Maximum and Minimum Points

Name: ______________	Date: ______________	Score: ______ / 24

Quick Review

Every quadratic $f(x) = ax^2 + bx + c$ has exactly one extreme value at its vertex. The sign of a tells you whether it's a maximum or a minimum.

Direction. $a > 0$: parabola opens up, vertex is the *minimum*. $a < 0$: opens down, vertex is the *maximum*. Cubics, exponentials, and other curves can have local extrema too, but quadratics always have exactly one global extremum.

Where is the vertex? For $f(x) = ax^2 + bx + c$, the vertex's x-coordinate is $x = -\frac{b}{2a}$. Plug back into f to get the y-coordinate (the extreme value itself).

Vertex form. If $f(x) = a(x - h)^2 + k$, the vertex is (h, k) directly – no formula needed. The squared term contributes ≥ 0 for $a > 0$ (so k is the minimum), or ≤ 0 for $a < 0$ (so k is the maximum).

Extreme value vs. extreme point. Some problems ask for the x-value where the extremum occurs (the location); some ask for the actual y-value (the extreme value). Read the prompt carefully. "What price gives the max profit?" wants x; "what is the maximum profit?" wants $f(x)$.

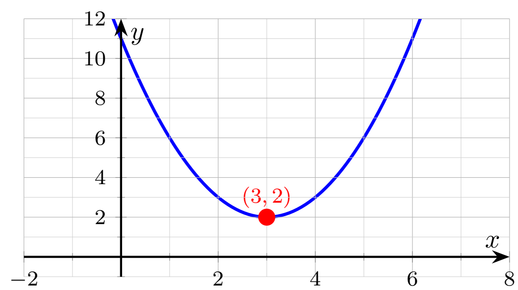
Reading from data. A small table or a graph can reveal a local extremum without algebra. Values decrease, hit a low point, then increase – that's a local min. Reverse for a local max.

Common slips. Reporting the x-coordinate when the question wants the y-coordinate (or vice versa). Forgetting the negative sign in $-\frac{b}{2a}$. Treating a maximum like a minimum because the formula gives the same vertex regardless of a's sign.

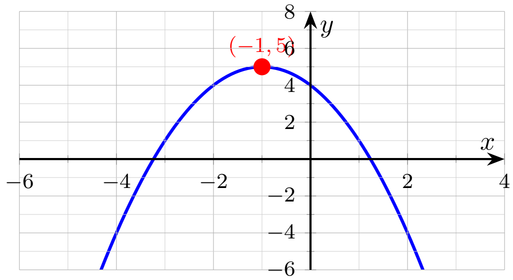
PRACTICE

For each function, find the vertex, classify it as a max or min, and give the requested coordinate. Read the prompt for whether it wants the location or the value.

1. $f(x) = (x - 3)^2 + 2$ has a (max/min) of $?$ at $x = ?$. ________



2. $g(x) = -(x+1)^2 + 5$ has a (max/min) of ? at $x = ?$. ________



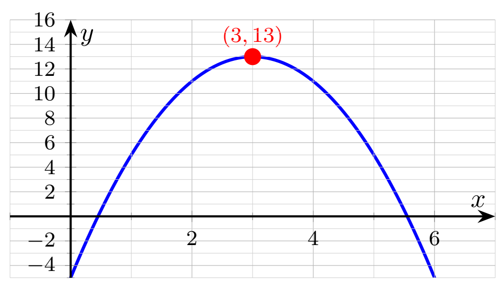
3. Find the minimum value of $f(x) = x^2 - 6x + 11$. ________

4. The table below shows $f(x)$ for $x = -1, 0, 1, 2, 3$. Identify the local minimum point. ________

x	-1	0	1	2	3
$f(x)$	5	1	0	1	5

5. For profit $P(x) = -x^2 + 8x - 7$, find the price x that maximizes profit. ________

6. Find the maximum value of $f(x) = -2x^2 + 12x - 5$. ________



7. A projectile's height is $h(t) = -16t^2 + 48t + 5$ ft. Find the maximum height. ________

8. Mark TRUE or FALSE: A parabola $f(x) = ax^2 + bx + c$ with $a > 0$ has a maximum at its vertex. ________

9. For $h(x) = -0.5(x-6)^2 + 18$, find the maximum value of h. ________

10. Find the minimum value of $f(x) = 2x^2 - 8x + 9$. ________

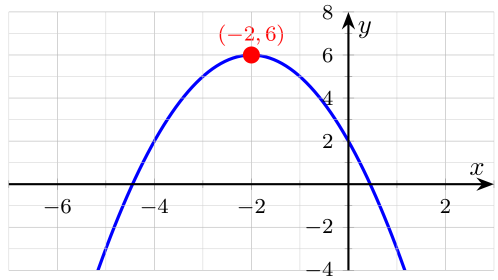
11. Find the vertex of $f(x) = x^2 + 4x - 1$ and classify. ________

12. Find the vertex of $g(x) = -3x^2 + 6x + 2$ and classify. ________

13. For $f(x) = (x-5)^2 + 7$, what is the minimum value? ________

14. For $f(x) = -2(x+3)^2 - 4$, what is the maximum value? ________

15. A function $f(x) = -(x+2)^2 + 6$ is graphed below. Identify the absolute maximum point. ________



16. For $f(x) = x^2 - 2x + 5$, find the minimum value. ________

17. For $g(x) = -x^2 + 4x$, find the maximum value. ________

18. Mark TRUE or FALSE: The maximum of $f(x) = x^2 + 1$ is at $x = 0$. ________

19. The height of a rocket is $f(t) = -5(t-3)^2 + 25$ m. Find its maximum height. ________

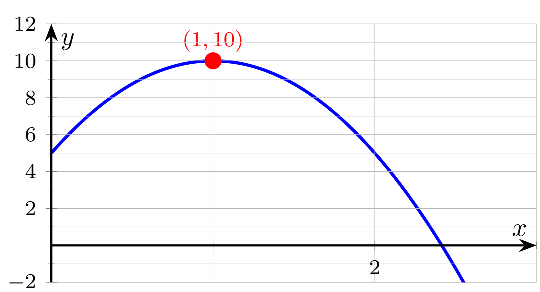
20. For $f(x) = 3x^2 - 12x + 1$, find the minimum value. ________

♦ Word Problems

21. A rancher has 100 ft of fencing to enclose a rectangular pen against a barn wall (so only three sides need fencing). Express the area as a function of one variable, find the dimensions that maximize the area, and state the maximum area. ____

22. A company's revenue from selling a product at price p dollars is $R(p) = -2p^2 + 80p$ thousand dollars. Find the price that maximizes revenue and state the maximum revenue. ____

23. A ball is thrown from the top of a 5-m building. Its height in meters is $h(t) = -5(t-1)^2 + 10$, where t is in seconds. Find the maximum height and the time at which it occurs. ____



24. From a table, a function's outputs are: ________

x	1	2	3	4	5
$f(x)$	-2	1	2	1	-2

Identify the local extreme point and classify it.

Answer Keys

1. min 2 at $x = 3$
2. max 5 at $x = -1$
3. 2 at $x = 3$
4. $(1, 0)$
5. $x = 4$
6. 13
7. 41 ft
8. FALSE
9. 18
10. 1
11. $(-2, -5)$ min
12. $(1, 5)$ max
13. 7
14. -4
15. $(-2, 6)$
16. 4
17. 4
18. FALSE
19. 25 m
20. -11
21. 25 ft $\times$ 50 ft $= 1250$ ft^2
22. $p = \$20,\ R_{\max} = \$800$ thousand
23. max height 10 m at $t = 1$ s
24. $(3, 2)$ local maximum

Step-by-Step Explanations

1. In vertex form $a(x-h)^2+k$, the vertex is read off directly as $(h,k) = (3,2)$. Since $a = 1 > 0$ the parabola opens up, so that vertex is the lowest point – a minimum value of 2, reached at $x = 3$.

2. Rewrite $-(x+1)^2+5$ as $a(x-h)^2+k$ with $h = -1$, $k = 5$ (note $x+1 = x-(-1)$). The vertex is $(-1,5)$, and $a = -1 < 0$ means the parabola opens down, so the vertex is the highest point – a maximum of 5 at $x = -1$.

3. One steady path is: $x_v = -\dfrac{-6}{2(1)} = 3$. $f(3) = 9 - 18 + 11 = 2$. Since $a = 1 > 0$, it's a minimum. That gives a quick check on the answer.

4. Values decrease until $x = 1$, then increase. The valley is at $(1,0)$ – a local minimum.

5. A careful way to see it: $x_v = -\dfrac{8}{2(-1)} = 4$. Since $a = -1 < 0$, the vertex is a maximum – the profit-maximizing price is \$4. (The maximum profit itself is $P(4) = -16 + 32 - 7 = 9$ thousand dollars.) That gives a quick check on the answer.

6. Keep the rule visible: $x_v = -\dfrac{12}{2(-2)} = 3$. $f(3) = -18 + 36 - 5 = 13$. Max because $a < 0$. The plot's vertex is exactly at $(3,13)$. That gives a quick check on the answer.

7. One steady path is: $t_v = -\dfrac{48}{2(-16)} = 1.5$ s. $h(1.5) = -36 + 72 + 5 = 41$ ft. (The launch height was 5 ft; the projectile peaks 36 ft higher than that at $t = 1.5$.) That gives a quick check on the answer.

8. Start with the key idea: $a > 0$ opens up – the vertex is a minimum. Maximum requires $a < 0$. That gives a quick check on the answer.

9. A careful way to see it: Vertex form: vertex is $(6,18)$ with $a = -0.5 < 0$, so the max is 18 (at $x = 6$). That gives a quick check on the answer.

10. Keep the rule visible: $x_v = -\dfrac{-8}{2(2)} = 2$. $f(2) = 8 - 16 + 9 = 1$. Min because $a = 2 > 0$. That gives a quick check on the answer.

11. One steady path is: $x_v = -\dfrac{4}{2} = -2$. $f(-2) = 4 - 8 - 1 = -5$. $a > 0$: min. This is the part to check before moving on, because it keeps the answer tied to the original question.

12. Start with the key idea: $x_v = -\dfrac{6}{2(-3)} = 1$. $g(1) = -3 + 6 + 2 = 5$. $a < 0$: max. This is the part to check before moving on, because it keeps the answer tied to the original question.

13. In vertex form, the squared part $(x-5)^2$ is never negative, so the smallest the function gets is when it's zero (at $x = 5$). That leaves just the constant $k = 7$. Because $a = 1 > 0$ the vertex is a minimum, so the minimum value is 7.

14. The vertex is $(-3,-4)$. Since $a = -2 < 0$ the parabola opens down, so the vertex is the highest point: the maximum value is $k = -4$ (reached at $x = -3$). A negative maximum is fine – it just means the whole curve sits at or below -4.

15. Vertex form gives $(-2,6)$ as the vertex; $a = -1 < 0$ means it's the absolute maximum.

16. Start with the key idea: $x_v = -\dfrac{-2}{2} = 1$. $f(1) = 1 - 2 + 5 = 4$. Min. This is the part to check before moving on, because it keeps the answer tied to the original question.

17. A careful way to see it: $x_v = -\dfrac{4}{2(-1)} = 2$. $g(2) = -4 + 8 = 4$. Max. This is the part to check before moving on, because it keeps the answer tied to the original question.

18. Keep the rule visible: $a = 1 > 0$, so the vertex $(0,1)$ is a *minimum*, not a maximum. (The parabola has no maximum – it grows without bound.) That gives a quick check on the answer.

19. One steady path is: Vertex form: max is $k = 25$ at $t = 3$ seconds. This is the part to check before moving on, because it keeps the answer tied to the original question.

20. Start with the key idea: $x_v = -\dfrac{-12}{6} = 2$. $f(2) = 12 - 24 + 1 = -11$. Min. This is the part to check before moving on, because it keeps the answer tied to the original question.

21. Let x be the depth (perpendicular to the barn) in ft. Then the side parallel to the barn is $100 - 2x$ ft. Area: $A(x) = x(100 - 2x) = -2x^2 + 100x$. Vertex: $x_v = -\dfrac{100}{2(-2)} = 25$ ft. $A(25) = 25 \cdot 50 = 1250$ ft^2. Dimensions: 25 ft by 50 ft, max area 1250 ft^2. (Sanity: at $x = 20$ ft, area $= 20 \cdot 60 = 1200$; at $x = 30$ ft, area $= 30 \cdot 40 = 1200$. The vertex really is the peak.)

22. Vertex: $p_v = -\dfrac{80}{2(-2)} = 20$. $R(20) = -2(400) + 80(20) = -800 + 1600 = 800$ thousand. Maximum because $a < 0$. The price of \$20 yields \$800,000 in revenue. (Sanity at \$15: $-2(225) + 1200 = -450 + 1200 = \$750{,}000$; at \$25: $-2(625) + 2000 = -1250 + 2000 = \$750{,}000$. Symmetric about \$20, consistent with the vertex.)

23. Vertex form: vertex $(1,10)$, $a = -5 < 0$, so the max height is 10 m at $t = 1$ s. (The ball started at height 5 m at $t = 0$, rose for 1 second to its peak of 10 m, then fell.)

24. Values rise from -2 to 1 to 2, then fall back to 1 and -2. The peak is at $(3,2)$, a local maximum. The symmetric drop on either side – 1 at both $x = 2$ and $x = 4$, -2 at both $x = 1$ and $x = 5$ – is the signature of a quadratic with vertex at $x = 3$. (You could even reconstruct: $f(x) = -(x-3)^2 + 2$, which checks out: $f(1) = -4 + 2 = -2$, $f(2) = -1 + 2 = 1$, $f(3) = 2$, $f(4) = 1$, $f(5) = -2$.)

Scan Me