Multiplying Rational Expressions

Name:	Date:	Score: ____ / 35

Quick Review

Multiplying rational expressions follows the same recipe as multiplying numeric fractions: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. The polynomial version just has more opportunities to factor and cancel.

Step 1 – factor everything first. Multiply nothing until every polynomial is in factored form. This is where the savings live.

Step 2 – cancel common factors across the product. A factor in any numerator can cancel a matching factor in any denominator. Cross-cancel before multiplying out – the arithmetic stays clean.

Step 3 – state every restriction. Each original denominator contributes its restrictions, and so does each factor that cancels. Quick check: in $\frac{x^2-4}{x+1} \cdot \frac{x+1}{x+2}$, the algebra cancels $x+1$ and $x+2$, but you still need $x \neq -1$ and $x \neq -2$ in the answer.

Sign-flipped factors. $a - b = -(b - a)$. So $\frac{x-3}{2} \cdot \frac{4}{3-x} = \frac{x-3}{2} \cdot \frac{4}{-(x-3)} = -\frac{4}{2} \cdot 1 = -2$. Always factor out the -1 before canceling.

Common slips. Trying to cancel terms across a sum or difference. Forgetting that canceled factors still set restrictions. Dropping a sign when one denominator is sign-flipped from the other.

PRACTICE

Factor every polynomial, cancel common factors across the product, then multiply. State every restriction.

1. $\frac{2}{x} \cdot \frac{x}{5}$ ________

2. Multiply $\frac{3x^2}{4} \cdot \frac{8}{x}$. The table evaluates the product at a few inputs. ________

x	1	2	3
$\frac{3x^2}{4} \cdot \frac{8}{x}$	6	12	18

3. Multiply $\frac{x^2-4}{x+1} \cdot \frac{x+1}{x+2}$. The table samples the product away from the excluded values. ________

x	0	1	3
product	-2	-1	1

4. $\frac{x-3}{x+2} \cdot \frac{x+2}{3-x}$ ________

5. $\frac{x^2-x-6}{x+4} \cdot \frac{x^2+3x-4}{x^2-9}$ ________

6. $\frac{x^2-4}{x+1} \cdot \frac{2-x}{x-2}$ ________

7. Multiply $\frac{x^2+5x+6}{x^2-1} \cdot \frac{x-1}{x+3}$. The table evaluates the product at several inputs. ________

x	0	2	3
product	2	$\frac{4}{3}$	$\frac{5}{4}$

8. $\frac{x^2-1}{x^2+5x+6} \cdot \frac{x+3}{x-1}$ ________

9. $\frac{x^2-4x}{x^2-16} \cdot \frac{x+4}{x-2}$ ________

10. $\frac{x^2-4}{x+1} \cdot \frac{x+1}{x-2}$ ________

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11. $\frac{6a}{b^2} \cdot \frac{b}{12a^2}$ ________

12. $\frac{x^2+2x+1}{x^2-4} \cdot \frac{x-2}{x+1}$ ________

13. $\frac{x^2-9}{x^2-x-6} \cdot \frac{x+2}{x+3}$ ________

14. $\frac{x}{x^2-1} \cdot \frac{x-1}{x^2+x}$ ________

15. $\frac{2x^2-2}{x^2+3x+2} \cdot \frac{x+2}{4x-4}$ ________

16. Mark TRUE or FALSE: $\frac{a-b}{b-a} = 1$. ________

17. $\frac{5}{x^2-1} \cdot (x+1)$ ________

18. $\frac{x^2-25}{x+5} \cdot \frac{1}{x-5}$ ________

19. $\frac{2x+10}{x^2-25} \cdot \frac{x-5}{4}$ ________

20. $\frac{x^2+7x+12}{x^2-9} \cdot \frac{x-3}{x+4}$ ________

♦ Word Problems

21. A rectangle has dimensions $\frac{x^2-4}{x+3}$ by $\frac{x+3}{x-2}$. Find the simplified area and state the restrictions. ____

22. A scaling problem requires you to evaluate $\frac{x^2-1}{x+2} \cdot \frac{2x+4}{x-1}$ at $x = 5$. Simplify first, then evaluate – and compare with plugging $x = 5$ in directly. ____

23. A geometry problem produces $\frac{x^2-9}{x^2+4x+3} \cdot \frac{x+1}{x-3}$. Simplify, state restrictions, and explain why the simplified answer is just a constant. ____

24. A physics setup uses the ratio $\frac{x^2+5x+6}{2x+4} \cdot \frac{4}{x+3}$. Simplify to a clean closed form. ____

Additional Practice

25. Simplify $\frac{x^2-9}{x-3}$. ________

26. Excluded value of $\frac{1}{x+4}$. ________

27. Domain of $f(x) = \frac{x}{x-5}$. ________

28. Multiply $\frac{x}{3} \cdot \frac{6}{x}$. ________

29. Divide $\frac{x^2}{5} \div \frac{x}{10}$. ________

30. Add $\frac{3}{x} + \frac{5}{x}$. ________

31. Subtract $\frac{7}{x-1} - \frac{2}{x-1}$. ________

32. Solve $\frac{1}{x} = 4$. ________

33. Solve $\frac{x+2}{x-1} = 3$. ________

34. Vertical asymptote of $y = \frac{4}{x+8}$. ________

35. Horizontal asymptote of $y = \frac{3x+1}{x-2}$. ________

Answer Keys

1. $\frac{2}{5},\ x \neq 0$
2. $6x,\ x \neq 0$
3. $x-2,\ x \neq -1,\ -2$
4. $-1,\ x \neq -2,\ 3$
5. $\frac{(x+2)(x-1)}{x+3},\ x \neq -4,\ -3,\ 3$
6. $-\frac{(x-2)(x+2)}{x+1},\ x \neq -1,\ 2$
7. $\frac{x+2}{x+1},\ x \neq \pm 1,\ -3$
8. $\frac{x+1}{x+2},\ x \neq -2,\ -3,\ 1$
9. $\frac{x}{x-2},\ x \neq -4,\ 4,\ 2$
10. $x+2,\ x \neq -1,\ 2$
11. $\frac{1}{2ab},\ a \neq 0,\ b \neq 0$
12. $\frac{x+1}{x+2},\ x \neq \pm 2,\ -1$
13. $1,\ x \neq -2,\ -3,\ 3$
14. $\frac{1}{(x+1)^2},\ x \neq 0,\ \pm 1$
15. $\frac{1}{2},\ x \neq -2,\ -1,\ 1$
16. FALSE
17. $\frac{5}{x-1},\ x \neq \pm 1$
18. $1,\ x \neq \pm 5$
19. $\frac{1}{2},\ x \neq \pm 5$
20. $1,\ x \neq -4,\ \pm 3$
21. area $= x+2,\ x \neq -3,\ 2$
22. $2(x+1) = 12$ at $x = 5$; direct: also 12
23. $1,\ x \neq -3,\ -1,\ 3$
24. $2,\ x \neq -2,\ -3$

Additional Practice Answers

25. $x+3,\ x \neq 3$
26. $x = -4$
27. $x \neq 5$
28. 2
29. $2x$
30. $\frac{8}{x}$
31. $\frac{5}{x-1}$
32. $x = \frac{1}{4}$
33. $x = \frac{5}{2}$
34. $x = -8$
35. $y = 3$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. The x on the bottom of the first fraction cancels the x on top of the second, leaving $\frac{2}{5}$. Even though the x vanishes, it once sat in a denominator, so keep the restriction $x \neq 0$ in your answer.

2. Multiply: $\frac{24x^2}{4x} = 6x$. (Coefficient: $24/4 = 6$; powers: $x^2/x = x$. The table values match $6x$ at $x = 1, 2, 3$.)

3. Factor the top: $x^2 - 4 = (x-2)(x+2)$. Cross-cancel the $(x+1)$ on the first bottom with the $(x+1)$ on the second top, and the $(x+2)$ on the first top with the $(x+2)$ on the second bottom. Left with $x-2$. Keep $x \neq -1, -2$: both sat in original denominators even though their factors cancelled. (The table agrees with $x-2$ at $x = 0, 1, 3$.)

4. Cancel $(x+2)$ across the product. The leftover is $\frac{x-3}{3-x}$; rewrite $3-x = -(x-3)$, so it becomes $\frac{x-3}{-(x-3)} = -1$. Sign-flipped factors like these always collapse to -1. Keep $x \neq -2$ and $x \neq 3$ from the original denominators.

5. Factor every piece: $x^2-x-6 = (x-3)(x+2)$, $x^2+3x-4 = (x+4)(x-1)$, $x^2-9 = (x-3)(x+3)$. The $(x+4)$ on top of the second fraction cancels the $(x+4)$ denominator of the first, and one $(x-3)$ cancels the other. What remains is $\frac{(x+2)(x-1)}{x+3}$. List $x \neq -4, -3, 3$ from the original denominators.

6. Keep the rule visible: $x^2-4 = (x-2)(x+2)$ and $2-x = -(x-2)$. Multiply: $\frac{(x-2)(x+2)}{x+1} \cdot \frac{-(x-2)}{x-2} = -\frac{(x-2)(x+2)}{x+1}$. One $(x-2)$ cancels but the other stays. That gives a quick check on the answer.

7. Factor: $x^2+5x+6 = (x+2)(x+3)$, $x^2-1 = (x-1)(x+1)$. Cancel $(x+3)$ and $(x-1)$. (The table matches $\frac{x+2}{x+1}$ at $x = 0, 2, 3$.)

8. Factor: $x^2-1 = (x-1)(x+1)$ and $x^2+5x+6 = (x+2)(x+3)$. The $(x-1)$ on the second bottom cancels the $(x-1)$ up top, and the $(x+3)$ factors cancel, leaving $\frac{x+1}{x+2}$. Keep $x \neq -2, -3, 1$ from the original denominators.

9. Factor: $x^2-4x = x(x-4)$ and $x^2-16 = (x-4)(x+4)$. The $(x+4)$ on the second top cancels the $(x+4)$ on the first bottom, and the $(x-4)$ factors cancel too, leaving $\frac{x}{x-2}$. Keep $x \neq -4, 4, 2$ from the original denominators.

10. Factor the top of the first fraction: $x^2-4 = (x-2)(x+2)$. The $(x+1)$ on the first bottom cancels the $(x+1)$ on the second top, and the $(x-2)$ on the second bottom cancels the $(x-2)$ up top. Left with $x+2$. Keep $x \neq -1, 2$ from the original denominators.

11. Coefficients: $\frac{6}{12} = \frac{1}{2}$. Powers of a: $\frac{a}{a^2} = \frac{1}{a}$. Powers of b: $\frac{b}{b^2} = \frac{1}{b}$. Combine: $\frac{1}{2ab}$.

12. Start with the key idea: $x^2+2x+1 = (x+1)^2$ and $x^2-4 = (x-2)(x+2)$. The $(x-2)$ on the second top cancels the $(x-2)$ below, and the $(x+1)$ on the second bottom cancels one of the two $(x+1)$ factors up top. One $(x+1)$ survives on top, giving $\frac{x+1}{x+2}$. Keep $x \neq \pm 2, -1$. That gives a quick check on the answer.

13. Factor: $x^2-9 = (x-3)(x+3)$ and $x^2-x-6 = (x-3)(x+2)$. Writing it out, $\frac{(x-3)(x+3)}{(x-3)(x+2)} \cdot \frac{x+2}{x+3}$ – every factor pairs off, so the product is 1. Still keep $x \neq -2, -3, 3$ from the original denominators; the expression equals 1 everywhere *except* those points.

14. Keep the rule visible: $x^2-1 = (x-1)(x+1)$, $x^2+x = x(x+1)$. Cancel x and $(x-1)$. Left with $\frac{1}{(x+1)(x+1)} = \frac{1}{(x+1)^2}$. That gives a quick check

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on the answer.

15. One steady path is: $2x^2-2=2(x-1)(x+1)$; $x^2+3x+2=(x+1)(x+2)$; $4x-4=4(x-1)$. Cancel $(x-1)$, $(x+1)$, $(x+2)$, and reduce $\frac{2}{4}$ to $\frac{1}{2}$. That gives a quick check on the answer.

16. Sign-flipped factors give -1, not 1. $b-a=-(a-b)$, so the quotient is -1 (for $a\neq b$).

17. A careful way to see it: Treat $x+1$ as $\frac{x+1}{1}$. Factor $x^2-1=(x-1)(x+1)$ and cancel. That gives a quick check on the answer.

18. Factor $x^2-25=(x-5)(x+5)$. The $(x+5)$ on top cancels the $(x+5)$ below, and the $(x-5)$ on the second bottom cancels the $(x-5)$ up top. Both factors clear, leaving 1. Keep $x\neq\pm5$ from the original denominators.

19. One steady path is: $2x+10=2(x+5)$; $x^2-25=(x-5)(x+5)$. Cancel $(x+5)$ and $(x-5)$. Reduce $\frac{2}{4}=\frac{1}{2}$. That gives a quick check on the answer.

20. Factor: $x^2+7x+12=(x+3)(x+4)$ and $x^2-9=(x-3)(x+3)$. The $(x-3)$ on the second top cancels the $(x-3)$ below, the $(x+4)$ on the second bottom cancels the $(x+4)$ up top, and one $(x+3)$ cancels the other. Everything clears, so the product is 1. Keep $x\neq-4,\pm3$.

21. Area $=\frac{x^2-4}{x+3}\cdot\frac{x+3}{x-2}=\frac{(x-2)(x+2)}{x+3}\cdot\frac{x+3}{x-2}=x+2$. Cancel $(x-2)$ and $(x+3)$. Restrictions: $x\neq-3$ (from the first denominator) and $x\neq2$ (from the second). At $x=5$ the formula gives area $=7$ – and the original product is $\frac{21}{8}\cdot\frac{8}{3}=7$. Match.

22. Factor: $x^2-1=(x-1)(x+1)$, $2x+4=2(x+2)$. Product: $\frac{(x-1)(x+1)}{x+2}\cdot\frac{2(x+2)}{x-1}=2(x+1)$. At $x=5$: $2(6)=12$. Direct plug: $\frac{24}{7}\cdot\frac{14}{4}=\frac{336}{28}=12$. Match. (Simplifying first saves a lot of arithmetic, especially when the factors involved cancel cleanly.)

23. Factor: $x^2-9=(x-3)(x+3)$ and $x^2+4x+3=(x+1)(x+3)$. Then $\frac{(x-3)(x+3)}{(x+1)(x+3)}\cdot\frac{x+1}{x-3}=1$. Every factor matches up – the constant 1 emerges because the two fractions were already reciprocals in disguise. Restrictions: $x\neq-3$ and $x\neq-1$ (original denominators) and $x\neq3$ (from the second numerator's matching denominator). The constant answer is valid only on the allowed domain – which means the original expression equals 1 everywhere except those four exclusions.

24. Factor: $x^2+5x+6=(x+2)(x+3)$ and $2x+4=2(x+2)$. Then $\frac{(x+2)(x+3)}{2(x+2)}\cdot\frac{4}{x+3}=\frac{4}{2}=2$. The variable disappears because every factor cancels in pairs. Restrictions: $x\neq-2$ and $x\neq-3$ (from the original denominators). On its domain the expression equals the constant 2. (Sanity check at $x=1$: $\frac{12}{6}\cdot\frac{4}{4}=2\cdot1=2$.)