Long-Term Financial Decisions

Name: ____________ Date: ____________ Score: ______ / 35

Quick Review

Long-term planning is where compound growth, present value, and annuities all show up together. A handful of formulas covers most of it.

Future value (lump sum): $FV = P(1+r)^t$ for annual compounding. The growth factor $(1+r)$ stacks itself t times – that's why later years add more dollars than earlier ones.

Present value (lump sum): $PV = \frac{FV}{(1+r)^t}$. This discounts a future amount back to today's value. If $\$1{,}000$ is promised ten years from now, today's equivalent is smaller – how much smaller depends on the rate.

Rule of 72**.** A back-of-the-envelope estimate for doubling time: $t \approx \frac{72}{r\%}$. At 6%, money doubles in about 12 years. It's an approximation, not exact – but close enough for ballpark planning between 4% and 12%.

Ordinary annuity future value. For equal deposits PMT made at the end of each period, with periodic rate i over n periods, $FV = PMT \cdot \frac{(1+i)^n - 1}{i}$. The pattern is a geometric series in disguise – each deposit compounds for a different number of years.

Loan payment. The flip side: $PMT = P \cdot \frac{i}{1-(1+i)^{-n}}$. Same algebra, solved for PMT instead of FV.

Real return vs. nominal return. Subtract inflation from the nominal rate to get the real return. A 1.5% savings account in a 2.5% inflation world loses purchasing power – the real return is -1%.

Common slips. Plugging the rate as a percent instead of a decimal. Confusing present value with future value (which way is time moving?). Treating the Rule of 72 as exact. Ignoring inflation when comparing investments over decades.

PRACTICE

Apply future value, present value, the Rule of 72, or the annuity formula as needed.

1. $FV formula for annual compounding$ ________
2. $PV formula for annual compounding$ ________
3. The table lists three savings plans. Using the Rule of 72, estimate the doubling time for the 6% plan. ________

Plan	Deposit	Annual Rate
Plan 1	$\$2{,}000$	4%
Plan 2	$\$2{,}000$	6%
Plan 3	$\$2{,}000$	8%

4. Rule of 72 at $r = 8\%$ ________
5. FV: $P = \$5{,}000, r = 2.5\%, t = 12$, annual. ________
6. PV: $FV = \$50{,}000, r = 3\%, t = 15$, annual. ________
7. Annuity FV: $PMT = \$6{,}000, r = 5\%, n = 30$. Find FV. ________
8. Inflation runs at 2.5% per year. Using the table, find the real (inflation-adjusted) return on the bond fund. ________

Account	Nominal Rate	Inflation
Savings	1.5%	2.5%
Bond fund	5%	2.5%
Stock fund	7%	2.5%

9. Real return: nominal 1.5%, inflation 2.5% ________
10. Annuity: PMT formula meaning i ________
11. $Loan PMT formula$ ________

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12. This table shows a $\$10{,}000$ investment growing at 4% compounded annually. Use it to find the balance after year 20. ________

$Year$	0	5	10	15
$Balance$	$\$10{,}000$	$\$12{,}167$	$\$14{,}802$	$\$18{,}009$

13. PV: $FV = \$100{,}000, r = 6\%, t = 25$, annual. ________
14. Rule of 72 at $r = 4\%$ vs. $r = 12\%$ ________
15. FV: $P = \$2{,}500, r = 8\%, t = 10$, annual. ________
16. Annuity FV: $PMT = \$2{,}000, r = 6\%, n = 10$ ________
17. PV: $FV = \$10{,}000, r = 4\%, t = 5$ ________
18. FV at continuous: $P = \$3{,}000, r = 5\%, t = 10$ ________
19. Annuity: triple PMT, FV changes by ... ________
20. Is the Rule of 72 exact? ________

♦ Word Problems

21. A college student inherits \$$5{,}000$ and parks it in a savings account earning 2.5% per year, compounded annually. About how much is in the account after 12 years? ____
22. A 35-year-old worker plans to retire at age 65. She deposits \$$6{,}000$ at the end of each year into an investment account that earns 5% annually. About how much will be in the fund at retirement? ____
23. A financial advisor presents two options for a long-term investment: a guaranteed savings account at 1.5% annual return, or a stock fund expected to return 7% annually. Inflation is forecast at 2.5% per year. What is the real (inflation-adjusted) return for each option, and which option preserves purchasing power? ____
24. An investor wants to estimate how long it will take \$$10{,}000$ to double at 9% annual interest. First use the Rule of 72, then compute the exact doubling time using $t = \dfrac{\ln 2}{\ln(1+r)}$. How close is the approximation? ____

Additional Practice

25. Simple interest on $\$800$ at 5% for 3 years. ________
26. Total after simple interest: $\$500$ at 4% for 2 years. ________
27. Compound amount: $\$1000(1.06)^2$. ________
28. Monthly payment total: $\$250$ for 48 months. ________
29. Markup: cost $\$40$, selling price $\$55$. ________
30. Discount: 20% off $\$75$. ________
31. Sale price after 20% off $\$75$. ________
32. Percent increase from 80 to 100. ________
33. Find tax at 7% on $\$60$. ________
34. Total with 7% tax on $\$60$. ________
35. Continuous growth: principal in $A = Pe^{rt}$. ________

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Answer Keys

1. $FV = P(1+r)^t$
2. $PV = \dfrac{FV}{(1+r)^t}$
3. $\approx$ 12 years
4. $\approx$ 9 years
5. $\approx \$6{,}724$
6. $\approx \$32{,}093$
7. $\approx \$398{,}633$
8. 2.5%
9. -1%
10. periodic rate
11. $PMT = P \cdot \dfrac{i}{1-(1+i)^{-n}}$
12. $\approx \$21{,}911$
13. $\approx \$23{,}300$
14. 18 vs. 6 years
15. $\approx \$5{,}397$
16. $\approx \$26{,}362$
17. $\approx \$8{,}219$
18. $\approx \$4{,}946$
19. triples
20. No – it's an approximation
21. $\approx \$6{,}724$
22. $\approx \$398{,}633$
23. Savings: -1%; Stocks: 4.5%
24. Rule of 72: 8 yr; exact: ≈ 8.04 yr

Additional Practice Answers

25. $\$120$
26. $\$540$
27. $\$1123.60$
28. $\$12000$
29. $\$15$
30. $\$15$
31. $\$60$
32. 25%
33. $\$4.20$
34. $\$64.20$
35. P

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. In $FV = P(1+r)^t$, the principal P gets multiplied by the growth factor $(1+r)$ once per year, and the exponent t counts how many years that happens. More years means the factor stacks more times – that's the engine of compounding.

2. This is the future-value formula solved for P. Dividing FV by $(1+r)^t$ undoes the compounding, moving a future dollar amount *backward* to its value today. Because you're dividing by a number bigger than 1, present value is always smaller than the future amount.

3. The Rule of 72 estimates doubling time as $t \approx 72/r\%$, using the rate as a whole number, not a decimal. Read Plan 2's rate from the table (6%): $72/6 = 12$ years. For comparison, Plan 1 at 4% takes $72/4 = 18$ years and Plan 3 at 8% takes $72/8 = 9$ – a higher rate doubles your money faster.

4. Apply $t \approx 72/r\%$ with the rate as a whole number: $72/8 = 9$ years. Plug in 8, not 0.08 – the Rule of 72 uses the percent value directly. A higher rate means faster doubling, just as intuition suggests.

5. Use $FV = P(1+r)^t$ with $P = 5000$, $r = 0.025$, $t = 12$: $FV = 5000(1.025)^{12} \approx 5000 \times 1.34489 = 6{,}724$. The 2.5% rate looks slow, but compounding over 12 years still adds about $\$1{,}724$ – the growth factor builds on itself each year.

6. Present value divides the future amount by the growth factor: $PV = \dfrac{FV}{(1+r)^t} = \dfrac{50000}{(1.03)^{15}}$. Since $(1.03)^{15} \approx 1.55797$, $PV \approx \dfrac{50000}{1.55797} \approx 32{,}093$. That's the amount to invest today to reach $\$50{,}000$ in 15 years – less than the target because interest fills the gap.

7. For an ordinary annuity, $FV = PMT \cdot \dfrac{(1+i)^n - 1}{i}$ with $PMT = 6000$, $i = 0.05$, $n = 30$. First the factor: $(1.05)^{30} \approx 4.32194$, so $\dfrac{4.32194 - 1}{0.05} = 66.4388$. Then $FV = 6000 \times 66.4388 \approx 398{,}633$. You only deposited $6000 \times 30 = \$180{,}000$, so interest contributes more than the deposits themselves.

8. Real return is approximately nominal rate minus inflation. Read the bond fund's row: nominal 5%, inflation 2.5%, so the real return is $5\% - 2.5\% = 2.5\%$. For context, savings comes out to $1.5\% - 2.5\% = -1\%$ (losing purchasing power) and stocks to $7\% - 2.5\% = 4.5\%$ real.

9. Subtract inflation from the nominal rate: $1.5 - 2.5 = -1$, so the real return is -1%. Even though the dollar balance grows at 1.5%, prices rise faster, so the money buys less each year. A negative real return quietly erodes wealth – the headline rate being positive can fool you.

10. In the annuity and loan formulas, i is the interest rate for a single compounding period, not the annual rate. If the annual rate is r and there are n periods per year, then $i = r/n$. Always match the period to the payment frequency – monthly payments need a monthly i, or every term is off.

11. This solves the annuity equation for the level payment needed to pay off principal P in n periods at periodic rate i.

12. The table stops at year 15, so extend one more 5-year step by multiplying the year-15 balance by $(1.04)^5$: $18{,}009 \times 1.21665 \approx 21{,}911$. (Or go straight from the start: $FV = 10000(1.04)^{20} \approx 10000 \times 2.19112 = 21{,}911$.) Money more than doubles in 20 years at just 4% – consistent with the Rule of 72 ($72/4 = 18$ years to double).

13. Discount the future amount with $PV = \dfrac{FV}{(1+r)^t} = \dfrac{100000}{(1.06)^{25}}$. Here $(1.06)^{25} \approx 4.29187$, so $PV \approx \dfrac{100000}{4.29187} \approx 23{,}300$. Twenty-five years at 6% shrinks $\$100$k down to about $\$23$k in today's dollars – the longer the horizon, the steeper the discount.

14. Apply $t \approx 72/r\%$ to each rate: $72/4 = 18$ years and $72/12 = 6$ years. Tripling the rate roughly thirds the doubling time – the relationship is inverse, and the rule stays reliable across this 4% to 12% range.

15. Use $FV = P(1+r)^t$ with $P = 2500$, $r = 0.08$, $t = 10$: $FV = 2500(1.08)^{10} \approx 2500 \times 2.15892 = 5{,}397$. The money more than doubles, which fits the Rule of 72: at 8% it doubles in about $72/8 = 9$ years, so 10 years lands just past double.

16. Annuity future value is $FV = PMT \cdot \dfrac{(1+i)^n - 1}{i}$ with $PMT = 2000$, $i = 0.06$, $n = 10$. Since $(1.06)^{10} \approx 1.79085$, the factor is $\dfrac{1.79085 - 1}{0.06} = 13.1808$, so $FV = 2000 \times 13.1808 \approx 26{,}362$. You deposited $\$20{,}000$, and interest adds the other $\$6{,}362$.

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17. Divide the future amount by the growth factor: $PV = \frac{10000}{(1.04)^5}$. With $(1.04)^5 \approx 1.21665$, $PV \approx \frac{10000}{1.21665} \approx 8{,}219$. Five years at 4% trims about 18% off in present-value terms – that gap is exactly the interest the deposit would earn.

18. Continuous compounding uses $FV = Pe^{rt}$, the same formula from the last section. With $P = 3000$, $r = 0.05$, $t = 10$, the exponent is $rt = 0.05 \times 10 = 0.5$, so $FV = 3000e^{0.5} \approx 3000 \times 1.64872 = 4{,}946$. Keep $e^{0.5}$ at full precision before multiplying to land the dollars exactly.

19. Look at the formula $FV = PMT \cdot \frac{(1+i)^n - 1}{i}$: with i and n fixed, the whole fraction is just a constant multiplier on PMT. So FV is directly proportional to PMT – triple the payment and the future value triples too. This only works because the rate and period count don't change.

20. No – it's a handy estimate, not a formula. The true doubling time comes from $t = \frac{\ln 2}{\ln(1+r)}$. The Rule of 72 matches this closely for rates roughly between 4% and 12%, but it drifts at very low or very high rates, so use it for quick mental math and the log formula when you need precision.

21. This is a straight future-value problem. $FV = P(1+r)^t = 5000(1.025)^{12}$. $(1.025)^{12} \approx 1.34489$, so $FV \approx 5000 \times 1.34489 \approx 6{,}724$. Twelve years of 2.5% adds about $\$1{,}724$ – the modest rate still adds up because the interest keeps earning on itself. (A common slip: using simple interest $5000 + 5000 \times 0.025 \times 12 = 6500$, which under-counts by more than $\$200$.)

22. This is an ordinary annuity. Over 30 years at 5%, $FV = PMT \cdot \frac{(1+r)^n - 1}{r}$ with $PMT = 6000$, $r = 0.05$, $n = 30$. $(1.05)^{30} \approx 4.32194$, so the factor is $\frac{4.32194 - 1}{0.05} = \frac{3.32194}{0.05} = 66.4388$. Then $FV \approx 6000 \times 66.4388 \approx 398{,}633$. Total deposited: $6000 \times 30 = \$180{,}000$. The other $\$218{,}633$ is pure interest – compounding doing the heavy lifting over three decades.

23. Real return is roughly nominal minus inflation. Savings: $1.5\% - 2.5\% = -1\%$ – the dollars grow, but they buy less each year, so purchasing power is shrinking. Stocks: $7\% - 2.5\% = 4.5\%$ real return – purchasing power grows steadily. So the savings account is a wealth eroder over long horizons even though the headline rate is positive. (The takeaway: always compare a nominal rate to inflation, not to zero. A 1.5% "positive" rate can still be losing.)

24. Rule of 72: $t \approx 72/9 = 8$ years – mental math with no calculator. Exact: $t = \frac{\ln 2}{\ln(1.09)} = \frac{0.6931}{0.08618} \approx 8.043$ years. The approximation is off by about half a percent at this rate – very close. That's why the rule is useful: for typical investment rates between 4% and 12%, it gets you within a few weeks of the real answer without a calculator.

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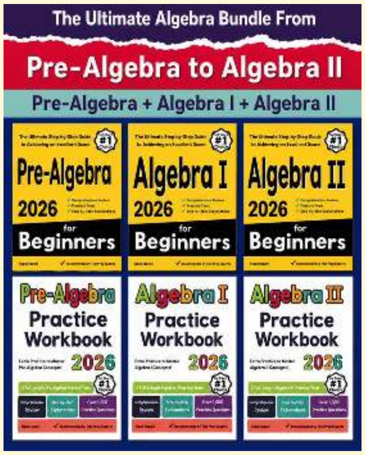

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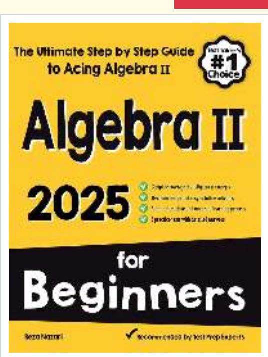

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