

# Linear vs Exponential Growth

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 28

## Q Quick Review

**Linear growth** adds a fixed amount per unit time — *constant absolute change*. The graph is a straight line:  $y = mx + b$  with  $m$  the per-unit increment.

**Exponential growth** multiplies by a fixed factor per unit time — *constant percent change*. The graph is a curve that gets steeper:  $y = a \cdot b^x$  with  $b > 1$ .

**Doubling and decay.** A 100% growth rate means  $b = 1 + 1 = 2$ , so the quantity *doubles* each step. A 10% decay rate gives  $b = 1 - 0.10 = 0.90$ , so the quantity keeps 90% of itself each step.

**The race.** For small  $x$ , a linear function with a steep slope often outpaces an exponential with a small initial value. But the exponential always wins eventually — for  $b > 1$ , the curve overtakes *any* straight line, given enough time. Quick check:  $L(x) = 100x + 1000$  vs  $E(x) = 10 \cdot 2^x$ . At  $x = 7$ ,  $L = 1700$  and  $E = 1280$  — linear still ahead. At  $x = 8$ ,  $L = 1800$  and  $E = 2560$  — exponential pulls ahead.

**Reading a table.** Constant first differences = linear. Constant ratio of consecutive outputs = exponential.

**Common slip.** “\$50 per year” and “5% per year” sound similar but live in different worlds: the first is linear, the second is exponential. The 5% account earns more dollars each year as the balance grows; the \$50 account earns the same 50 every year. Long term, 5% wins big.

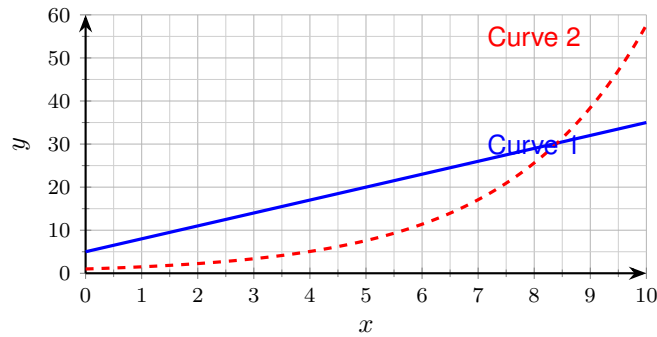
## PRACTICE

*Distinguish linear from exponential. Identify, compare, predict.*

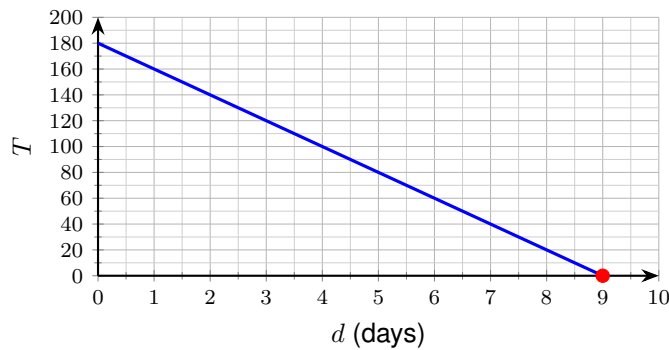
1. Linear growth: changes by a fixed \_\_\_\_\_ per unit time. \_\_\_\_\_
2. Exponential growth: changes by a fixed \_\_\_\_\_ per unit time. \_\_\_\_\_
3. Account A: +\$50 per year. Account B: +5% per year. Long-run winner? \_\_\_\_\_
4. Doubling time for 100% annual growth? \_\_\_\_\_
5. Doubling time for 25% annual growth (using rule of 72)? \_\_\_\_\_
6. From the table  $y = 3, 5, 7, 9, 11$ : linear or exponential? \_\_\_\_\_
7. From the table  $y = 2, 6, 18, 54$ : linear or exponential? \_\_\_\_\_
8.  $L(x) = 100x + 1000$ ,  $E(x) = 10 \cdot 2^x$ . Find  $L(7)$  and  $E(7)$ . \_\_\_\_\_
9.  $L(x) = 100x + 1000$ ,  $E(x) = 10 \cdot 2^x$ . Smallest integer  $x$  with  $E(x) > L(x)$ . \_\_\_\_\_
10. Decay factor for 12% annual decay. \_\_\_\_\_
11. Growth factor for 15% annual growth. \_\_\_\_\_



12. The graph below shows linear (solid) and exponential (dashed). Which is which? \_\_\_\_\_



- 13. A population starts at 600 and adds 120 per month. Model? \_\_\_\_\_
- 14. A population starts at 600 and grows by 20% per month. Model? \_\_\_\_\_
- 15. Plan E: starts at 200 points, grows 15% per week. Plan E value after 4 weeks (nearest point)? \_\_\_\_\_
- 16. Does an exponential function with  $b > 1$  always have constant percent change? \_\_\_\_\_
- 17. Population: 5000, 5300, 5600, 5900. Linear or exponential? \_\_\_\_\_
- 18. Population: 5000, 5500, 6050, 6655. Linear or exponential? \_\_\_\_\_
- 19. If linear adds 100/yr and exponential grows 10%/yr, both starting at 100, which is bigger at year 30? \_\_\_\_\_
- 20. The graph below tracks tickets remaining per day (linear decrease). What does the  $x$ -intercept represent? \_\_\_\_\_

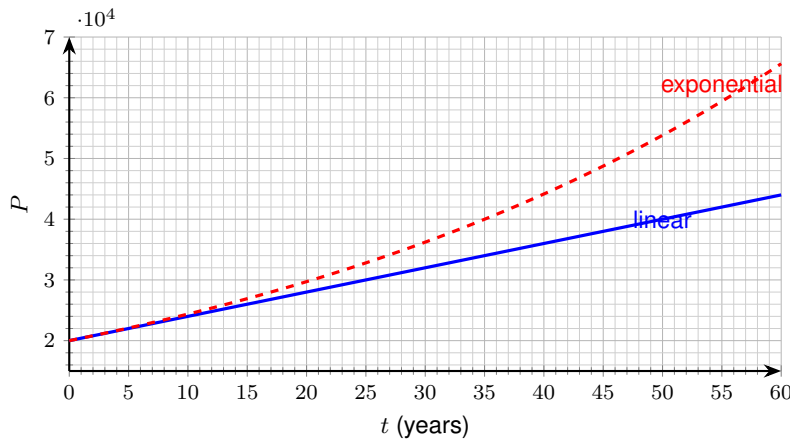


◆ Word Problems

21. Account L starts at \$5,000 and adds \$5,000 each year. Account E also starts at \$5,000 but grows by 10% each year. After how many whole years will Account E first exceed Account L? \_\_\_\_\_

$t$	0	10	20	30	40
$L(t)$	5,000	55,000	105,000	155,000	205,000
$E(t)$	5,000	12,968	33,637	87,247	226,296

22. Compare a city growing linearly at 400 people per year (starting at 20,000) with one growing exponentially at 2% per year (also starting at 20,000). Compute each city's population at  $t = 20$  years and at  $t = 60$  years. \_\_\_\_\_



23. A scientist plants 50 seedlings and finds that 10 new ones sprout each month from cuttings (linear increase). Across town, a competing nursery starts with 50 seedlings and doubles its inventory every 3 months (exponential). Which has more after 6 months? After 12 months? \_\_\_\_\_

24. A new app gains 200 users in the first week and then either (a) gains 200 new users every week thereafter, or (b) grows its weekly user count by 10% each week. Write a function for each scenario (letting  $t$  be weeks after launch with the first-week count as the initial value), and find each user count at  $t = 30$  weeks. \_\_\_\_\_

Additional Practice

25. Evaluate  $3 \cdot 2^4$ . \_\_\_\_\_

26. Find  $a$  in  $y = a \cdot 3^x$  if  $y(0) = 7$ . \_\_\_\_\_

27. Growth or decay:  $y = 12(0.8)^x$ . \_\_\_\_\_

28. Growth or decay:  $y = 5(1.12)^t$ . \_\_\_\_\_



## Answer Keys

1. amount (dollars/units)  
 2. percent  
 3. Account B  
 4. 1 year  
 5.  $\approx 2.88$  years  
 6. linear  
 7. exponential  
 8. 1700, 1280  
 9.  $x = 8$   
 10. 0.88  
 11. 1.15  
 12. Curve 1: linear; Curve 2: exponential
- Additional Practice Answers**
25. 48  
 26.  $a = 7$
13.  $P(m) = 600 + 120m$  (linear)  
 14.  $P(m) = 600(1.20)^m$  (exponential)  
 15. 350  
 16. yes  
 17. linear (slope 300)  
 18. exponential (factor 1.1)  
 19. linear  
 20. the day all tickets are sold  
 21.  $t = 39$   
 22.  $L(20) = 28,000$ ,  $E(20) \approx 29,719$   
 $L(60) = 44,000$ ,  $E(60) \approx 65,621$   
 23.  $L(6) = 110$ ,  $E(6) = 200$ ;  $L(12) = 170$ ,  $E(12) = 800$   
 24. (a)  $U(t) = 200 + 200t$ ,  $U(30) = 6,200$   
 (b)  $U(t) = 200(1.1)^t$ ,  $U(30) \approx 3,490$
27. decay  
 28. growth

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

1. Linear growth adds the same absolute amount each step — a fixed number of dollars or units — which is why its graph is a straight line with constant slope.  
 2. Exponential growth multiplies by the same factor each step, which is the same as changing by a fixed *percent*. That percent is constant even though the dollar change keeps rising.  
 3. Account A adds a fixed \$50 each year (linear), while Account B grows by a fixed percent (exponential). Because B's 5% is taken on an ever-larger balance, its yearly gain eventually outpaces a flat \$50, so B wins long-run.  
 4. A 100% growth rate gives a factor of  $1 + 1 = 2$ , so the amount is multiplied by 2 in a single year. Multiplying by 2 is doubling, so the doubling time is exactly 1 year.  
 5. Solve  $(1.25)^t = 2$ :  $t = \log 2 / \log 1.25 \approx 3.11$ . Rule of 72:  $72/25 \approx 2.88$ . Both round to about 3 years.  
 6. Check the first differences:  $5 - 3 = 2$ ,  $7 - 5 = 2$ ,  $9 - 7 = 2$ ,  $11 - 9 = 2$ . They are constant, so the data is linear (you are adding 2 each step, not multiplying).  
 7. The differences (4, 12, 36) are not constant, so test ratios:  $6/2 = 3$ ,  $18/6 = 3$ ,  $54/18 = 3$ . The constant ratio of 3 means exponential growth.  
 8. Substitute  $x = 7$  into each:  $L(7) = 100(7) + 1000 = 700 + 1000 = 1700$ , and  $E(7) = 10 \cdot 2^7 = 10 \cdot 128 = 1280$ . The linear function is still ahead at this point.  
 9. At  $x = 7$ ,  $E = 1280 < L = 1700$ , so test the next integer. At  $x = 8$ :  $L(8) = 100(8) + 1000 = 1800$  and  $E(8) = 10 \cdot 2^8 = 2560$ . Now  $E > L$ , so  $x = 8$  is the first integer where exponential overtakes linear.  
 10. For decay, the factor is  $1 - r$ . Here  $r = 0.12$ , so the factor is  $1 - 0.12 = 0.88$ , meaning 88% of the amount survives each year. (Don't use 0.12 itself — that's the rate, not the factor.)  
 11. For growth, the factor is  $1 + r$ . With  $r = 0.15$ , that gives  $1 + 0.15 = 1.15$  — you keep 100% and add another 15% each year.  
 12. The straight line is linear ( $y = 3x + 5$ ). The curve that bends upward is exponential ( $y = 1.5^x$ ). The exponential starts below the line but eventually overtakes it (around  $x = 10$ ).  
 13. Adding a fixed 120 each month is constant absolute change, so the model is linear: the starting 600 is the intercept and 120 is the slope, giving  $P(m) = 600 + 120m$ .  
 14. A fixed percent per month is constant multiplicative change, so the model is exponential. The factor is  $1 + 0.20 = 1.20$  and the initial value is 600, giving  $P(m) = 600(1.20)^m$ .  
 15. Growth factor  $1 + 0.15 = 1.15$ , so  $P(4) = 200(1.15)^4$ . Compute the power first:  $1.15^4 \approx 1.749$ , then  $200(1.749) \approx 349.8$ , which rounds to 350 points.  
 16. Yes — multiplying by the same  $b$  each step is the very definition of constant percent change. The percent is  $(b - 1) \cdot 100\%$  every step, no matter how large

the amount has grown.

17. The first differences are  $5300 - 5000 = 300$ ,  $5600 - 5300 = 300$ ,  $5900 - 5600 = 300$  — all equal. A constant difference means linear growth with slope 300.  
 18. The differences (500, 550, 605) are not constant, so check ratios:  $5500/5000 = 1.1$ ,  $6050/5500 = 1.1$ ,  $6655/6050 = 1.1$ . A constant ratio of 1.1 means exponential growth at 10% per step.  
 19. One steady path is:  $L(30) = 100 + 3000 = 3100$ .  $E(30) = 100(1.1)^{30} \approx 1745$ . So linear actually wins at year 30 — the crossover takes longer than thirty years. (At  $t \approx 47$ , the exponential overtakes.) The race depends on the numbers; exponential always wins eventually but sometimes very late. That gives a quick check on the answer.  
 20. Start with the key idea:  $x$ -intercept is where  $T = 0$ . Here, that's the day when every ticket has been sold. From the graph: day 9. That gives a quick check on the answer.  
 21. Set up both:  $L(t) = 5000 + 5000t$  adds a fixed \$5,000 each year, while  $E(t) = 5000(1.1)^t$  multiplies by 1.1 each year. To find the crossover, compare year by year near the end. At  $t = 38$ ,  $L = 5000 + 5000(38) = 195,000$  but  $E = 5000(1.1)^{38} \approx 187,022$ , so  $L$  still leads. At  $t = 39$ ,  $L = 200,000$  while  $E = 5000(1.1)^{39} \approx 205,724$ , so  $E$  pulls ahead. The first whole year with  $E > L$  is  $t = 39$ . Exponential growth waits nearly four decades, then overtakes for good.  
 22. The linear city adds a fixed 400 each year:  $L(t) = 20000 + 400t$ , so  $L(20) = 28,000$  and  $L(60) = 44,000$ . The exponential city multiplies by 1.02 each year:  $E(t) = 20000(1.02)^t$ .  $E(20) = 20000(1.02)^{20} \approx 20000(1.486) = 29,719$ , and  $E(60) = 20000(1.02)^{60} \approx 20000(3.281) = 65,621$ . At 20 years exponential is barely ahead; by 60 years it has nearly lapped linear, because each year's growth is taken from an ever-larger base.  
 23. Linear:  $L(m) = 50 + 10m$ .  $L(6) = 110$ ,  $L(12) = 170$ . Exponential (doubling every 3 months):  $E(m) = 50 \cdot 2^{m/3}$ .  $E(6) = 50 \cdot 2^2 = 200$ .  $E(12) = 50 \cdot 2^4 = 800$ . At 6 months, exponential is already ahead (200 vs 110). By 12 months, it's 800 vs 170 — the gap keeps widening because exponential growth accelerates, while linear growth holds its pace.  
 24. Linear: each week adds the same 200 users.  $U(t) = 200 + 200t$ , so  $U(30) = 200 + 6000 = 6,200$ . Exponential: each week multiplies by 1.10.  $U(t) = 200(1.1)^t$ , so  $U(30) = 200(1.1)^{30} \approx 200(17.45) = 3,490$ . Linear wins at  $t = 30$  here — the 10% growth needs more time to overtake 200/week. (The exponential crosses around  $t \approx 47$  weeks. After that, it races away. Real apps usually grow somewhere between the two, until growth saturates.)



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