = 1 + 5i$. Answer: $\frac{1 + 5i}{2} = \frac{1}{2} + \frac{5}{2}i$.

20. FOIL: $6 - 3i + 8i - 4i^2 = 6 + 5i + 4 = 10 + 5i$. (Geometrically, this multiplies the modulus of $3 + 4i$ by the modulus of $2 - i$ and adds the arguments — a scale-and-rotate.)

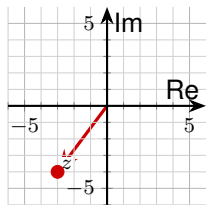
21. Conjugate product: $(3 + i)(3 - i) = 3^2 + 1^2 = 10$. That's exactly $|3 + i|^2$, because $|a + bi|^2 = a^2 + b^2$. (The conjugate trick is why modulus calculations come out clean.)

22. Multiplying by i twice is multiplying by $i^2 = -1$, a 180° rotation. So $(3 + 4i) \cdot i^2 = (3 + 4i)(-1) = -3 - 4i$. The point lands directly opposite the origin from the start.

Answer graph



Scan Me



23. One steady path is: $V = IZ = (2 + i)(3 - 4i) = 6 - 8i + 3i - 4i^2 = 6 - 5i + 4 = 10 - 5i$ volts. Engineers use complex multiplication this way every day — $i^2 = -1$ does real work. That gives a quick check on the answer.

24. Start with the key idea: $H = \frac{12 + 5i}{3 + 2i}$. Multiply by the conjugate $3 - 2i$.
 Bottom: $(3 + 2i)(3 - 2i) = 9 + 4 = 13$. Top: $(12 + 5i)(3 - 2i) = 36 - 24i + 15i - 10i^2 = 36 - 9i + 10 = 46 - 9i$. So $H = \frac{46 - 9i}{13} = \frac{46}{13} - \frac{9}{13}i$.
 That gives a quick check on the answer.



Build Algebra Confidence From Pre-Algebra Through Algebra II



The Complete Algebra Success Bundle

Pre-Algebra, Algebra I, and Algebra II in one clear path

Friendly lessons, focused practice, and full-review support for every stage.



Scan for the Bundle

6 Books
3 Courses
1 Path

Bundle Value: Six coordinated books help students review missing skills, learn new algebra topics, and practice until the steps feel natural.

Complete Course Path

- ✓ Starts with Pre-Algebra foundations
- ✓ Moves smoothly into Algebra I skills
- ✓ Extends learning through Algebra II topics
- ✓ Great for review, tutoring, and summer study

One bundle, one steady path.

Step-by-Step Lessons

- ✓ Plain-English explanations students can follow
- ✓ Worked examples that show every important step
- ✓ Common mistakes called out before they stick
- ✓ Skill-building practice after each lesson
- ✓ Helpful for independent study or class support

Less guessing. More understanding.

Practice That Sticks

- ✓ Matching practice workbooks for extra repetition
- ✓ Review sets to keep older skills fresh
- ✓ Answer explanations for checking thinking
- ✓ Strong support before tests and final exams
- ✓ Designed to build fluency and confidence

Practice today. Remember tomorrow.

STUDENT FAVORITE • Master Algebra II From the Ground Up



Algebra II for Beginners

Written by a top math teacher & aligned with national and state Algebra II courses. From polynomial functions to logarithms, trigonometry, and rational expressions — explained the easy way.

- ✓ **Complete coverage** of every Algebra II concept — perfect companion to these worksheets
- ✓ **Step-by-step explanations** with worked examples on every topic
- ✓ **QR codes in every chapter** for free video lessons & bonus practice
- ✓ **2 full-length practice tests** with detailed answer keys

- ✓ 100% Guaranteed
- ✓ Lifetime Support
- ✓ Trusted by Teachers

Start Your Algebra Journey Today! →

★ STUDENT'S #1 CHOICE ★

Teacher-recommended • 12,000+ Happy Students

PDF EDITION



Instant download • any device

PAPERBACK



Paperback on Amazon

Hold it in your hands

Pair these free worksheets with *Algebra II for Beginners* and you have a complete self-paced course — concept lessons, daily practice, and full exam-style reviews, all in one path. → EffortlessMath.com/product/algebra-ii-for-beginners