Linear vs Exponential Growth

Name: ____________	Date: ____________	Score: ______ / 28

Quick Review

Linear growth adds a fixed amount per unit time — *constant absolute change*. The graph is a straight line: $y = mx + b$ with m the per-unit increment.

Exponential growth multiplies by a fixed factor per unit time — *constant percent change*. The graph is a curve that gets steeper: $y = a \cdot b^x$ with $b > 1$.

Doubling and decay. A 100% growth rate means $b = 1 + 1 = 2$, so the quantity *doubles* each step. A 10% decay rate gives $b = 1 - 0.10 = 0.90$, so the quantity keeps 90% of itself each step.

The race. For small x, a linear function with a steep slope often outpaces an exponential with a small initial value. But the exponential always wins eventually — for $b > 1$, the curve overtakes *any* straight line, given enough time. Quick check: $L(x) = 100x + 1000$ vs $E(x) = 10 \cdot 2^x$. At $x = 7$, $L = 1700$ and $E = 1280$ — linear still ahead. At $x = 8$, $L = 1800$ and $E = 2560$ — exponential pulls ahead.

Reading a table. Constant first differences = linear. Constant ratio of consecutive outputs = exponential.

Common slip. "\$50 per year" and "5% per year" sound similar but live in different worlds: the first is linear, the second is exponential. The 5% account earns more dollars each year as the balance grows; the \$50 account earns the same 50 every year. Long term, 5% wins big.

PRACTICE

Distinguish linear from exponential. Identify, compare, predict.

1. Linear growth: changes by a fixed _____ per unit time. ________
2. Exponential growth: changes by a fixed _____ per unit time. ________
3. Account A: $+\$50$ per year. Account B: $+5\%$ per year. Long-run winner? ________
4. Doubling time for 100% annual growth? ________
5. Doubling time for 25% annual growth (using rule of 72)? ________
6. From the table $y = 3, 5, 7, 9, 11$: linear or exponential? ________
7. From the table $y = 2, 6, 18, 54$: linear or exponential? ________
8. $L(x) = 100x + 1000$, $E(x) = 10 \cdot 2^x$. Find $L(7)$ and $E(7)$. ________
9. $L(x) = 100x + 1000$, $E(x) = 10 \cdot 2^x$. Smallest integer x with $E(x) > L(x)$. ________
10. Decay factor for 12% annual decay. ________
11. Growth factor for 15% annual growth. ________

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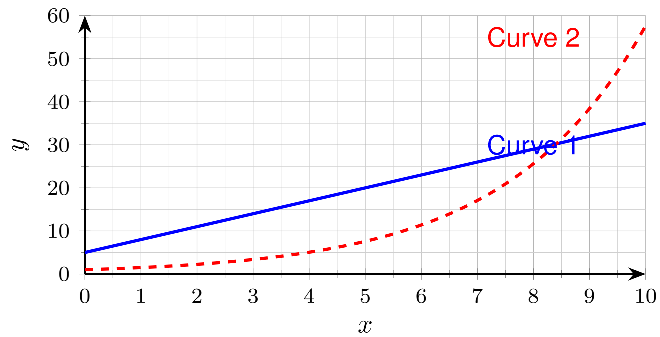
12. The graph below shows linear (solid) and exponential (dashed). Which is which? ________



13. A population starts at 600 and adds 120 per month. Model? ________

14. A population starts at 600 and grows by 20% per month. Model? ________

15. Plan E: starts at 200 points, grows 15% per week. Plan E value after 4 weeks (nearest point)? ________

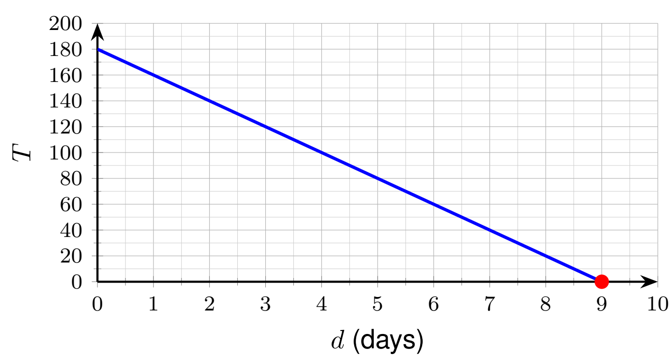
16. Does an exponential function with $b > 1$ always have constant percent change? ________

17. Population: $5000,\ 5300,\ 5600,\ 5900$. Linear or exponential? ________

18. Population: $5000,\ 5500,\ 6050,\ 6655$. Linear or exponential? ________

19. If linear adds 100/yr and exponential grows 10%/yr, both starting at 100, which is bigger at year 30? ________

20. The graph below tracks tickets remaining per day (linear decrease). What does the x-intercept represent? ________



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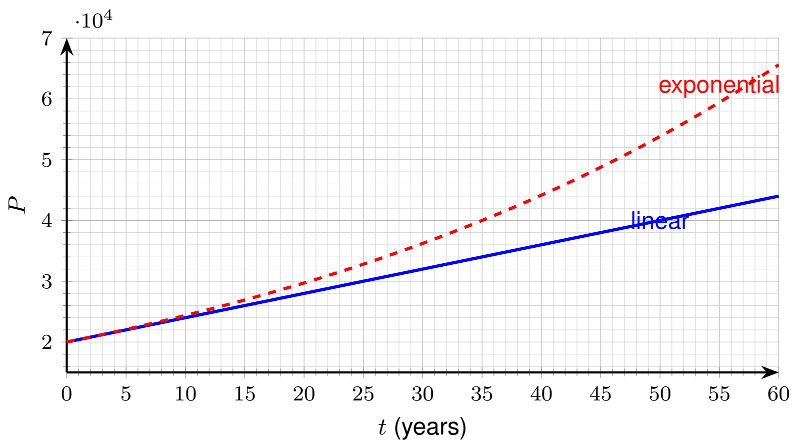
Word Problems

21. Account L starts at $\$5{,}000$ and adds $\$5{,}000$ each year. Account E also starts at $\$5{,}000$ but grows by 10% each year. After how many whole years will Account E first exceed Account L? ________

t	0	10	20	30	40
$L(t)$	5,000	55,000	105,000	155,000	205,000
$E(t)$	5,000	12,968	33,637	87,247	226,296

22. Compare a city growing linearly at 400 people per year (starting at $20{,}000$) with one growing exponentially at 2% per year (also starting at $20{,}000$). Compute each city's population at $t = 20$ years and at $t = 60$ years. ________



23. A scientist plants 50 seedlings and finds that 10 new ones sprout each month from cuttings (linear increase). Across town, a competing nursery starts with 50 seedlings and doubles its inventory every 3 months (exponential). Which has more after 6 months? After 12 months? ____

24. A new app gains 200 users in the first week and then either (a) gains 200 new users every week thereafter, or (b) grows its weekly user count by 10% each week. Write a function for each scenario (letting t be weeks after launch with the first-week count as the initial value), and find each user count at $t = 30$ weeks. ____

Additional Practice

25. Evaluate $3 \cdot 2^4$. ________

26. Find a in $y = a \cdot 3^x$ if $y(0) = 7$. ________

27. Growth or decay: $y = 12(0.8)^x$. ________

28. Growth or decay: $y = 5(1.12)^t$. ________

Answer Keys

1. amount (dollars/units)
2. percent
3. Account B
4. 1 year
5. ≈ 2.88 years
6. linear
7. exponential
8. $1700,\ 1280$
9. $x = 8$
10. 0.88
11. 1.15
12. Curve 1: linear; Curve 2: exponential
13. $P(m) = 600 + 120m$ (linear)
14. $P(m) = 600(1.20)^m$ (exponential)
15. 350
16. yes
17. linear (slope 300)
18. exponential (factor 1.1)
19. linear
20. the day all tickets are sold
21. $t = 39$
22. $L(20){=}28{,}000,\ E(20){\approx}29{,}719$; $L(60){=}44{,}000,\ E(60){\approx}65{,}621$
23. $L(6) = 110,\ E(6) = 200;\ L(12) = 170,\ E(12) = 800$
24. $(a)\ U(t){=}200{+}200t,\ U(30){=}6{,}200$; $(b)\ U(t){=}200(1.1)^t,\ U(30){\approx}3{,}490$

Additional Practice Answers

25. 48
26. $a = 7$
27. decay
28. growth

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Linear growth adds the same absolute amount each step — a fixed number of dollars or units — which is why its graph is a straight line with constant slope.

2. Exponential growth multiplies by the same factor each step, which is the same as changing by a fixed *percent*. That percent is constant even though the dollar change keeps rising.

3. Account A adds a fixed $\$50$ each year (linear), while Account B grows by a fixed percent (exponential). Because B's 5% is taken on an ever-larger balance, its yearly gain eventually outpaces a flat $\$50$, so B wins long-run.

4. A 100% growth rate gives a factor of $1 + 1 = 2$, so the amount is multiplied by 2 in a single year. Multiplying by 2 is doubling, so the doubling time is exactly 1 year.

5. Solve $(1.25)^t = 2$: $t = \log 2/\log 1.25 \approx 3.11$. Rule of 72: $72/25 \approx 2.88$. Both round to about 3 years.

6. Check the first differences: $5-3 = 2, 7-5 = 2, 9-7 = 2, 11-9 = 2$. They are constant, so the data is linear (you are adding 2 each step, not multiplying).

7. The differences $(4, 12, 36)$ are not constant, so test ratios: $6/2 = 3$, $18/6 = 3$, $54/18 = 3$. The constant ratio of 3 means exponential growth.

8. Substitute $x = 7$ into each: $L(7) = 100(7) + 1000 = 700 + 1000 = 1700$, and $E(7) = 10 \cdot 2^7 = 10 \cdot 128 = 1280$. The linear function is still ahead at this point.

9. At $x = 7$, $E = 1280 < L = 1700$, so test the next integer. At $x = 8$: $L(8) = 100(8) + 1000 = 1800$ and $E(8) = 10 \cdot 2^8 = 2560$. Now $E > L$, so $x = 8$ is the first integer where exponential overtakes linear.

10. For decay, the factor is $1 - r$. Here $r = 0.12$, so the factor is $1 - 0.12 = 0.88$, meaning 88% of the amount survives each year. (Don't use 0.12 itself — that's the rate, not the factor.)

11. For growth, the factor is $1 + r$. With $r = 0.15$, that gives $1 + 0.15 = 1.15$ — you keep 100% and add another 15% each year.

12. The straight line is linear ($y = 3x + 5$). The curve that bends upward is exponential ($y = 1.5^x$). The exponential starts below the line but eventually overtakes it (around $x = 10$).

13. Adding a fixed 120 each month is constant absolute change, so the model is linear: the starting 600 is the intercept and 120 is the slope, giving $P(m) = 600 + 120m$.

14. A fixed percent per month is constant multiplicative change, so the model is exponential. The factor is $1 + 0.20 = 1.20$ and the initial value is 600, giving $P(m) = 600(1.20)^m$.

15. Growth factor $1 + 0.15 = 1.15$, so $P(4) = 200(1.15)^4$. Compute the power first: $1.15^4 \approx 1.749$, then $200(1.749) \approx 349.8$, which rounds to 350 points.

16. Yes — multiplying by the same b each step is the very definition of constant percent change. The percent is $(b-1) \cdot 100\%$ every step, no matter how large the amount has grown.

17. The first differences are $5300 - 5000 = 300$, $5600 - 5300 = 300$, $5900 - 5600 = 300$ — all equal. A constant difference means linear growth with slope 300.

18. The differences $(500, 550, 605)$ are not constant, so check ratios: $5500/5000 = 1.1$, $6050/5500 = 1.1$, $6655/6050 = 1.1$. A constant ratio of 1.1 means exponential growth at 10% per step.

19. One steady path is: $L(30) = 100 + 3000 = 3100$. $E(30) = 100(1.1)^{30} \approx 1745$. So linear actually wins at year 30 — the crossover takes longer than thirty years. (At $t \approx 47$, the exponential overtakes.) The race depends on the numbers; exponential always wins eventually but sometimes very late. That gives a quick check on the answer.

20. Start with the key idea: x-intercept is where $T = 0$. Here, that's the day when every ticket has been sold. From the graph: day 9. That gives a quick check on the answer.

21. Set up both: $L(t) = 5000 + 5000t$ adds a fixed \$5,000 each year, while $E(t) = 5000(1.1)^t$ multiplies by 1.1 each year. To find the crossover, compare year by year near the end. At $t = 38$, $L = 5000 + 5000(38) = 195{,}000$ but $E = 5000(1.1)^{38} \approx 187{,}022$, so L still leads. At $t = 39$, $L = 200{,}000$ while $E = 5000(1.1)^{39} \approx 205{,}724$, so E pulls ahead. The first whole year with $E > L$ is $t = 39$. Exponential growth waits nearly four decades, then overtakes for good.

22. The linear city adds a fixed 400 each year: $L(t) = 20000 + 400t$, so $L(20) = 28{,}000$ and $L(60) = 44{,}000$. The exponential city multiplies by 1.02 each year: $E(t) = 20000(1.02)^t$. $E(20) = 20000(1.02)^{20} \approx 20000(1.486) = 29{,}719$, and $E(60) = 20000(1.02)^{60} \approx 20000(3.281) = 65{,}621$. At 20 years exponential is barely ahead; by 60 years it has nearly lapped linear, because each year's growth is taken from an ever-larger base.

23. Linear: $L(m) = 50 + 10m$. $L(6) = 110$, $L(12) = 170$. Exponential (doubling every 3 months): $E(m) = 50 \cdot 2^{m/3}$. $E(6) = 50 \cdot 2^2 = 200$. $E(12) = 50 \cdot 2^4 = 800$. At 6 months, exponential is already ahead (200 vs 110). By 12 months, it's 800 vs 170 — the gap keeps widening because exponential growth accelerates, while linear growth holds its pace.

24. Linear: each week adds the same 200 users. $U(t) = 200 + 200t$, so $U(30) = 200 + 6000 = 6{,}200$. Exponential: each week multiplies by 1.10. $U(t) = 200(1.1)^t$, so $U(30) = 200(1.1)^{30} \approx 200(17.45) = 3{,}490$. Linear wins at $t = 30$ here — the 10% growth needs more time to overtake 200/week. (The exponential crosses around $t \approx 47$ weeks. After that, it races away. Real apps usually grow somewhere between the two, until growth saturates.)

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