Logarithmic Scales and Applications

Name: ______________________	Date: ______________	Score: _____ / 28

Quick Review

Why logarithmic scales exist. Some quantities span so many *orders of magnitude* that a linear scale becomes useless: earthquake amplitudes range from microscopic to city-leveling; sound intensities span 10^{12} in the human hearing range; hydrogen-ion concentrations cover 14 orders of magnitude in everyday chemistry. Logarithmic scales *compress* that range into a friendly handful of digits.

Richter scale (earthquakes). Magnitude M relates to amplitude A via $M = \log_{10}(A/A_0)$, so each *unit* of magnitude is a $\times 10$ in amplitude. Magnitudes 5 and 7 differ by 2 units, i.e., $10^2 = 100$ times in amplitude. Energy scales with the *cube* of amplitude, so a $+2$ magnitude jump is $100^3 = 10^6$ times more energy — a million-fold.

Decibel scale (sound). $L = 10\log_{10}(I/I_0)$, with $I_0 = 10^{-12}$ W/m^2 the threshold of hearing. A $+10$ dB jump is a $\times 10$ in intensity; a $+20$ dB jump is a $\times 100$. (Perceived loudness roughly doubles every $+10$ dB, which is why a 80 dB blender sounds about twice as loud as a 70 dB radio — even though it's 10 times the actual intensity.)

pH scale (chemistry). pH $= -\log_{10}[H^+]$, where $[H^+]$ is the hydrogen-ion concentration in moles per liter. Lower pH means *more* acidic. pH 5 has $10^2 = 100$ times the hydrogen-ion concentration of pH 7 (a difference of 2 pH units).

Orders of magnitude. A jump of 1 order of magnitude is a factor of 10. A bacterium (10^{-12} kg) and a human (10^2 kg) differ by 14 orders of magnitude — a number you can write down, where "10^{14} times heavier" would be incomprehensible.

Common slips. Reading "twice the magnitude" as twice the amplitude (no — $+2$ magnitude is $\times 100$ amplitude). Confusing dB difference with dB ratio (a 20 dB *difference* means $100\times$ intensity, not $20\times$). Treating pH 0 as "no acidity" (no, pH 0 is extremely acidic — 1 M H$^+$).

PRACTICE

Use the right log-scale formula. Convert between the scale value and the underlying quantity.

1. An increase of 1 in Richter magnitude corresponds to what change in amplitude? ________
2. A solution at pH 5 has how many times the hydrogen-ion concentration of pH 7? ________
3. Using $L = 10\log_{10}(I/I_0)$, how many times more intense is 80 dB vs 60 dB? ________
4. Bacterium mass $\approx 10^{-12}$ kg, human mass $\approx 10^2$ kg. How many orders of magnitude apart? ________
5. The table records two earthquakes on the Richter scale. By what factor does B exceed A in amplitude? ________

Earthquake	Magnitude M
A	5.0
B	7.0

6. $[H^+] = 1 \times 10^{-4}$ M. Find pH. ________
7. Earthquake X: magnitude 4.5. Earthquake Y: magnitude 6.5. Amplitude ratio? ________
8. Which statements about logarithmic scales are TRUE? Compress wide ranges; Richter/decibel/pH all base 10; equal differences match equal ratios; pH 0 means no acidity. ________
9. Two sound sources are measured below. Using $L = 10\log_{10}(I/I_0)$, find the intensity ratio of the louder source to the quieter one. ________

Source	Level (dB)
Radio	70
Mixer	90

10. pH 3.2. Find $[H^+]$ to two significant digits in scientific notation. ________
11. How many orders of magnitude separate 10^{-9} from 10^6? ________

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12. Two solutions are listed with their pH values. How many times greater is the hydrogen-ion concentration of the more acidic solution? ________

Solution	pH
Coffee	5.0
Pure water	7.0

13. What is $[H^+]$ when pH $= 7$? (Pure water.) ________

14. Sound at 40 dB is how many times the threshold intensity I_0? ________

15. Earthquake magnitudes 6 and 8. Energy ratio? (Energy $\propto$ amplitude^3.) ________

16. A solution has $[H^+] = 2.5 \times 10^{-5}$ M. Find pH to two decimals. ________

17. A jet engine at 130 dB is how many times more intense than a conversation at 60 dB? ________

18. On a log scale, equal differences correspond to equal _____ of the underlying quantity. ________

19. If a battery has charge 10^4 times the leakage current of another, how many orders of magnitude apart are they? ________

20. A magnitude 6.0 earthquake is how many times the amplitude of magnitude 4.0? How many times the energy? ________

♦ Word Problems

21. Earthquake A registers magnitude 5.0 on the Richter scale; Earthquake B registers 7.5. By what factor does B exceed A in amplitude? In energy? (Energy scales as the cube of amplitude.) ____

22. A noisy fan reads 75 dB. A vacuum nearby reads 85 dB. By what factor is the vacuum more intense than the fan? Then determine the combined dB level if both sources operate together (intensities add). ____

23. Lemon juice has pH 2.0. Tap water has pH 7.0. How many times more acidic is lemon juice, by hydrogen-ion concentration? ____

24. A laboratory measures $[H^+] = 3.5 \times 10^{-9}$ M in a sample. Compute the pH (to two decimals). Is the sample acidic, neutral, or basic? ____

Additional Practice

25. Evaluate $\log_2 32$. ________

26. Evaluate $\log_5 125$. ________

27. Rewrite $\log_3 81 = 4$ exponentially. ________

28. Solve $\log_4 x = 3$. ________

Answer Keys

1. $\times 10$	**13.** 10^{-7} M
2. 100	**14.** 10^4
3. 100	**15.** 10^6
4. 14	**16.** ≈ 4.60
5. 100	**17.** 10^7
6. 4	**18.** ratios
7. 100	**19.** 4
8. first three	**20.** 100 amplitude; 10^6 energy
9. 100	**21.** amplitude $10^{2.5} \approx 316$; energy $10^{7.5} \approx 3.16 \times 10^7$
10. 6.3×10^{-4} M	**22.** 10 times; ≈ 85.4 dB
11. 15	**23.** $100{,}000$ (a factor of 10^5)
12. 100	**24.** pH ≈ 8.46; basic

Additional Practice Answers

25. 5	**27.** $3^4 = 81$
26. 3	**28.** $x = 64$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Richter is base-10 logarithmic: each unit step is a factor of 10 in amplitude. So $+1$ in magnitude is $10\times$ the wave.

2. Keep the rule visible: ΔpH $= 2$, so the ratio is $10^2 = 100$. (Lower pH = more acidic = higher $[H^+]$.) That gives a quick check on the answer.

3. One steady path is: $\Delta L = 20$ dB means $\log(I_1/I_2) = 2$, so the ratio is $10^2 = 100$. Note: 20 dB *difference*, not $20\times$. That gives a quick check on the answer.

4. Orders of magnitude are the difference of the exponents on base 10: $2 - (-12) = 14$. So a human is roughly 10^{14} times the mass of a bacterium — fourteen powers of ten.

5. A careful way to see it: $\Delta M = 7.0 - 5.0 = 2$, so amplitude ratio $= 10^2 = 100$. This is the part to check before moving on, because it keeps the answer tied to the original question.

6. Apply the pH formula pH $= -\log_{10}[H^+]$. With $[H^+] = 10^{-4}$, the log reads off the exponent: $\log_{10}(10^{-4}) = -4$. Negate it: pH $= -(-4) = 4$.

7. One steady path is: $\Delta M = 2.0$, so the ratio is $10^2 = 100$. (Energy ratio would be $100^3 = 10^6$ — a million.) That gives a quick check on the answer.

8. The last is false: pH 0 is extremely acidic ($[H^+] = 1$ M). The other three define how log scales work.

9. A careful way to see it: $\Delta L = 90 - 70 = 20$ dB, so the intensity ratio is $10^{20/10} = 10^2 = 100$. That gives a quick check on the answer.

10. Keep the rule visible: $\log_{10}[H^+] = -3.2$, so $[H^+] = 10^{-3.2} \approx 6.31 \times 10^{-4}$ M. This is the part to check before moving on, because it keeps the answer tied to the original question.

11. One steady path is: $6 - (-9) = 15$. Each order of magnitude is a factor of 10. That gives a quick check on the answer.

12. Start with the key idea: ΔpH $= 7.0 - 5.0 = 2$, and since pH $= -\log_{10}[H^+]$, each pH unit is a factor of 10 in $[H^+]$. So the ratio is $10^2 = 100$ (the lower-pH coffee is the more acidic one). That gives a quick check on the answer.

13. Start from pH $= -\log_{10}[H^+] = 7$, so $\log_{10}[H^+] = -7$. Undo the base-10 log by raising 10 to each side: $[H^+] = 10^{-7}$ M. This is the neutral value — pure water.

14. Set the decibel formula equal to 40: $40 = 10\log_{10}(I/I_0)$. Divide by 10: $\log_{10}(I/I_0) = 4$. Undo the base-10 log: $I/I_0 = 10^4$, so the sound is 10^4 times the threshold intensity.

15. One steady path is: Amplitude ratio $10^2 = 100$. Energy ratio $100^3 = 10^6$ — a millionfold. That gives a quick check on the answer.

16. Start with the key idea: pH $= -\log_{10}(2.5 \times 10^{-5}) = -(\log 2.5 - 5) = 5 - \log 2.5 \approx 5 - 0.398 \approx 4.60$. This is the part to check before moving on, because it keeps the answer tied to the original question.

17. A careful way to see it: $\Delta L = 70$ dB, so the intensity ratio is $10^{70/10} = 10^7$ — ten million times. That gives a quick check on the answer.

18. That's the defining feature of a logarithmic scale. Equal additive steps on the scale = equal *multiplicative* steps in the quantity.

19. The ratio of the two quantities is 10^4, and the order-of-magnitude gap is the base-10 log of that ratio: $\log_{10}(10^4) = 4$. One order of magnitude for each power of 10.

20. Start with the key idea: $\Delta M = 2$ gives $\times 100$ amplitude. Energy $\propto$ amplitude^3, so $\times 100^3 = 10^6$. That gives a quick check on the answer.

21. The Richter scale gives $\log_{10}(\text{amplitude ratio}) = \Delta M = 2.5$. So the amplitude ratio is $10^{2.5} = 10^2 \cdot 10^{0.5} \approx 100 \cdot 3.162 \approx 316$. Energy scales with the cube of amplitude, so the energy ratio is $(10^{2.5})^3 = 10^{7.5} \approx 3.16 \times 10^7$ — B releases about 31.6 million times the energy of A. (A common shortcut: each $+0.5$ magnitude step is roughly a $\times 3.16$ in amplitude and roughly $\times 31.6$ in energy.)

22. For the ratio: $\Delta L = 10$ dB, so the intensity ratio is $10^{10/10} = 10^1 = 10$. The vacuum is $10\times$ the fan's intensity. For the combined level, treat the intensities directly. Let $I_{\text{fan}} = I_0 \cdot 10^{7.5}$ and $I_{\text{vac}} = I_0 \cdot 10^{8.5}$. Their sum: $I_{\text{tot}} = I_0(10^{7.5} + 10^{8.5}) = I_0 \cdot 10^{7.5}(1 + 10) = 11 \cdot 10^{7.5} I_0$. The level: $L = 10\log_{10}(11 \cdot 10^{7.5}) = 10(\log 11 + 7.5) \approx 10(1.041 + 7.5) \approx 85.4$ dB. (Adding a quieter source to a louder one barely budges the total — the louder source dominates.)

23. One steady path is: ΔpH $= 7.0 - 2.0 = 5.0$. Since pH $= -\log_{10}[H^+]$, a *lower* pH means a higher $[H^+]$. Each unit of pH difference is a factor of 10 in $[H^+]$, so 5 units is $10^5 = 100{,}000$. Lemon juice has $100{,}000$ times the hydrogen-ion concentration of tap water. (Don't be fooled by the size of the pH numbers — a small pH difference hides a huge chemical difference.) That gives a quick check on the answer.

24. Use pH $= -\log_{10}[H^+]$. $\log_{10}(3.5 \times 10^{-9}) = \log_{10}(3.5) + (-9) \approx 0.544 - 9 = -8.456$. Negate: pH ≈ 8.46. Since $8.46 > 7$, the sample is *basic* (alkaline). Neutral water is exactly 7; anything above is basic, anything below is acidic. (Common bases like baking soda solutions sit around pH 8–9, so this reading is plausible for a mild basic solution.)

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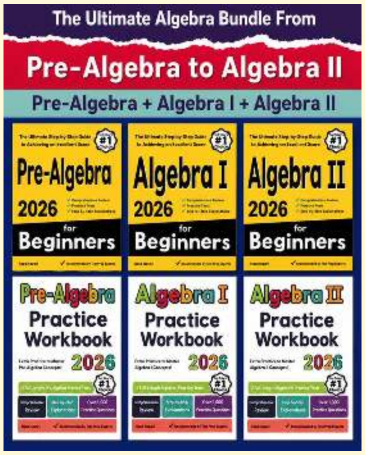

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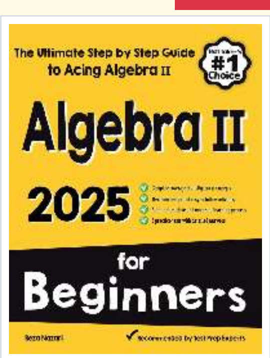

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