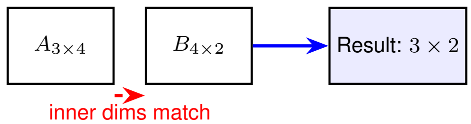
Matrix Multiplication

Name: ____________	Date: ____________	Score: ______ / 28

Quick Review

Matrix multiplication is *not* entrywise. To compute AB, you take **row of** A **dotted with column of** B for each entry of the result. So entry (i, j) of AB is $\sum_k a_{i,k}\, b_{k,j}$ — multiply matching positions along the row and column, then add.

Dimension rule. If A is $m \times n$ and B is $p \times q$, the product AB is defined only when $n = p$ — the *inner* dimensions match. The result is $m \times q$, the *outer* dimensions. Check this first every single time; multiplying matrices of incompatible shapes is the most common error in the chapter.



Non-commutative. $AB \neq BA$ in general. Sometimes only one of the two products is even defined. So order matters: writing BA when the problem says AB changes the answer, often dramatically.

Scalar multiplication is the simple cousin: a number times a matrix multiplies every entry. The **identity matrix** I_n acts like 1 for multiplication: $AI = IA = A$ (when dimensions are compatible). Matrix multiplication *is* associative ($(AB)C = A(BC)$) and distributes over addition ($A(B + C) = AB + AC$) — those two facts let you rearrange and simplify even when you can't commute.

PRACTICE

Compute each product or scalar multiple. Write "undefined" if dimensions don't match.

1. $3\begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$ ________

2. Find the dimensions of AB when A is 3×4 and B is 4×2. ________

3. Matrix M records units made by two machines for two products (rows are machines, columns are products); matrix N records hours each product needs at two plants. Compute the product MN where $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $N = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$. ________

Matrix	Row	Col 1	Col 2
M	1	1	2
M	2	3	4
N	1	5	6
N	2	7	8

4. $\begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ ________

5. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ ________

6. AB where $A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 \\ -2 & 5 \end{bmatrix}$ ________

7. Compute AB by finishing the row-by-column diagram below.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1\cdot2+2\cdot1 & | & 1\cdot0+2\cdot3 \\ \hline 3\cdot2+4\cdot1 & | & 3\cdot0+4\cdot3 \end{bmatrix}$$

8. BA where $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

9. AB where $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

10. A coefficient matrix C (rows are recipes, columns are three ingredients) multiplies a column D of ingredient amounts. Compute CD where $C = \begin{bmatrix} 2 & 0 & -1 \\ 4 & 3 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 5 \\ -2 \\ 6 \end{bmatrix}$.

Recipe	Ing. 1	Ing. 2	Ing. 3
1	2	0	−1
2	4	3	1

11. $\begin{bmatrix} 3 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

12. $\begin{bmatrix} 4 \\ 2 \end{bmatrix} \begin{bmatrix} 3 & 5 \end{bmatrix}$

13. $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 5 \end{bmatrix}$

14. $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

15. AI where $A = \begin{bmatrix} 4 & 7 \\ -2 & 3 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

16. A club sells two items. The quantity row vector is $Q = \begin{bmatrix} 12 & 8 \end{bmatrix}$ and the price column vector is $P = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ (in dollars). Compute the product QP.

Item	Quantity	Price ($)
1	12	3
2	8	5

17. A^2 where $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

18. Compare $(A+B)C$ and $AC+BC$ for any compatible matrices.

19. $5A - 3A$ for any matrix A

20. $\begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

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Word Problems

21. A fundraiser sells two snack packs at prices $P = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ dollars. A class sells the quantity row vector $Q = \begin{bmatrix} 12 & 8 \end{bmatrix}$ (12 of pack 1, 8 of pack 2). What is QP, and what does it represent? ______

22. A bakery's daily output is the column vector $D = \begin{bmatrix} 40 \\ 30 \\ 25 \end{bmatrix}$ (croissants, muffins, scones). The price vector per item (row) is $P = \begin{bmatrix} 3.5 & 2.5 & 4 \end{bmatrix}$ dollars. Find PD — the day's total revenue — and interpret. ______

23. A linear transformation matrix $T = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ acts on the point $\vec{v} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ by $T\vec{v}$. Find $T\vec{v}$ and describe what T does geometrically. ______

24. Two matrices represent factory output and unit cost. $F = \begin{bmatrix} 100 & 80 \\ 60 & 90 \end{bmatrix}$ gives units produced (rows: factories; columns: products A and B). The cost-per-unit column vector is $C = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ dollars. Compute FC and interpret each entry. ______

Additional Practice

25. State the dimensions of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$. ________

26. Add $\begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 5 & 6 \end{bmatrix}$. ________

27. Subtract $\begin{bmatrix} 7 & 2 \\ 1 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ -2 & 1 \end{bmatrix}$. ________

28. Find $\det \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$. ________

Answer Keys

1. $\begin{bmatrix} 6 & -3 \\ 0 & 12 \end{bmatrix}$
2. 3×2
3. $\begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$
4. $\begin{bmatrix} 12 \\ 6 \\ 18 \end{bmatrix}$
5. $\begin{bmatrix} 4 & 5 \\ 10 & 11 \end{bmatrix}$
6. $\begin{bmatrix} 4 & 3 \\ 3 & 12 \end{bmatrix}$
7. $\begin{bmatrix} 4 & 6 \\ 10 & 12 \end{bmatrix}$
8. $\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$
9. $\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$
10. $\begin{bmatrix} 4 \\ 20 \end{bmatrix}$
11. $[22]$
12. $\begin{bmatrix} 12 & 20 \\ 6 & 10 \end{bmatrix}$
13. undefined
14. undefined
15. $\begin{bmatrix} 4 & 7 \\ -2 & 3 \end{bmatrix}$
16. $[76]$
17. $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$
18. equal (distributive)
19. $2A$
20. $\begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}$
21. $76 total revenue
22. $315
23. $\begin{bmatrix} 8 \\ 15 \end{bmatrix}$, stretch by 2 in x and 3 in y
24. $FC = \begin{bmatrix} 1140 \\ 1020 \end{bmatrix}$

Additional Practice Answers

25. 2×3
26. $\begin{bmatrix} 4 & 3 \\ 7 & 6 \end{bmatrix}$
27. $\begin{bmatrix} 4 & -2 \\ 3 & 4 \end{bmatrix}$
28. 2

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. This is scalar (not matrix) multiplication: the 3 distributes to every entry. $3(2) = 6, 3(-1) = -3, 3(0) = 0, 3(4) = 12$. Positions stay put; only the values scale. Don't confuse this with a matrix product — a lone number always just multiplies each entry.

2. Inner dimensions (4 and 4) match, so the product is defined. The result picks up the outer dimensions: 3 (from A) by 2 (from B). Always check inner-match first.

3. Each entry is a row-dot-column. $(1,1)$: $(1)(5) + (2)(7) = 19$. $(1,2)$: $(1)(6) + (2)(8) = 22$. $(2,1)$: $(3)(5) + (4)(7) = 43$. $(2,2)$: $(3)(6) + (4)(8) = 50$. Stack: $[19, 22; 43, 50]$.

4. Check shapes first: 3×2 times 2×1 has matching inner dims, so AB is 3×1. Each entry dots a row of A with the single column of B: row 1 gives $(2)(5) + (1)(2) = 12$, row 2 gives $(0)(5) + (3)(2) = 6$, row 3 gives $(4)(5) + (-1)(2) = 18$. The result is a column vector.

5. Inner dims match ($3 = 3$), so 2×3 times 3×2 gives 2×2. Each entry dots a row of the left with a column of the right: $(1,1)$: $(1)(1) + (2)(0) + (3)(1) = 4$; $(1,2)$: $(1)(0) + (2)(1) + (3)(1) = 5$; $(2,1)$: $(4)(1) + (5)(0) + (6)(1) = 10$; $(2,2)$: $(4)(0) + (5)(1) + (6)(1) = 11$.

6. Keep the rule visible: $(1,1)$: $(2)(1) + (-1)(-2) = 2 + 2 = 4$. $(1,2)$: $(2)(4) + (-1)(5) = 8 - 5 = 3$. $(2,1)$: $(3)(1) + (0)(-2) = 3$. $(2,2)$: $(3)(4) + (0)(5) = 12$. Two double-negatives in $(1,1)$ make the $+2$. That gives a quick check on the answer.

7. Each entry is row-of-A dotted with column-of-B. $(1,1)$: row 1 $(1,2)$ with column 1 $(2,1)$ gives $(1)(2) + (2)(1) = 4$. $(1,2)$: $(1)(0) + (2)(3) = 6$. $(2,1)$: $(3)(2) + (4)(1) = 10$. $(2,2)$: $(3)(0) + (4)(3) = 12$. Multiply along the row and down the column, then add.

8. Start with the key idea: BA: row 1 of B is $(0,1)$, so $(0 \cdot 1 + 1 \cdot 0, 0 \cdot 2 + 1 \cdot 1) = (0,1)$. Row 2 of B is $(1,0)$, so $(1 \cdot 1 + 0 \cdot 0, 1 \cdot 2 + 0 \cdot 1) = (1,2)$. (Compute AB on the same matrices for comparison — the products differ, showing matrix mult isn't commutative.) That gives a quick check on the answer.

9. A careful way to see it: AB: row 1 of A is $(1,2)$, so $(1 \cdot 0 + 2 \cdot 1, 1 \cdot 1 + 2 \cdot 0) = (2,1)$. Row 2 is $(0,1)$, giving $(0 \cdot 0 + 1 \cdot 1, 0 \cdot 1 + 1 \cdot 0) = (1,0)$. Different from BA above — a clean confirmation of non-commutativity. That gives a quick check on the answer.

10. Keep the rule visible: C is 2×3 and D is 3×1, so CD is 2×1. Top: $(2)(5) + (0)(-2) + (-1)(6) = 10 + 0 - 6 = 4$. Bottom: $(4)(5) + (3)(-2) + (1)(6) = 20 - 6 + 6 = 20$. That gives a quick check on the answer.

11. One steady path is: 1×2 times 2×1 gives 1×1 — a single number wrapped in brackets. $(3)(4) + (5)(2) = 12 + 10 = 22$. (This is the dot product of two vectors, presented as a matrix product.) That gives a quick check on the answer.

12. Now reverse: 2×1 times 1×2 gives 2×2. $(4)(3) = 12$, $(4)(5) = 20$, $(2)(3) = 6$, $(2)(5) = 10$. Same vectors, totally different product — order matters.

13. This is addition, and addition needs identical shapes. A 1×3 vector and a 1×2 vector have no way to pair up every position, so the sum is undefined. The right move is to check dimensions first and refuse to compute when they clash.

14. Keep the rule visible: 2×2 times 1×3: the inner dimensions are 2 and 1, which don't match. So AB is undefined. (For this row vector to be multiplied by a 2×2 matrix, it would need to sit on the *left and* have 2 columns — neither holds.) That gives a quick check on the answer.

15. One steady path is: I is the identity, so AI returns A unchanged — it's the matrix version of multiplying by 1. Confirm with the top-left entry: row 1 of A dotted with column 1 of I is $(4)(1) + (7)(0) = 4$, exactly A's top-left. Every other entry lands the same way. That gives a quick check on the answer.

16. Start with the key idea: 1×2 times 2×1 gives 1×1: $(12)(3) + (8)(5) = 36 + 40 = 76$. In context, this is the total revenue from Q items at prices P. That gives a quick check on the answer.

17. A careful way to see it: $A^2 = A \cdot A$. Entries: $(1)(1) + (1)(0) = 1$, $(1)(1) + (1)(1) = 2$, $(0)(1) + (1)(0) = 0$, $(0)(1) + (1)(1) = 1$. The 1 in position $(1,2)$ kept incrementing — a pattern that makes A^n predictable later. That gives a quick check on the answer.

18. Matrix multiplication distributes over addition on the right: $(A + B)C = AC + BC$. This identity makes simplifying matrix expressions feel a lot like simplifying polynomial expressions — as long as you don't try to commute.

19. Scalars combine on a single matrix: $5A - 3A = (5 - 3)A = 2A$. You're not multiplying matrices here, just scalars, so all the usual rules apply.

20. Multiplying by the 2×2 identity on either side returns the original matrix — the identity is the matrix analog of the number 1. Verify with one entry:

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$(2)(1) + (-3)(0) = 2$, matching.
21. A careful way to see it: Q is 1×2 and P is 2×1, so QP is 1×1. Compute: $(12)(3) + (8)(5) = 36 + 40 = 76$. The single entry 76 represents the total revenue — each quantity is paired with its price, then summed, exactly what the dot product does. That gives a quick check on the answer.
22. Keep the rule visible: P is 1×3 and D is 3×1, so PD is 1×1. Compute: $(3.5)(40)+(2.5)(30)+(4)(25) = 140+75+100 = 315$. So the day's revenue is $\$315$. Matrix multiplication bundles *multiply-then-sum* into one operation — ideal for cost/revenue work. That gives a quick check on the answer.
23. One steady path is: $T\vec{v}$: $(2)(4) + (0)(5) = 8$, $(0)(4) + (3)(5) = 15$. So $\vec{v}$ maps to $(8, 15)$. Because T has only diagonal entries, it stretches the x-coordinate by 2 and the y-coordinate by 3 independently — a non-uniform scale. (Diagonal matrices always act this way.) That gives a quick check on the answer.
24. Start with the key idea: F is 2×2 and C is 2×1, so FC is 2×1. Row 1: $(100)(5) + (80)(8) = 500 + 640 = 1140$. Row 2: $(60)(5) + (90)(8) = 300 + 720 = 1020$. So factory 1's total cost is $\$1140$ and factory 2's is $\$1020$. Each row-column dot product collapses *units* $\times$ *price* into a single total for that factory. That gives a quick check on the answer.

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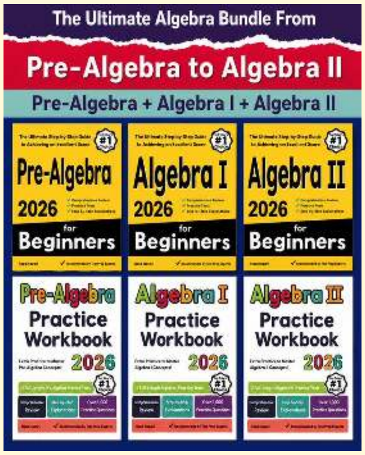

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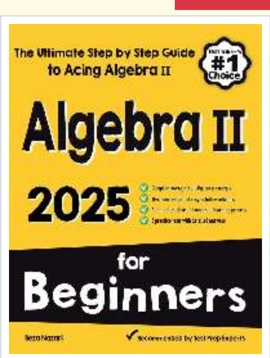

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