Multiplying Radical Expressions

Name: ____________________	Date: ______________	Score: _______ / 32

Quick Review

Multiplying radicals leans on one rule, then FOIL handles the rest.

The product rule. For $a, b \geq 0$, $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$. Coefficients sit outside and multiply on their own: $(2\sqrt{5})(3\sqrt{2}) = 6\sqrt{10}$. The rule extends to the same index for any radical: $\sqrt[3]{a} \cdot \sqrt[3]{b} = \sqrt[3]{ab}$. (*Different indices* don't combine directly — $\sqrt{2} \cdot \sqrt[3]{2}$ needs a rational-exponent rewrite.)

Distribute / FOIL with radicals. Treat each radical like any other algebraic term and distribute or FOIL: $\sqrt{2}(\sqrt{2}+1) = (\sqrt{2})^2 + \sqrt{2} = 2 + \sqrt{2}$. With binomials use FOIL exactly as you would for $(x+a)(x+b)$, then simplify each radical product.

Two patterns to memorize. *Square of a radical sum:* $(\sqrt{a}+b)^2 = a + 2b\sqrt{a} + b^2$. *Conjugate pair (difference of squares):* $(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b}) = a - b$. The middle terms cancel and the radicals vanish — this is why the conjugate strategy works for rationalizing.

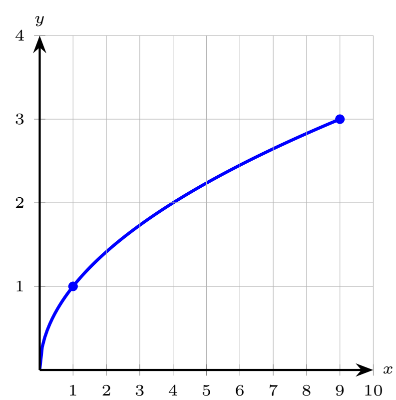
Always simplify the result. The radical you get from multiplying often still hides a perfect square: $\sqrt{6} \cdot \sqrt{8} = \sqrt{48} = 4\sqrt{3}$. Stop only when every radical's radicand is square-free.

Common slips. Adding radicands instead of multiplying ($\sqrt{a} \cdot \sqrt{b} \neq \sqrt{a+b}$). Forgetting that $(\sqrt{a})^2 = a$, not a^2. Skipping the final simplify after a successful FOIL.

PRACTICE

Multiply and simplify. Combine radicands under one radical, then pull out perfect squares.

1. First multiply $\sqrt{2} \cdot \sqrt{8}$ to get a single number N. Then use the graph of $f(x) = \sqrt{x}$ below to read $f(N)$. ________



2. Multiply: $\sqrt{2}(\sqrt{2}+1)$. ________

3. Multiply: $(2\sqrt{5})(3\sqrt{2})$. ________

4. FOIL: $(\sqrt{3}+2)(\sqrt{3}-5)$. ________

5. Expand: $(\sqrt{5}+1)^2$. ________

6. Multiply: $(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})$. ________

7. Expand: $(2\sqrt{6}-\sqrt{3})(\sqrt{6}+4\sqrt{3})$. ________

8. Mark TRUE or FALSE: $\sqrt{a} \cdot \sqrt{b} = \sqrt{a+b}$ for $a, b \geq 0$. ________

9. Multiply: $(4+\sqrt{11})(4-\sqrt{11})$. ________

10. Expand: $(3\sqrt{2}+\sqrt{5})(2\sqrt{2}-4\sqrt{5})$. ________

11. Multiply: $\sqrt{6} \cdot \sqrt{8}$. ________

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12. The table gives $g(x) = \sqrt{x}$ at several perfect squares. Use the pattern to find $g(81)$. ______

x	9	16	49	64
$g(x)$	3	4	7	8

13. Multiply: $\sqrt{3}(\sqrt{12} - \sqrt{27})$. ______
14. Expand: $(\sqrt{x} + 2)(\sqrt{x} - 3)$ for $x \geq 0$. ______
15. Multiply: $2\sqrt{6} \cdot 5\sqrt{15}$. ______
16. Expand: $(\sqrt{a} - \sqrt{b})^2$ for $a, b \geq 0$. ______
17. Multiply: $(\sqrt{10} + 3)(\sqrt{10} - 3)$. ______
18. Multiply: $\sqrt[3]{4} \cdot \sqrt[3]{6}$. ______
19. Mark TRUE or FALSE: $(\sqrt{a})^2 = a$ for $a \geq 0$. ______
20. Multiply: $(2 + \sqrt{3})(5 - \sqrt{3})$. ______

♦ Word Problems

21. A rectangle has length $\sqrt{18}$ cm and width $\sqrt{50}$ cm. Find the exact area in simplified radical form. ______
22. A square has side length $\sqrt{5} + 2$ inches. Find the exact area in simplified radical form. ______
23. Two segments have lengths $\sqrt{7} + \sqrt{3}$ and $\sqrt{7} - \sqrt{3}$. Find their product, in simplified form. ______
24. A pendulum's period (seconds) is roughly $T = 2\pi\sqrt{\frac{L}{g}}$. Two pendulums have lengths $L_1 = 2$ m and $L_2 = 8$ m. Find the exact ratio T_2/T_1 in simplified radical form. ______

Additional Practice

25. Simplify $\sqrt{72}$. ______
26. Simplify $\sqrt{45}$. ______
27. Simplify $\sqrt[3]{64}$. ______
28. Solve $\sqrt{x + 5} = 9$. ______
29. Solve $\sqrt{x} - 3 = 4$. ______
30. Domain of $y = \sqrt{x - 6}$. ______
31. Add $3\sqrt{5} + 2\sqrt{5}$. ______
32. Multiply $\sqrt{3} \cdot \sqrt{12}$. ______

Answer Keys

1. $\boxed{f(4)=2}$	13. $\boxed{-3}$
2. $\boxed{2+\sqrt{2}}$	14. $\boxed{x-\sqrt{x}-6}$
3. $\boxed{6\sqrt{10}}$	15. $\boxed{30\sqrt{10}}$
4. $\boxed{-7-3\sqrt{3}}$	16. $\boxed{a-2\sqrt{ab}+b}$
5. $\boxed{6+2\sqrt{5}}$	17. $\boxed{1}$
6. $\boxed{4}$	18. $\boxed{2\sqrt[3]{3}}$
7. $\boxed{21\sqrt{2}}$	19. TRUE
8. FALSE	20. $\boxed{7+3\sqrt{3}}$
9. $\boxed{5}$	21. $\boxed{30 \text{ cm}^2}$
10. $\boxed{-8-10\sqrt{10}}$	22. $\boxed{9+4\sqrt{5} \text{ in}^2}$
11. $\boxed{4\sqrt{3}}$	23. $\boxed{4}$
12. $\boxed{9}$	24. $\boxed{2}$

Additional Practice Answers

25. $\boxed{6\sqrt{2}}$	29. $\boxed{x=49}$
26. $\boxed{3\sqrt{5}}$	30. $\boxed{x\geq 6}$
27. $\boxed{4}$	31. $\boxed{5\sqrt{5}}$
28. $\boxed{x=76}$	32. $\boxed{6}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Combine radicands: $\sqrt{2}\cdot\sqrt{8}=\sqrt{16}=4$, so $N=4$. Trace the curve at $x=4$: the height is 2, i.e. $f(4)=\sqrt{4}=2$. (The marks $(1,1)$ and $(9,3)$ calibrate the square-root shape so the read is exact.)

2. Distribute the $\sqrt{2}$ to each term inside: $\sqrt{2}\cdot\sqrt{2}+\sqrt{2}\cdot 1$. The first product is $(\sqrt{2})^2=2$ (a square root squared returns its radicand), and the second stays $\sqrt{2}$, so the result is $2+\sqrt{2}$. These are unlike terms, so they don't combine further.

3. Multiply the coefficients together and the radicands together, separately: $2\cdot 3=6$ on the outside, and $\sqrt{5}\cdot\sqrt{2}=\sqrt{10}$ by the product rule. Combine to $6\sqrt{10}$, and since 10 is square-free it's already simplest.

4. F: $\sqrt{3}\cdot\sqrt{3}=3$. O: $\sqrt{3}\cdot(-5)=-5\sqrt{3}$. I: $2\cdot\sqrt{3}=2\sqrt{3}$. L: $2\cdot(-5)=-10$. Combine: $(3-10)+(-5+2)\sqrt{3}=-7-3\sqrt{3}$.

5. Use $(a+b)^2=a^2+2ab+b^2$ with $a=\sqrt{5}$ and $b=1$: $(\sqrt{5})^2+2(\sqrt{5})(1)+1^2=5+2\sqrt{5}+1=6+2\sqrt{5}$. Don't drop the middle term $2\sqrt{5}$ — that's the classic squaring-a-sum slip.

6. Conjugates: $(\sqrt{7})^2-(\sqrt{3})^2=7-3=4$. The middle terms cancel — that's the whole point of the conjugate pattern.

7. FOIL: $12+8\sqrt{18}-\sqrt{18}-12=7\sqrt{18}$. Then $\sqrt{18}=3\sqrt{2}$, so the answer is $21\sqrt{2}$. (Notice the rational parts cancelled — a sign you FOILed correctly.)

8. The product rule turns $\sqrt{a}\sqrt{b}$ into $\sqrt{ab}$, not $\sqrt{a+b}$. Counter: $\sqrt{4}\sqrt{9}=2\cdot 3=6$ and $\sqrt{4\cdot 9}=\sqrt{36}=6$. But $\sqrt{4+9}=\sqrt{13}\neq 6$.

9. These are conjugates, so use the difference-of-squares pattern $(a+b)(a-b)=a^2-b^2$ with $a=4$, $b=\sqrt{11}$: $4^2-(\sqrt{11})^2=16-11=5$. The middle terms cancel and the radical disappears, leaving a clean integer.

10. FOIL: $(3\sqrt{2})(2\sqrt{2})=12$; $(3\sqrt{2})(-4\sqrt{5})=-12\sqrt{10}$; $(\sqrt{5})(2\sqrt{2})=2\sqrt{10}$; $(\sqrt{5})(-4\sqrt{5})=-20$. Combine: $12-12\sqrt{10}+2\sqrt{10}-20=-8-10\sqrt{10}$.

11. Combine under one radical by the product rule: $\sqrt{6}\cdot\sqrt{8}=\sqrt{48}$. Then simplify — the largest perfect square in 48 is 16, so $\sqrt{48}=\sqrt{16}\sqrt{3}=4\sqrt{3}$. Multiplying first often produces a radicand that still needs simplifying.

12. Each x is a perfect square sent to its root: $g(64)=8$ since $8^2=64$. Because $81=9^2$, the pattern gives $g(81)=9$. (The value at $x=81$ is left out on purpose — continue the pattern.)

13. Distribute the $\sqrt{3}$ across both terms: $\sqrt{3}\cdot\sqrt{12}=\sqrt{36}=6$ and $\sqrt{3}\cdot\sqrt{27}=\sqrt{81}=9$. Then subtract: $6-9=-3$. Each product happened to land on a perfect square, so the radicals vanished entirely.

14. FOIL just as you would with binomials, treating $\sqrt{x}$ as the variable: F $=\sqrt{x}\cdot\sqrt{x}=x$, O $=-3\sqrt{x}$, I $=2\sqrt{x}$, L $=-6$. Combine the like middle terms $-3\sqrt{x}+2\sqrt{x}=-\sqrt{x}$, giving $x-\sqrt{x}-6$.

15. One steady path is: Coefficients: $2\cdot 5=10$. Radicals: $\sqrt{6}\sqrt{15}=\sqrt{90}=3\sqrt{10}$. Combine: $10\cdot 3\sqrt{10}=30\sqrt{10}$. That gives a quick check on the answer.

16. Apply $(p-q)^2=p^2-2pq+q^2$ with $p=\sqrt{a}$ and $q=\sqrt{b}$: $(\sqrt{a})^2-2\sqrt{a}\sqrt{b}+(\sqrt{b})^2=a-2\sqrt{ab}+b$, using $\sqrt{a}\sqrt{b}=\sqrt{ab}$. The middle term is the piece students most often forget.

17. A conjugate pair, so use difference of squares with $a=\sqrt{10}$, $b=3$: $(\sqrt{10})^2-3^2=10-9=1$. The radical cancels out cleanly.

18. Keep the rule visible: $\sqrt[3]{4\cdot 6}=\sqrt[3]{24}$. Now $24=8\cdot 3$, so $\sqrt[3]{24}=2\sqrt[3]{3}$. (Same-index radicals combine; different indices need rational exponents.) That gives a quick check on the answer.

19. Squaring a principal square root returns the radicand — the radical and the square are inverses on non-negative inputs.

20. FOIL each pair: F $=2\cdot 5=10$, O $=2\cdot(-\sqrt{3})=-2\sqrt{3}$, I $=\sqrt{3}\cdot 5=5\sqrt{3}$, L $=\sqrt{3}\cdot(-\sqrt{3})=-3$. Combine the rational parts $10-3=7$ and the radical parts $-2\sqrt{3}+5\sqrt{3}=3\sqrt{3}$, giving $7+3\sqrt{3}$.

21. Area $=\sqrt{18}\cdot\sqrt{50}=\sqrt{900}=30$ cm^2. (The radicands multiplied to a perfect square, so the area is a clean integer. When you see two radicals whose radicands share prime factors, expect that.)

22. Area $=(\sqrt{5}+2)^2=5+4\sqrt{5}+4=9+4\sqrt{5}$ in^2. Use $(a+b)^2=a^2+2ab+b^2$ with $a=\sqrt{5}$, $b=2$ — don't fall for the lazy $(\sqrt{5})^2+2^2$ trap that drops the middle term.

23. Conjugate pair: $(\sqrt{7})^2-(\sqrt{3})^2=7-3=4$. The product is a clean integer because the radicals cancel. (Whenever you spot a conjugate pair, the difference-of-squares pattern guarantees the radicals will vanish.)

24. Start with the key idea: $\dfrac{T_2}{T_1}=\dfrac{2\pi\sqrt{L_2/g}}{2\pi\sqrt{L_1/g}}=\sqrt{\dfrac{L_2}{L_1}}=\sqrt{\dfrac{8}{2}}=\sqrt{4}=2$. The constants 2π and g cancel, leaving a clean ratio. Quadrupling the length doubles the period — a classic physics-class fact that falls out of one radical step. That gives a quick check on the answer.

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